Rešitve nalog pri predmetu Klasična fizika

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Uvod

- Zbirka vsebuje rešitve nalog, ki smo jih rešili pri predmetu Klasična fizika v študijskem letu 2020/21.
- Vrstni red nalog v večji meri sledi zbirki nalog, objavljeni v spletni učilnici, osnovo katere je zasnovala Saša Prelovšek Komelj leta 2013. Ta zbirka se iz leta v leto posodablja tako, da oštevilšenje ni nujno več enako, kot je bilo v času tega zapisa
- Označevanje nalog:
  - naloge, ki so podane v omenjeni zbirki so označene z obkroženo številko, ki označuje poglavje in številko naloge, ki sledi kratici *nal.*
  - naloge, ki jih omenjena zbirka samo navaja glede na originalen izvor - to so kolokvijske naloge in naloge iz Zbirke 9: Naloge iz fizike; M. Gros, M. Hribar, A. Kodre in J. Strnad, pa so označene z izvorno oznako.

Kazalo

1 Kinematika v 1D

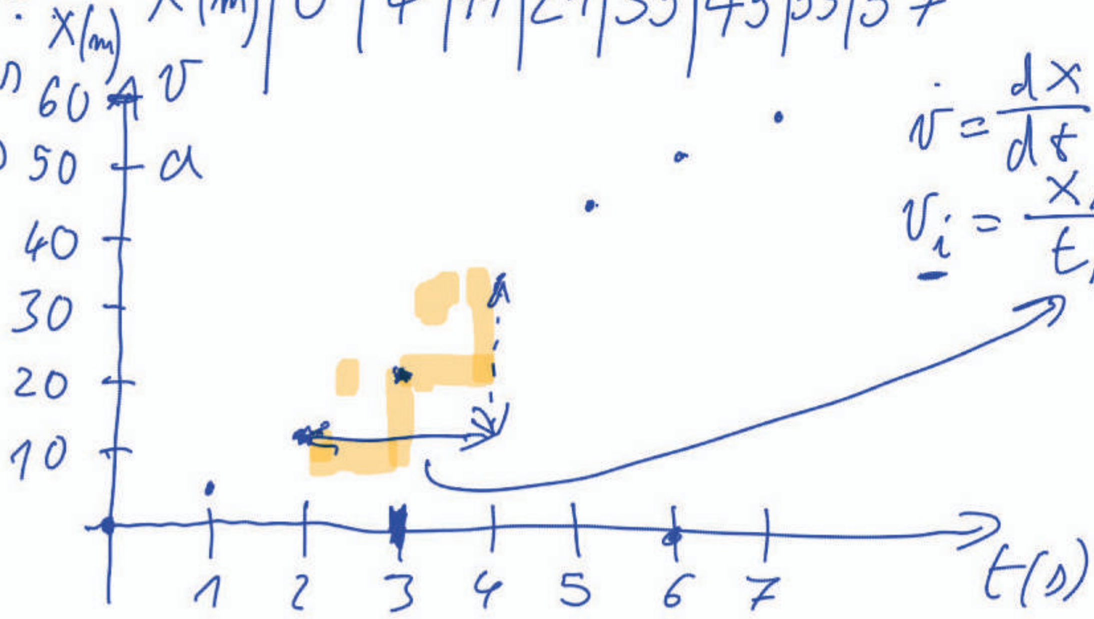
(1)

2.) $\bar{v} = ?$ $\bar{a} = ?$

$t_1 = 3s$

$t_2 = 6s$

t (s)	0	1	2	3	4	5	6	7
X (m)	0	4	11	21	35	45	53	57



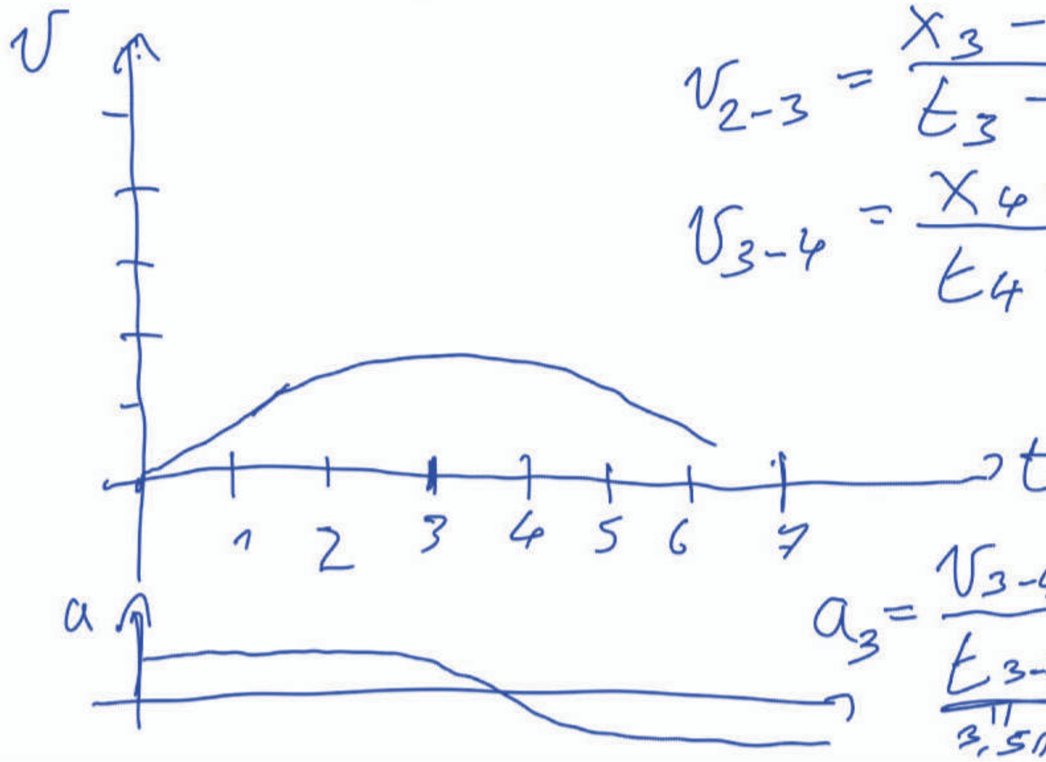
$$v = \frac{dx}{dt}$$

$$v_i = \frac{x_{i+1} - x_{i-1}}{t_{i+1} - t_{i-1}}$$

$$\bar{v}(t=3s) = \frac{35m - 11m}{4s - 2s} = \frac{24\,m}{2s} = 12\ m/s$$

$$\bar{v}(t=6s) = \frac{57m - 45m}{7s - 5s} = \frac{12\,m}{2\,m} = 6\ m/s$$

$$a = \frac{dv}{dt} \longrightarrow a_i = \frac{v_{i+1} - v_{i-1}}{t_{i+1} - t_{i-1}} = \frac{\frac{x_{i+2} - x_i}{t_{i+2} - t_i} - \frac{x_i - x_{i-2}}{t_i - t_{i-2}}}{\underbrace{t_{i+1} - t_{i-1}}_{\Delta t}}$$



$$v_{2-3} = \frac{x_3 - x_2}{t_3 - t_2} = 10\,m/s$$

$$v_{3-4} = \frac{x_4 - x_3}{t_4 - t_3} = 14\,m/s$$

$$a_3 = \frac{v_{3-4} - v_{2-3}}{\underset{3{,}5s}{\underset{\shortparallel}{t_{3-4}}} - \underset{2.5s}{\underset{\shortparallel}{t_{2-3}}}}$$

$$a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

$$a_3 = \frac{14\,m/s - 10\,m/s}{3.5\,s - 2.5\,s} = \underline{4\,m/s^2}$$

$$a_i = \frac{\frac{x_{i+2} - x_i}{t_{i+2} - t_i} - \frac{x_i - x_{i-2}}{t_i - t_{i-2}}}{\underbrace{t_{i+1} - t_{i-1}}_{\Delta t}} = \frac{x_{i+2} - x_i - x_i + x_{i-2}}{\Delta t \cdot \Delta t} = \frac{x_{i+2} - 2x_i + x_{i-2}}{\Delta t^2}$$

$$\Delta t = t_{i+2} - t_i$$

$\Delta t \Rightarrow$ SKRAJŠAMO $\Rightarrow \boxed{\Delta t = t_{i+1} - t_i}$

$$\hookrightarrow \boxed{a_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{\Delta t^2}}$$

$$a_6 = \frac{57\,m - 2 \cdot 53\,m + 45\,m}{1\,s^2} = \underline{-4\,m/s^2}$$

(1)

