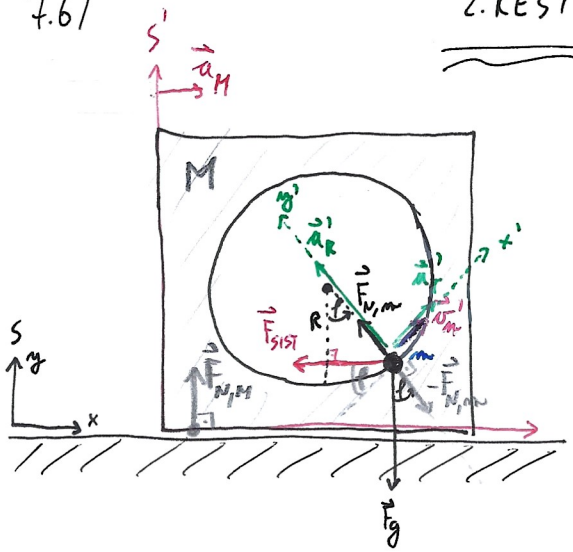
7.6)

2. REŠITEV S SILAMI S PRIBLIŽKI LE NA KONCU



- HITROST KLADE (MASE m) V POSPEŠENEM SISTEMU S' KLANCA:

$$v_m' = R \cdot \frac{d\varphi}{dt} = R \cdot \dot{\varphi}$$

- TANGENTNI POSPEŠEK KLADE (MASE m) V SISTEMU S':

$$a_T' = \frac{dv_m'}{dt} = R \cdot \ddot{\varphi}$$

- RADIALNI POSPEŠEK KLADE (MASE m) V SISTEMU S' (KLADA POSPEŠENO KROŽI V TEM SISTEMU):

$$a_R' = \frac{v_m'^2}{R} = \frac{R^2 \dot{\varphi}^2}{R} = R \cdot \dot{\varphi}^2$$

- 2. NEWTONOV ZAKON ZA KLADO (MASE m) V POSPEŠENEM SISTEMU S':

→ TANGENTNA SMER (x'):

$$m a_T' = -F_{g,x'} - F_{SIST,x'} \quad ; \quad \left[F_{g,x'} = F_g \cdot \sin\varphi \;\; F_g = m \cdot g \;\; F_{SIST,x'} = F_{SIST} \cdot \cos\varphi \;\; F_{SIST} = m \cdot a_M \right]$$

$$m R \ddot{\varphi} = -m g \sin\varphi - m a_M \cos\varphi$$

$$\text{①}: \quad \boxed{R\ddot{\varphi} = -g\sin\varphi - a_M \cos\varphi}$$

→ RADIALNA SMER (y'):

$$m a_R' = F_{N,m} - F_{g,y'} + F_{SIST,y'} \quad ; \quad \left[F_{g,y'} = F_g \cdot \cos\varphi \;\; F_{SIST,y'} = F_{SIST} \cdot \sin\varphi \right]$$

$$m R \dot{\varphi}^2 = F_{N,m} - m g \cos\varphi + m a_M \sin\varphi \quad / : m$$

$$\text{②}: \quad \boxed{R\dot{\varphi}^2 = \frac{F_{N,m}}{m} - g\cos\varphi + a_M \sin\varphi}$$

- 2. NEWTONOV ZAKON ZA KLANEC (MASE M) V MIRUJOČEM INERCIALNEM SISTEMU S V HORIZONTALNI SMERI (x):

$$M a_M = F_{N,m,x} \quad ; \quad \left[F_{N,m,x} = F_{N,m} \cdot \sin\varphi \right]$$

$$M a_M = F_{N,m} \cdot \sin\varphi$$

$$\text{③}: \quad \boxed{F_{N,m} = \frac{M a_M}{\sin\varphi}}$$

- VSTAVIMO ENAČBO ③ V ENAČBO ②:

$$R\dot{\varphi}^2 = \frac{M a_M}{m \sin\varphi} - g\cos\varphi + a_M \sin\varphi = \left(\sin\varphi + \frac{M}{m \sin\varphi} \right) \cdot a_M - g\cos\varphi$$

$$\text{④}: \quad \boxed{a_M = \frac{g\cos\varphi + R\dot{\varphi}^2}{\frac{M}{m\sin\varphi} + \sin\varphi} \; / \cdot \sin\varphi = \frac{g\cos\varphi + R\dot{\varphi}^2}{\frac{M}{m} + \sin^2\varphi} \cdot \sin\varphi}$$

- KONČNO, VSTAVIMO TO ENAČBO (4) V ENAČBO (1):

$$R\ddot{\varphi} = -\left[g + \frac{g\cos\varphi + R\dot{\varphi}^2}{\frac{M}{m} + \sin^2\varphi}\cdot\cos\varphi\right]\cdot\sin\varphi$$

$\downarrow$ $/ : R$

$$(5):\quad \ddot{\varphi} + \left[\frac{g}{R} + \frac{\frac{g}{R}\cdot\cos\varphi + \dot{\varphi}^2}{\frac{M}{m} + \sin^2\varphi}\cdot\cos\varphi\right]\cdot\sin\varphi = 0$$

- V LIMITI MALIH KOTOV $|\varphi| \ll 1$ IŠČEMO HARMONIČNA NIHANJA, KI IMAJO ENAČBO NIHANJA:

$$\ddot{\varphi} + \omega_0^2\cdot\varphi \cong 0$$

- EKSAKTNO ENAČBO (5) MORAMO TAKO RAZVITI (LE) DO 1. REDA V $\varphi, \dot{\varphi}, \ddot{\varphi}$ (!)

$\downarrow$

UPORABIMO:

$$\sin\varphi \approx \underbrace{\varphi}_{1.\text{RED}}$$

$$\cos\varphi \approx \underbrace{1}_{0.\text{RED}} \gg \underbrace{-\frac{\varphi^2}{2}}_{2.\text{RED}} \approx \underbrace{1}_{0.\text{RED}}$$

- TAKO:

$$\ddot{\varphi} + \left[\frac{g}{R} + \frac{\frac{g}{R}\cdot\overbrace{\cos\varphi}^{(\approx 1 - \frac{\varphi^2}{2})} \gg \dot{\varphi}^2}{\frac{M}{m} \gg \underbrace{\sin^2\varphi}_{(\approx\varphi^2)}}\cdot\underbrace{\cos\varphi}_{(\approx 1 - \frac{\varphi^2}{2})\,(\approx 1)}\right]\cdot\underbrace{\sin\varphi}_{(\approx\varphi)} = 0$$

(števec $\approx \frac{g}{R}$, imenovalec $\approx \frac{M}{m}$)

$\Downarrow$

$$\ddot{\varphi} + \underbrace{\left[\frac{g}{R}\left(1 + \frac{m}{M}\right)\right]}_{(\equiv\,\omega_0^2)}\cdot\varphi \approx 0$$

$\downarrow$ $/\sqrt{\ }$

$$\left(\frac{2\pi}{t_0} =\right)\ \omega_0 \cong \sqrt{\frac{g}{R}\left(1 + \frac{m}{M}\right)} \quad \checkmark \quad \Longrightarrow \quad t_0 = \frac{2\pi}{\omega_0} \cong \frac{2\pi}{\sqrt{\left(1 + \frac{m}{M}\right)\cdot\frac{g}{R}}} \quad \checkmark$$