3 mol

X = 2

t[s]	0		1		2		3
v[m/s]		1		2		3	
x	0		1		3		6

$$v \neq v(t)$$
$$v = konst$$

$$x = \int_{t_1}^{t_2} v(t)\,dt = v \cdot \Delta t$$

ZA $v =$ konst.

$$x_{1-2} = 2\,m/s \cdot 1\,s = \underline{2\,m}$$

① 4mol: $X(t) = At^2 - Bt^3$, $A = 3\,m/s^2$, $B = 2\,m/s^3$

a) $\bar{v} = ?$
$\bar{a} = ?$
$t = [0.6, 0.8]$

b) $v = ?$
$a = ?$
$t = 0.7\,s$

a) $\bar{v} = \frac{X_2 - X_1}{t_2 - t_1}$ $\rightarrow$ $\bar{v} = \frac{3\,m/\cancel{s^2} \cdot 0.8^2 \cancel{s^2} - 2\,m/\cancel{s^3} \cdot 0.8^3 \cancel{s^3} - 3\,m/\cancel{s^2} \cdot 0.6^2 \cancel{s^2} + 2\,m/\cancel{s^3} \cdot 0.6^3 \cancel{s^3}}{0.2\,s}$

$\bar{v} = \frac{\int_1^2 v(t)\,dt}{\Delta t}$ $= \boxed{1.24\,m/s}$

$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1}$ $\qquad v = \frac{dx}{dt} = 2At - 3Bt^2$

$= \frac{2\cdot 3\,m/s^2 \cdot 0{,}8\cancel{s} - 3\cdot 2\,m/s^3 \cdot 0.8^2 \cancel{s^2} - 2\cdot 3\,m/s^2 \cdot 0.6\cancel{s} + 3\cdot 2\,\frac{m}{s^3}\, 0.6^2 \cancel{s^2}}{0.2\,s}$

$= -2.4\,m/s^2$

b) $v = 2At - 3Bt^2 = 2\cdot 3\,m/s^2 \cdot 0{,}7\,s - 3\cdot 2\,m/s^3 \cdot 0.7^2 s^2 = 6 \cdot 0{,}7(1 - 0.7)\,\frac{m}{s}$

$= 1{,}26\,m/s$

$a = \frac{dv}{dt} = 2A - 6\cdot Bt = 2\cdot 3\,m/s^2 - 6\cdot 2\,m/s^3 \cdot 0.7\,s$

$= -2{,}4\,m/s^2$

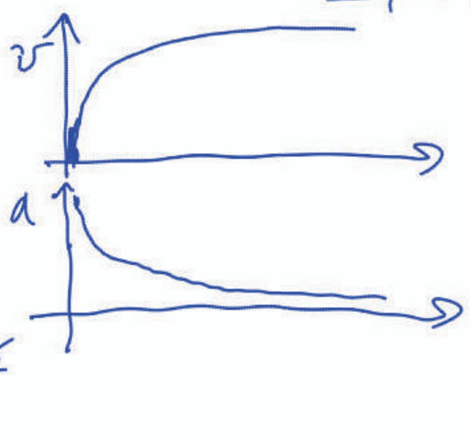
ZBIRKA 9: 6mol/st 9

$v = k\cdot\sqrt{t}$
$k = 5\,m/s^{3/2}$
$t = 30\,s$

$a = ?$
$x = ?$
$a(t=0) = ?$

$a = \frac{dv}{dt} = k\,\frac{1}{2}\,t^{-1/2} = 5\,m/s^{3/2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{30}\,s^{1/2}} = 0.46\,\frac{m}{s^2}$

$a(t=0) = \frac{k}{2}\,\frac{1}{\sqrt{t}} \xrightarrow{t \to 0} \infty$



$X = \int_0^t v\,dt = \int_0^t k\,\underset{t^{1/2}}{\underset{\parallel}{\sqrt{t}}}\,dt = k\,\frac{t^{3/2}}{3/2}\Big|_0^t$

$= \frac{2k}{3}\left(t^{3/2} - 0\right) = \frac{2\cdot 5\,m}{3\,\cancel{s^{3/2}}} \cdot 30^{3/2} \cdot \cancel{s^{3/2}}$

$= 547{,}7\,m$

ZBIRKA 9 5mal/stg

$v_0 = 4\,m/s$

$t = 10\,s$

$a = -k v^2$

$k = 0.65\,m^{-1}$

$x = ?$

$v = ?$

kdaj bi se ustavil?

$\int \frac{dx}{x^2} = -\frac{1}{x}$

$\ln A - \ln B = \ln \frac{A}{B}$

$$\underline{v = \int_0^t a\,dt}$$

$$a = \frac{dv}{dt}$$

$$-kv^2 = \frac{dv}{dt}$$

$$-\int_0^t dt = \int_{v_0}^{v(t)} \frac{dv}{kv^2}$$

$$+t = \frac{1}{k}\left(+\frac{1}{v}\right)\Big|_{v_0}^{v(t)}$$

$$t = \frac{1}{k}\left(\frac{1}{v} - \frac{1}{v_0}\right) \quad / \cdot k$$

$$kt = \frac{1}{v} - \frac{1}{v_0}$$

$$\frac{1}{v} = kt + \frac{1}{v_0}$$

$$\underline{v = \frac{1}{kt + 1/v_0} = v(t)}$$

$$v = \frac{1}{0.65\,\frac{1}{m}\cdot 10\,s + \frac{1\cdot s}{4\,m}}$$

$$\underline{\underline{v = 0.148\,m/s}}$$

$$x = \int_0^t v(t)\,dt$$

$$= \int_0^t \underbrace{\frac{1}{kt + \frac{1}{v_0}}}\,dt$$

$$\underline{u = kt + \frac{1}{v_0}}$$

$$du = k\,dt + 0$$

$$\hookrightarrow \underline{dt = \frac{1}{k}\cdot du}$$

$$= \int_{\frac{1}{v_0}}^{kt+\frac{1}{v_0}} \frac{1}{u}\,\frac{1}{k}\cdot du$$

$$= \frac{1}{k}\ln u\Big|_{1/v_0}^{kt+\frac{1}{v_0}}$$

$$= \frac{1}{k}\ln\frac{kt + \frac{1}{v_0}}{1/v_0}$$

$$\underline{= \frac{1}{k}\ln(ktv_0 + 1)}$$

$$= \frac{1}{0.65\,m^{-1}}\ln\left(0.65\,\frac{1}{\cancel{m}}\cdot 10\,\cancel{s}\cdot 4\,\frac{\cancel{m}}{\cancel{s}} + 1\right)$$

$$\underline{\underline{= 5.07\,m}}$$

① nal 8 ⓗ

$l = 2m$

$\Delta t = 0.25 s$

$h = ?$

$z = z_0 + v_0 t + \frac{gt^2}{2}$

→ ZAČETNA TOČKA

$z_0 = h$

$v_0 = ?$

$v_0 = \sqrt{2gh}$

ENAKOMERNO POSPEŠENO

$a = konst.$ $a = \frac{dv}{dt} \cdot \frac{dx}{dx} = v$

$a = v \frac{dv}{dx}$

$\int_{x_0}^{x} a \cdot dx = \int_{v_0}^{v} v\, dv$

$a \underbrace{(x - x_0)}_{s = POT} = \frac{1}{2}(v^2 - v_0^2)$

$2as = v^2 - v_0^2$

↳ ČE ZAČETNA HITROST = 0

↳ $\sqrt{2as} = v$

⇒ KO PADE NA DNO OKNA

$\cancel{h} + l = \cancel{h} + \sqrt{2gh}\,\Delta t + \frac{g \Delta t^2}{2}$

$l - \frac{g\Delta t^2}{2} = \sqrt{2gh}\,\Delta t \;/^2$

$\frac{\left(l - \frac{g\Delta t^2}{2}\right)^2}{2g\Delta t^2} = h = \frac{\left(2m - \frac{10m \cdot 0.25^2 s^2}{s^2 \cdot 2}\right)^2}{2 \cdot 10 m/s^2 \cdot 0.25^2 s^2}$

$h = 2{,}28 m$

ZBIRKA 9 nal 12/str 10

$t = 5 s$

$v_z = 340 m/s$

$h = ?$

PADEC: $h = \frac{g t_p^2}{2}$

ZVOK: $h = v_z \cdot t_z \rightarrow t_z = \frac{h}{v_z}$

ČAS: $t = t_p + t_z = t_p + \frac{h}{v_z} \rightarrow t_p = t - \frac{h}{v_z}$

$h = \frac{g}{2}\left(t - \frac{h}{v_z}\right)^2$

$\frac{2h}{g} = t^2 - 2\frac{ht}{v_z} + \frac{h^2}{v_z^2}$

$0 = \frac{1}{v_z^2} \cdot h^2 - 2\left(\frac{1}{g} + \frac{t}{v_z}\right)h + t^2$

$0 = ax^2 + bx + c$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$h = \frac{2\left(\frac{1}{g} + \frac{t}{v_z}\right) \pm \sqrt{4\left(\frac{1}{g} + \frac{t}{v_z}\right)^2 - 4\frac{1}{v_z^2} \cdot t^2}}{2 \cdot \frac{1}{v_z^2}}$

$= v_z^2\left(\frac{1}{g} + \frac{t}{v_z}\right) \pm \left(\sqrt{\frac{1}{g^2} + \frac{2t}{v_z g} + \frac{t^2}{v_z^2} - \frac{t^2}{v_z^2}}\right) \cdot v_z^2$

$= v_z^2\left[\frac{1}{g} + \frac{t}{v_z} \pm \sqrt{\frac{1}{g^2} + \frac{2t}{v_z g}\frac{g}{g}}\right]$

$= v_z t + \frac{v_z^2}{g} \pm \frac{v_z^2}{g}\sqrt{1 + \frac{2tg}{v_z}} = v_z t + \frac{v_z^2}{g}\left(1 \pm \sqrt{1 + \frac{2gt}{v_z}}\right)$

$h \rightarrow$ IZBEREMO $-$

$h = v_z \cdot t + \frac{v_z^2}{g}\left(1 - \sqrt{1 + \frac{2gt}{v_z}}\right) = \frac{340 m}{s} 5 s + \frac{340^2 m^2 s^2}{s^2 \cdot 10 m}\left(1 - \sqrt{1 + \frac{2 \cdot 10 m \cdot s \cdot 5 s}{s^2 \cdot 340 m}}\right)$

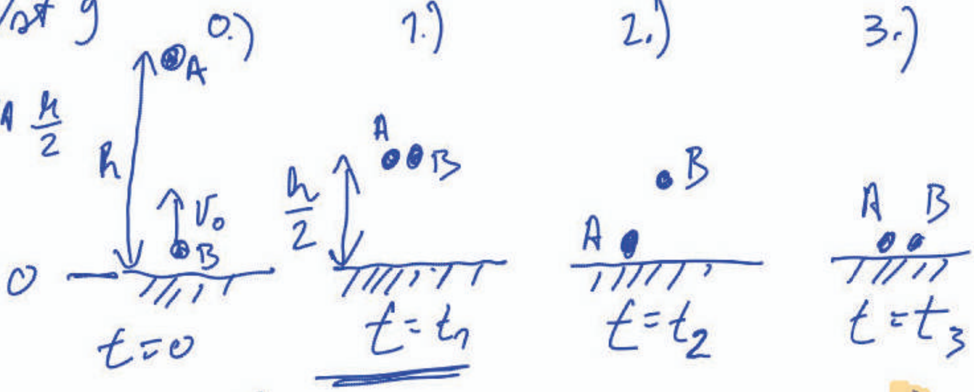
$h = 109{,}4 m$

ZBIRKA 9 7. rel./st. 9

$v_0 = 10\,m/s$, SREČATA NA $\frac{h}{2}$

$h = ?$ ✓

$t = t_3 - t_2 = ?$



POLOŽAJI: A $y_A = h - \frac{gt^2}{2}$

B $y_B = v_0 \cdot t - \frac{gt^2}{2}$

• OB SREČANJU PRI 1.)

$$y_A = y_B = \frac{h}{2}$$

$$h - \cancel{\frac{g t_1^2}{2}} = v_0 t_1 - \cancel{\frac{g t_1^2}{2}}$$

$$h = v_0 t_1$$

A: $\frac{h}{2} = h - \frac{g t_1^2}{2}$

$$-\frac{h}{2} = -\frac{g t_1^2}{2} \Rightarrow t_1^2 = \frac{h}{g}$$

$$h = v_0 \cdot \sqrt{\frac{h}{g}} \quad /^2 \quad \to 1\,s$$

$$h^{\cancel{2}} = v_0^2 \frac{\cancel{h}}{g}$$

$$h = \frac{v_0^2}{g} = \frac{100\,m^2/s^2}{10\,m/s^2} = 10\,m$$

PRI 2.) $y_A = 0 = h - \frac{g t_2^2}{2}$

$$\to t_2 = \sqrt{\frac{2h}{g}} = \sqrt{2}\,s$$

PRI 3.) $y_B = 0 = v_0 t_3 - \frac{g t_3^2}{2}$

$$\frac{2v_0}{g} = t_3 = 2\,s$$

$$t = t_3 - t_2 = (2 - \sqrt{2})\,s$$

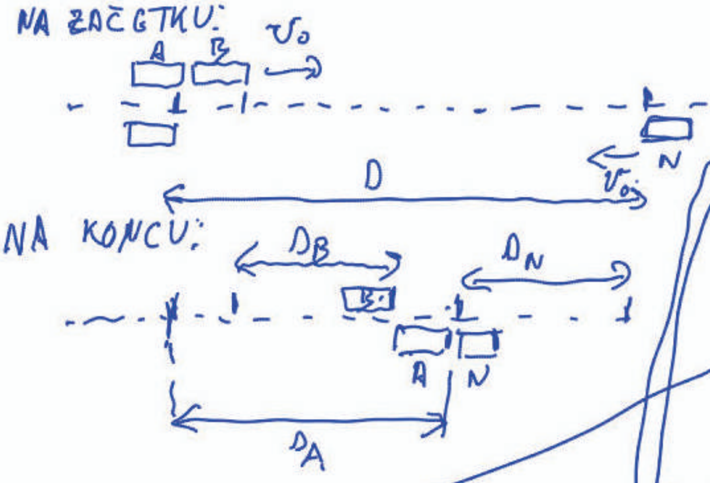
2 nal 12

$v_0 = 80\,\mathrm{km/h}$

$s = 160\,\mathrm{m}$

$v_N = v_0$, $l = 4\,\mathrm{m}$

$a = ?$



$s_B = s_N$

$s_A = s_B + 2l$

$s = s_N + s_B + 2l = 2s_B + 2l$

$s_B = v_0 t = s_N \longrightarrow t = \dfrac{s_B}{v_0}$

$s_A = v_0 \cdot t + \dfrac{at^2}{2}$

$\rightarrow s - 2l = 2 s_B \Rightarrow s_B = \dfrac{s-2l}{2}$

$s_B = \dfrac{152}{2}\,\mathrm{m}$

$s_B = 76\,\mathrm{m}$

$s_A = s_B + 2l = 84\,\mathrm{m}$

$$s_A = \cancel{v_0}\,\frac{s_B}{\cancel{v_0}} + \frac{a\left(\frac{s_B}{v_0}\right)^2}{2}$$

$$s_A - s_B = \frac{a\left(\frac{s_B}{v_0}\right)^2}{2}$$

$$\rightarrow a = \frac{2\overbrace{(s_A - s_B)}^{2l}}{s_B^2}\cdot v_0^2 = \frac{2\cdot 8\,\cancel{\mathrm{m}}\cdot 80^2\cdot 3{,}6^{-2}\,\frac{\mathrm{m}^{\cancel{2}}}{\mathrm{s}^2}}{76^2\,\cancel{\mathrm{m}^2}}$$

$a = 1{,}37\,\mathrm{m/s^2}$

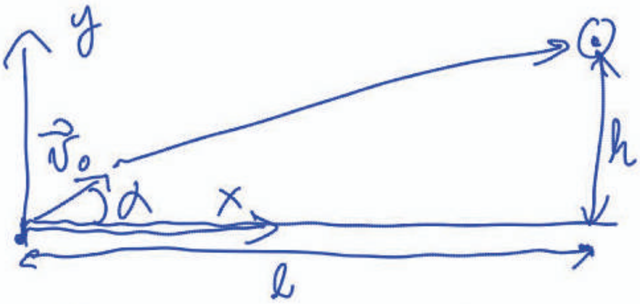
2 Kinematika v 2D

② nal 1.

h, l, v_0

$\alpha = ?$ ✓

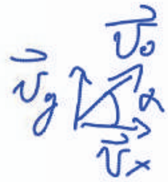
$\beta = ?$



IZSTRELEK

$v_x = v_0 \cdot \cos\alpha$

$v_y = \underbrace{v_0 \cdot \sin\alpha}_{v_{y_0}} - gt$



POLOŽAJI: IZSTRELEK:

$x_1 = v_x \cdot t = v_0 \cdot \cos\alpha \cdot t$

$y_1 = 0 + v_{y_0} \cdot t - \frac{gt^2}{2} = v_0 \cdot \sin\alpha \cdot t - \frac{gt^2}{2}$

TARČA:

$x_T = l$

$y_T = h - \frac{gt^2}{2}$

Y → ENAKOMERNO POSPEŠENO

$y = \int v\,dt$

• OB ZADETKU: y: $v_0 \sin\alpha \cdot t - \cancel{\frac{gt^2}{2}} = h - \cancel{\frac{gt^2}{2}}$

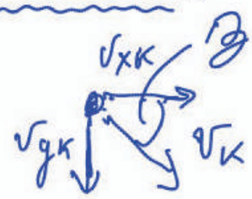
$h = v_0 \sin\alpha \cdot t$

x: $v_0 \cdot \cos\alpha \cdot t = l$ → $\frac{h}{l} = \frac{\cancel{v_0} \sin\alpha \cdot \cancel{t}}{\cancel{v_0} \cos\alpha \cdot \cancel{t}}$

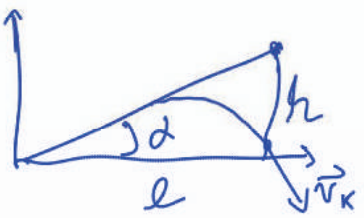
$\boxed{tg\alpha = \frac{h}{l}}$

$t = \frac{l}{v_0 \cos\alpha}$

SMER HITROSTI $\vec{v}_K$



$tg\beta = \frac{v_{yK}}{v_{xK}} = \frac{v_0 \sin\alpha - gt}{v_0 \cdot \cos\alpha} =$



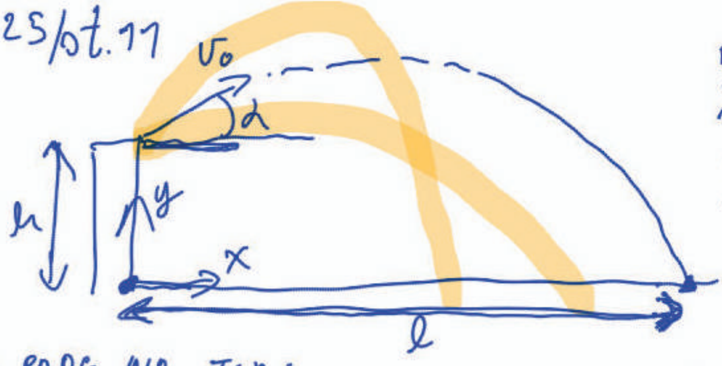
$$\boxed{tg\beta = \frac{v_0 \sin\alpha - g\, l \frac{1}{v_0 \cos\alpha}}{v_0 \cdot \cos\alpha}}$$

ZBIRKA 9 nal 25/st. 11

$v_0 = 10\,m/s$

$h = 12\,m$

$\alpha(l = l_{max}) = ?$



POLOŽAJ: $v_x = v_0 \cdot \cos\alpha = konst.$

$$x = \int v_x\,dt = v_0 \cdot \cos\alpha \cdot t$$

$$y = h + v_0 \sin\alpha \cdot t - \frac{g t^2}{2}$$

KO PADE NA TLA:

$$x = l = v_0 \cdot \cos\alpha \cdot t \longrightarrow t = \frac{l}{v_0 \cdot \cos\alpha}$$

$$y = 0 = h + v_0 \cdot \sin\alpha \cdot t - \frac{g t^2}{2}$$

$$0 = h + \cancel{v_0} \sin\alpha \frac{l}{\cancel{v_0} \cdot \cos\alpha} - g \frac{l^2}{2 v_0^2 \cos^2\alpha}$$

$$0 = h + l \cdot tg\alpha - \frac{g l^2}{2 v_0^2}(1 + tg^2\alpha)$$

$$0 = -\frac{g l^2}{2 v_0^2} \cdot tg^2\alpha + l\, tg\alpha + h - \frac{g l^2}{2 v_0^2}$$

$$\boxed{tg\alpha = \frac{-l \pm \sqrt{l^2 + 4 \frac{g l^2}{2 v_0^2} \cdot \left(h - \frac{g l^2}{2 v_0^2}\right)}}{-2 \cdot \frac{g l^2}{2 v_0^2}}}$$

$$\cos^2\alpha + \sin^2\alpha = 1$$

$$\cos^2\alpha = 1 - \sin^2\alpha$$

$$1 = \frac{1}{\cos^2\alpha} - tg^2\alpha$$

$$\boxed{\frac{1}{\cos^2\alpha} = 1 + tg^2\alpha}$$

DA DOSEŽEMO $l = l_{max}$ MORA IMETI $tg\alpha$ SAMO ENO REŠITEV

↳ DISKRIMINANTA = 0

$$\hookrightarrow l^2 + \frac{\cancel{4}^2 g l^2}{\cancel{2} v_0^2}\left(h - \frac{g l^2}{2 v_0^2}\right) = 0 \quad / \cdot \frac{1}{l^2}$$

$$1 + \frac{2g}{v_0^2} \cdot h - \frac{\cancel{2} g}{v_0^2} \cdot \frac{g l^2}{\cancel{2} v_0^2} = 0 \quad / \cdot v_0^4$$

$$v_0^4 + 2 g v_0^2 \cdot h = g^2 l^2 \Rightarrow \boxed{l = \frac{v_0 \sqrt{v_0^2 + 2gh}}{g}} =$$

$$= \frac{10\,m/s \sqrt{100\,m^2/s^2 + 2 \cdot 10\,m/s^2 \cdot 12\,m}}{10\,m/s^2}$$

$$\boxed{l_{max} = 18{,}4\,m}$$

$$tg\alpha = \frac{v_0 \sqrt{v_0^2 + 2gh}}{g \cdot \cancel{2} \frac{g l^2}{\cancel{2} v_0^2}} = \frac{v_0^{\cancel{4}\,3} \sqrt{v_0^2 + 2gh} \cdot \cancel{g^2}}{\cancel{g^2}\, \cancel{v_0^2}\,(v_0^2 + 2gh)} = \frac{v_0}{\sqrt{v_0^2 + 2gh}} = \frac{10\,m/s}{\sqrt{100\,m^2/s^2 + 2 \cdot 10\,m/s^2 \cdot 12\,m}}$$

$$\Rightarrow \boxed{\alpha = 28{,}5^\circ}$$

↳ ZA $h = 0 \Rightarrow tg\alpha = 1$

↳ $\alpha = 45^\circ$

(2) nal 3

φ

$n = 1$

$n = n$ ODBOJI

$\zeta = ?$

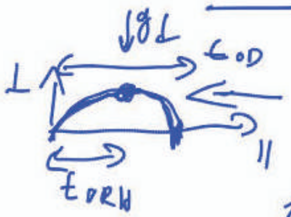
→ ZA ODBOJ NAZAJ → $v_\parallel = 0$ PRI ODBOJU

→ ODBOJ POMENI $v_\perp$ OBRNE PREDZNAK

HITROSTI: $\parallel: v_\parallel = v_0 \cdot \cos\zeta - g \cdot \sin\varphi \cdot t$

$\perp: v_\perp = v_0 \cdot \sin\zeta - g \cos\varphi \cdot t$

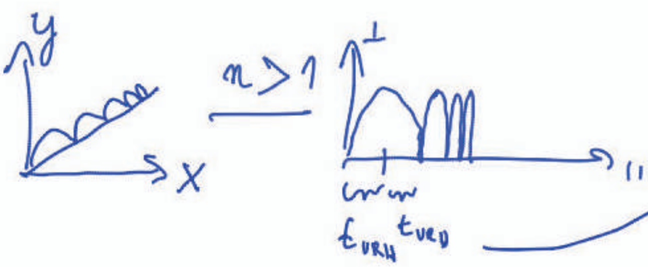
PRI ODBOJU: $v_\parallel = 0 = v_0 \cdot \cos\zeta - g \sin\varphi \cdot t_{OD} \Rightarrow t_{OD} = \frac{v_0 \cos\zeta}{g \sin\varphi}$



NA VRHU: $v_\perp = 0 = v_0 \sin\zeta - g \cos\varphi \cdot t_{VRH} \Rightarrow t_{VRH} = \frac{v_0 \sin\zeta}{g \cos\varphi}$

ZA $n = 1$: $t_{OD} = 2 t_{VRH}$ $\Rightarrow \frac{\cancel{v_0} \cos\zeta}{\cancel{g} \sin\varphi} = \frac{2 \sin\zeta \cancel{v_0}}{\cancel{g} \cos\varphi}$

$$\tg\zeta = \frac{1}{2} \cdot \ctg\varphi$$



$t_{OD} = 2 \cdot n \, t_{VRH} \rightarrow \tg\zeta = \frac{1}{2n} \cdot \ctg\varphi$

(2) nal 4.

$v_y = 3\,m/s$

$v_x = \alpha \cdot y$

$\alpha = 0.5\,s^{-1}$

$y(x) = ?$

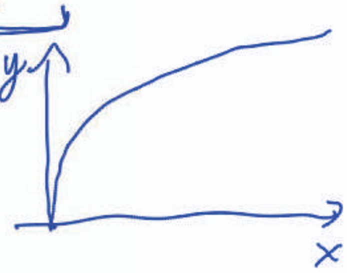
$v_y = \frac{dy}{dt} = konst. \Rightarrow y = v_y \cdot t$

$v_x \neq konst \Rightarrow x = \int_0^t v_x \cdot dt = \int_0^t \alpha \cdot y\, dt = \alpha v_y \int_0^t t \cdot dt$

$x = \alpha \cdot v_y \frac{t^2}{2}$

$\hookrightarrow t = \sqrt{\frac{2x}{\alpha \cdot v_y}}$

$y = v_y \sqrt{\frac{2x}{\alpha \cdot v_y}} = \sqrt{\frac{2 v_y}{\alpha}} \cdot \sqrt{x}$



ZBIRKA 9 nal 31/str. 11

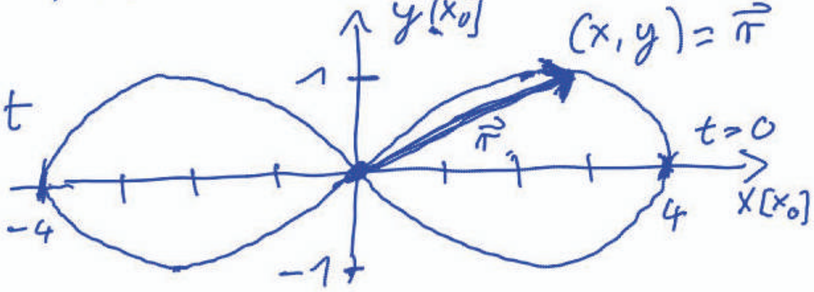
$x = 4x_0 \cos\omega t$

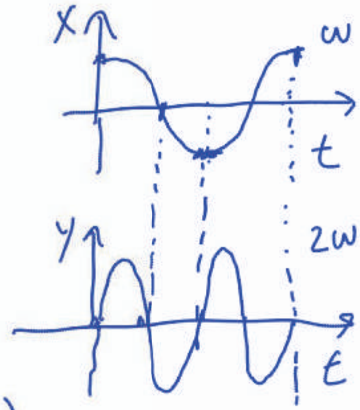
$y = x_0 \cdot \sin 2\omega t$

$x_0 = 10\,cm$

$\omega = 0.1\,s^{-1}$

(x, y) za a = max ?





$\vec{r} = (x, y) = x_0(4\cos\omega t, \sin 2\omega t)$

$\vec{v} = (\dot{x}, \dot{y}) = x_0(-4\omega\sin\omega t, 2\omega\cos 2\omega t)$

$\vec{a} = (\ddot{x}, \ddot{y}) = x_0(-4\omega^2\cos\omega t, -4\omega^2\sin 2\omega t) =$

$= -4\omega^2 x_0 (\cos\omega t, \sin 2\omega t)$

ZA MAKSIMUM:

$\frac{d|\vec{a}|}{dt} = 0$

$|\vec{a}|^2$

$|\vec{a}| = 4\omega^2 x_0 \sqrt{\cos^2\omega t + \sin^2 2\omega t}$

ISTO VELJA TUDI ZA $|\vec{a}|^2 \Rightarrow \frac{d|\vec{a}|^2}{dt} = 0$

$\frac{d(\cos^2\omega t + \sin^2 2\omega t)}{dt} = 0$

$\sin 2\alpha = 2\cos\alpha\sin\alpha \Rightarrow$

$2\cos\omega t(-\sin\omega t)\cdot\cancel{\omega} + 2\sin 2\omega t\cos 2\omega t \cdot 2\cancel{\omega} = 0$

$-\cancel{\sin 2\omega t} + 4\cancel{\sin 2\omega t}\cos 2\omega t = 0$

$1 = 4\cos 2\omega t$

$2\omega t = \pm\arccos\frac{1}{4}$

$\omega t = \pm\frac{1}{2}\arccos\frac{1}{4}$

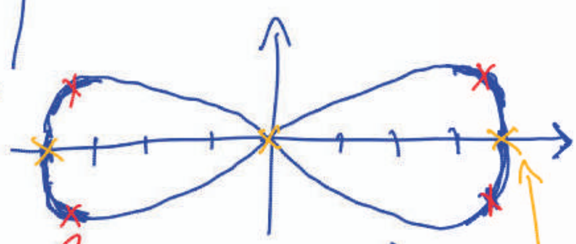
$\sin 2\omega t = 0$

$2\omega t = n\pi$

$\omega t = \frac{n\pi}{2}$

$x = 4x_0\cos\omega t = \pm 31{,}6\,cm$

$y = x_0\sin 2\omega t = \pm 9{,}7\,cm$



$\vec{a} = 4\omega^2 x_0\sqrt{\cos^2\omega t + \sin^2 2\omega t}$

maximum

$a > 0$

minimum

$a = 0$

91/92 / kol 1./nal 2.

$A = 2\,m$

$B = 1\,m$

$t_0 = 0.5\,s$

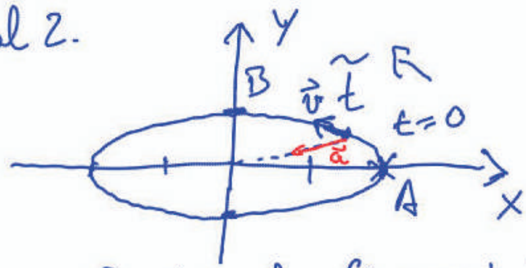
$x = A \cdot \cos 2\pi t/t_0$

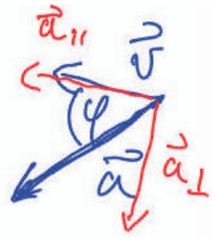
$y = B \cdot \sin 2\pi t/t_0$

$\tilde{t} = t_0/12$

$a_{\parallel}, a_{\perp}$ (OB $\tilde{t}$) = ?

SMER SILE ?





$$\vec{r} = (x, y) = (A\cos\omega t, B\sin\omega t) \;;\; \omega = \frac{2\pi}{t_0}$$

$$\vec{v} = (\dot{x}, \dot{y}) = (-A\omega\sin\omega t, B\omega\cos\omega t)$$

$$\vec{a} = (\ddot{x}, \ddot{y}) = (-A\omega^2\cos\omega t, -B\omega^2\sin\omega t) = -\omega^2\vec{r}$$

$$\boxed{\begin{aligned} a_{\parallel} &= a\cdot\cos\varphi \\ a_{\perp} &= a\cdot\sin\varphi \end{aligned}}$$

$$\vec{a}\cdot\vec{v} = |a|\cdot|v|\cdot\cos\varphi$$

$$\boxed{\cos\varphi = \frac{\vec{a}\cdot\vec{v}}{|a||v|}}$$

OB ČASU $t = \tilde{t}$

$$\vec{a}\cdot\vec{v} = A^2\omega^3\sin\omega t\cos\omega t - B^2\omega^3\sin\omega t\cos\omega t$$

$$= \omega^3\cos\omega t\sin\omega t\,(A^2 - B^2)$$

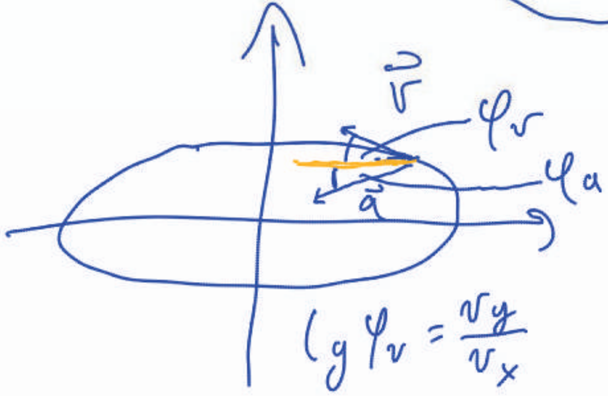
$$= 2577\,m^2/s^3$$

$$|a| = \sqrt{A^2\omega^4\cos^2\omega t + B^2\omega^4\sin^2\omega t}$$

$$= 284\,m/s^2$$

$$|v| = \sqrt{A^2\omega^2\sin^2\omega t + B^2\omega^2\cos^2\omega t}$$

$$= 16{,}6\,m/s$$



$$\boxed{\begin{aligned} \cos\varphi &= 0.546 \\ a_{\parallel} &= 155\,m/s^2 \\ a_{\perp} &= 239\,m/s^2 \end{aligned}}$$

$$tg\,\varphi_v = \frac{v_y}{v_x}$$

$$tg\,\varphi_a = \frac{a_y}{a_x}$$

$$\varphi = |\varphi_v - \varphi_a|$$

SILA KAŽE V SMERI POSPEŠKA!

ZBIRKA 9 nal 19/st 10

$\nu = 0.5\ s^{-1}$

$\varphi_0 = 30°$

$r = 10\ cm$

$\tilde{\varphi} = 10°$

$|\vec{a}| = ?$

β KOT PROTI r ?

$\varphi = \varphi_0 \sin \omega_0 t$

$\omega_0 = 2\pi\nu$

$|a| = \sqrt{a_r^2 + a_t^2}$

$v = \omega r$ KROŽENJE

$a_t = \dot{\omega} r = \alpha r$

TANGENCIALNI POSPEŠEK

$a_r = \omega^2 r$

RADIALNI POSPEŠEK

$\dot{\varphi} = \omega = \varphi_0 \omega_0 \cos \omega_0 t$

$\ddot{\varphi} = \dot{\omega} = \alpha = -\varphi_0 \omega_0^2 \sin \omega_0 t$

$\tilde{\varphi} \to \tilde{t}$: $\tilde{\varphi} = \varphi_0 \sin \omega_0 \tilde{t}$

$\omega_0 \tilde{t} = \arcsin \frac{\tilde{\varphi}}{\varphi_0} = \arcsin \frac{1}{3} = 19.47°$

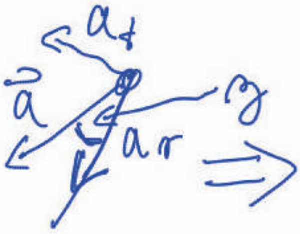
$a_t = -\varphi_0 \omega_0^2 \sin \omega_0 \tilde{t} \cdot r$

$=$

$a_r = \varphi_0^2 \omega_0^2 \cos^2 \omega_0 t \cdot r$

$=$

$|a| = 0,19\ m/s^2$



$\Rightarrow a_r = |a| \cdot \cos\beta$

$\to \cos\beta = \frac{a_r}{|a|} \Rightarrow \beta = 36.6°$

(2) nal 8.

$R = 1\,m$

$\omega = \dfrac{k}{\sqrt{\varphi}}$

$k = 1\,s^{-1}$

- a_r, a_t,
- ∡ med $\vec{v}$ in $\vec{a}$

PO 1 OBHODU

$\varphi = 2\pi$

$$a_r = \omega^2 r = \frac{k^2}{\varphi} \cdot R = \frac{1 \cdot 1\,m}{s^2\, 2\pi} = \frac{1}{2\pi}\ m/s^2$$

$$a_t = \alpha \cdot r = -\frac{k^2}{2\varphi^2} \cdot R = -\frac{1 \cdot 1\,m}{s^2\, 2 \cdot 4\pi^2} = \frac{-1}{8\pi^2}\ m/s^2$$

$v = \omega \cdot r$

$a_t = \alpha \cdot r$

$\dfrac{1}{\sqrt{\varphi}} = \varphi^{-\frac{1}{2}}$

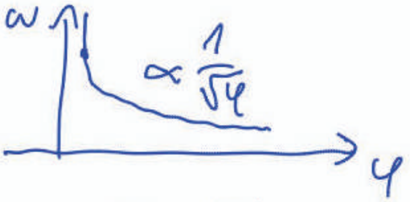
- $$\alpha = \frac{d\omega}{dt}\overbrace{\frac{d\varphi}{d\varphi}}^{=\omega} = \omega \frac{d\omega}{d\varphi} = \underbrace{\frac{k}{\sqrt{\varphi}}}_{\varphi^{-\frac{1}{2}}}\, k\left(-\frac{1}{2}\right)\varphi^{-\frac{3}{2}} = -\frac{k^2}{2\varphi^2}$$

φ^{-2}

- $$\operatorname{tg} \vartheta = \frac{|a_t|}{|a_r|} = \frac{\cancel{k^2}\cancel{R}\,\varphi}{2\varphi^{\cancel{2}}\,\cancel{k^2}\cancel{R}} = \frac{1}{2\varphi} = \frac{1}{4\pi} \Rightarrow \vartheta = 4{,}6^\circ$$

$$\measuredangle_{\vec{v},\vec{a}} = \vartheta + 90^\circ = 94{,}6^\circ$$

φ_0



$$\alpha = \frac{d\omega}{dt} \cdot \overbrace{\frac{d\varphi}{d\varphi}}^{=\omega} \qquad \omega = \frac{d\varphi}{dt}$$

ZBIRKA 9 nal 11/st. 9

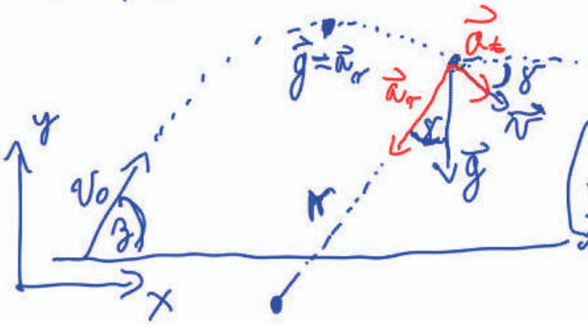
$\vartheta = 60^\circ$

$v_0 = 20\,m/s$

$t = 3\,s$

$a_r, a_t = ?$

$r = ?$



$\vec a_t \parallel \vec v \,,\ \vec a_r \perp \vec v$

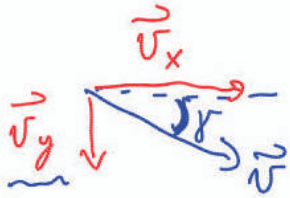
$\underline{a_r = g\cdot\cos\gamma} = \omega^2 r = \frac{v^2}{r^2} r = \underline{\frac{v^2}{r}}$

$\underline{a_t = g\cdot\sin\gamma}$

$r = \frac{v^2}{a_r} = \frac{v^2}{g\cdot\cos\gamma} = \frac{(v_x^2+v_y^2)}{g\cos\gamma}$

$a_r = |\vec a_r|$

$\vec a_r = (\ _x\ ,\ _y\ )$



HITROST: $\underline{v_x} = v_0 \cdot \cos\vartheta = 20\,m/s \cdot \frac{1}{2} = \underline{10\,m/s}$

$v = \sqrt{v_x^2+v_y^2}$

$v_y = v_0 \cdot \sin\vartheta - g t =$

$= 20\,m/s\,\frac{\sqrt{3}}{2} - 10\,\frac{m}{\cancel{s^2}} \cdot 3\,\cancel{s} = \underline{-12{,}7\,m/s}$

$\tan\gamma = \frac{|v_y|}{|v_x|} = \frac{12{,}7}{10} \Rightarrow \gamma = 51{,}8^\circ$

$a_r = 6{,}3\,m/s^2,\ a_t = 7{,}5\,m/s^2,\ r = 39\,m$

ZBIRKA 9 nal 17/st 10

$\omega_0 = 6 s^{-1}$

$\alpha = konst. < 0$

$\omega_{10} = 0$

$\Delta t = t_5 - t_4 = ?$

$v^2 = v_0^2 + 2as$ ← PREMO GIBANJE ENAKOMERNO POSPEŠENO

$\omega^2 = \omega_0^2 + 2\alpha\varphi$ ← ENAKOMERNO POSPEŠENO KROŽENJE

N=10: $\omega_{10}^2 = \omega_0^2 + 2\alpha\varphi_{10}$; $\varphi_{10} = 10 \cdot 2\pi$

$0 = \omega_0^2 + 2\alpha\varphi_{10}$

$\alpha = -\frac{\omega_0^2}{2\varphi_{10}} = \frac{-\cancel{36}^{9}\, s^{-2}}{\cancel{2} \cdot 10 \cdot \cancel{2}\pi} = -\frac{9}{10}\frac{1}{\pi} s^{-2}$

N=4,5: $\omega_{4,5} = \omega_0 + \alpha t_{4,5}$; $\omega_{4,5}^2 = \omega_0^2 + 2\alpha\varphi_{4,5} \quad /\sqrt{\ }$

$\omega_{4,5} = \sqrt{\omega_0^2 + 2\alpha\varphi_{4,5}}$

$\sqrt{\omega_0^2 + 2\alpha\varphi_{4,5}} = \omega_0 + \alpha t_{4,5}$

$t_{4,5} = \frac{1}{\alpha}\left(\sqrt{\omega_0^2 + 2\alpha\varphi_{4,5}} - \omega_0\right)$

$t_4 = 4,72 s$

$t_5 = 6,13 s$ $\quad \Delta t = 1,4 s$

3 Newtonov zakon, sistemske sile, energija

ZBIRKA 9 nal 2/str 12

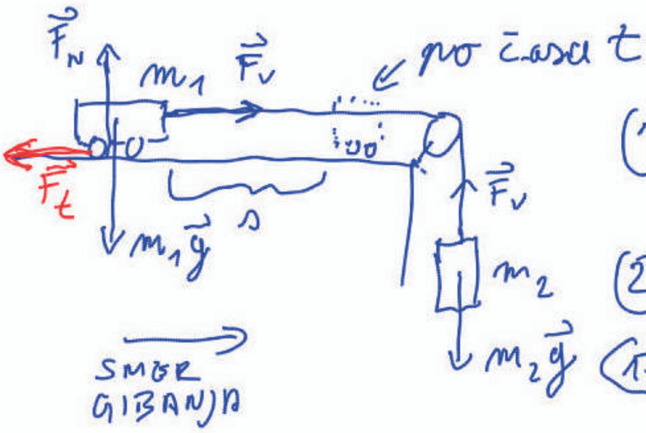
$m_1 = 1{,}2\ kg$

$m_2 = 0{,}4\ kg$

$s = 1\ m$

$t = 1{,}1\ s$

$F_t = ?$



(1) $\underbrace{F_v - F_t = m_1 a}$

$F_N - m_1 g = 0$

(2) $\underbrace{m_2 g - F_v = m_2 a}$

(1+2) $\Rightarrow m_2 g - F_t = (m_1 + m_2) a$

$F_t = m_2 g - (m_1 + m_2) a$

$F_t = 4N - 1{,}6\ kg \cdot 1{,}65\ \frac{m}{s^2}$

$\underline{F_t = 1{,}3\ N}$

ENAKOMERNO POSPEŠENO:

$s = \frac{a t^2}{2} \Rightarrow a = \frac{2s}{t^2} = \frac{2m}{1{,}21 s^2}$

$\underline{a = 1{,}65\ m/s^2}$

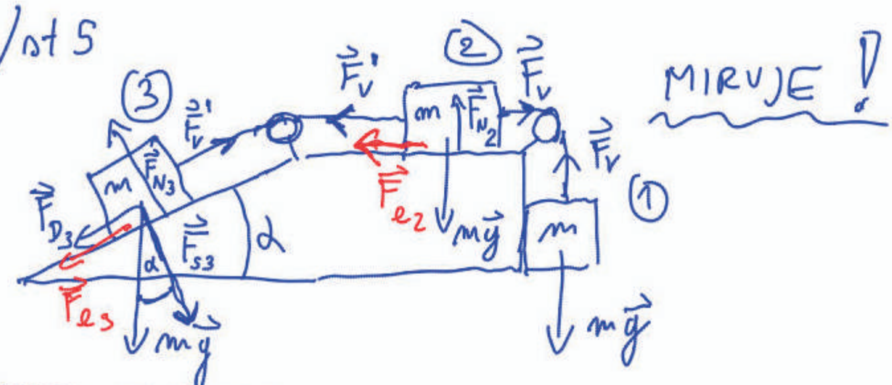
ZBIRKA 9 nal 1/st 5

$\alpha = 30^\circ$

$k_t = ?$

$\parallel$

k_e



V SMERI VRVICE:

(1) $F_V - mg = 0 \Rightarrow F_V = mg$

(2) $F_V' + F_{e_2} - F_V = 0$; $F_{e_2} = k_e \cdot F_{N2} = k_e \cdot mg$

$F_V' + k_e \cdot mg - mg = 0$

(3) $F_{D3} + F_{e_3} - F_V' = 0$; $F_{D3} = mg \cdot \sin\alpha$; $F_{N_3} = F_{S3}$

$mg(\sin\alpha + k_e \cos\alpha) - F_V' = 0$; $F_{e_3} = k_e \cdot F_{N_3} = k_e \cdot mg \cdot \cos\alpha$

$+ \quad (k_e - 1)\cancel{mg} + \cancel{mg}(\sin\alpha + k_e \cdot \cos\alpha) = 0$

$k_e(1 + \cos\alpha) - 1 + \sin\alpha = 0$

$k_e = \dfrac{1 - \sin\alpha}{(1 + \cos\alpha)} = \dfrac{1 - \frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} = \dfrac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$

$k_e = \dfrac{1}{2+\sqrt{3}} = 2 - \sqrt{3}$

ZBIRKA 9 nal 7/str 12

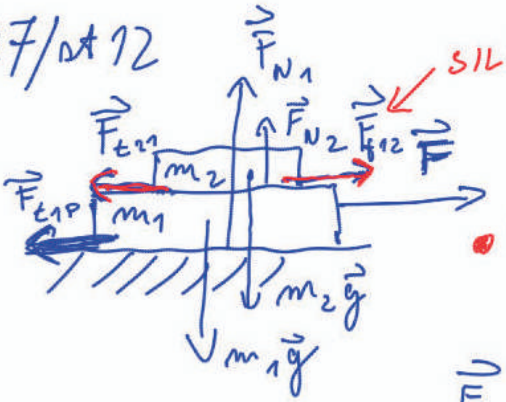
$m_1 = 2\,kg$

$m_2 = 1\,kg$

$k_{1P} = 0,4$

$k_{12} = 0,3$

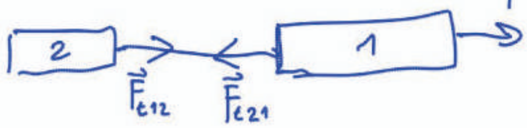
$F = ?$



SILA TRENJA TELESA 1 NA TELO 2

$\Downarrow$

• F_{t21} DELUJ V NASPROTNO SMER NA TELO 1



$|\vec{F}_{t21}| = |\vec{F}_{t12}|$

$\vec{F}_{t21} = -\vec{F}_{t12}$

(1) $F - F_{t1P} - F_{t21} = m_1 a_1$; $F_{t21} = F_{N2} \cdot k_{12} = m_2 g k_{12}$

$F_{t1P} = F_{N1} \cdot k_{1P} = (m_1 + m_2) g k_{1P}$

(2) $F_{t12} = m_2 a_2$

$\Downarrow$

$$a = \frac{\cancel{m_2} g k_{12}}{\cancel{m_2}} = \underline{g k_{12}}$$

TIK PREDEN ZDRSNE

$a_1 = a_2 = a$

$F = m_1 g k_{12} + (m_1 + m_2) g k_{1P} + m_2 g k_{12}$

$F = (m_1 + m_2) g (k_{12} + k_{1P})$

$F = 3\,kg \cdot 10\,m/s^2 \cdot 0,7 = \underline{\underline{20,6\,N}}$

ZBIRKA 9   nal 18/st 7

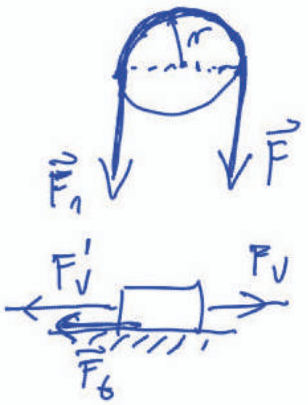
$r = 10\,cm$

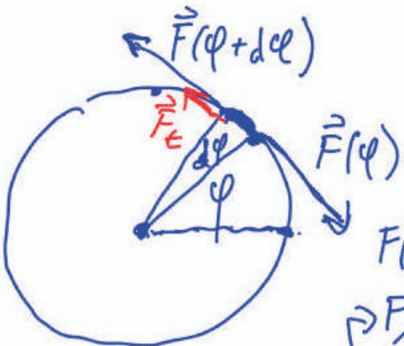
$F = 2000\,N$

$k_t = 0{,}6$

$F_1 = ?$

$M = ?$





$F(\varphi) = ?$

$F(\varphi+d\varphi)+F_t-F(\varphi)=0$

$\Rightarrow \cancel{F(\varphi)}+dF+F_t-\cancel{F(\varphi)}=0$

$\underline{F(\varphi+d\varphi)\sim F(\varphi)+dF}$

$dF+F_t=0$

$dF + k_t F\, d\varphi = 0$

$dF = -k_t F\, d\varphi$

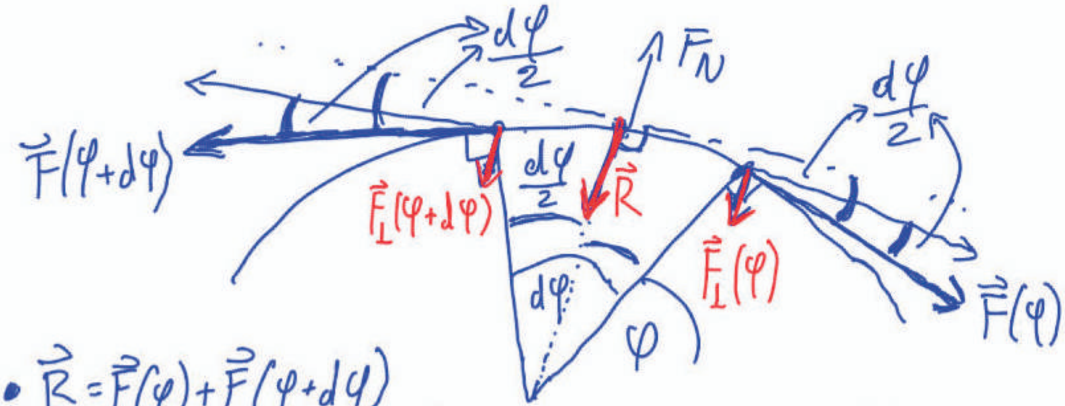
$$\int_{F_0}^{F_1}\frac{dF}{F} = -k_t\int_0^{\varphi_1} d\varphi$$

$$\ln\frac{F_1}{F_0} = -k_t\varphi_1$$

$$F_1 = F_0\, e^{-k_t\varphi_1}$$

$$= 2000\,N\cdot e^{-0{,}6\cdot\pi}$$

$$F_1 = 304\,N$$



- $\vec{R} = \vec{F}(\varphi)+\vec{F}(\varphi+d\varphi)$

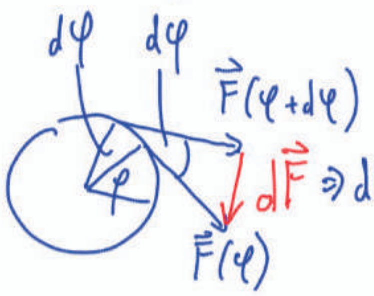
  $\vec{R} \sim \vec{F}_\perp(\varphi)+\vec{F}_\perp(\varphi+d\varphi)$

ZA MAJHEN $d\varphi$

- $F_\perp(\varphi+d\varphi) = F(\varphi+d\varphi)\cdot\sin\frac{d\varphi}{2} = [F(\varphi)+dF]\cdot\frac{d\varphi}{2} = F(\varphi)\cdot\frac{d\varphi}{2} + \underbrace{dF\frac{d\varphi}{2}}_{0}$
- $F_\perp(\varphi) = F(\varphi)\cdot\sin\frac{d\varphi}{2} = F(\varphi)\frac{d\varphi}{2}$

$\Rightarrow R = \cancel{2}F(\varphi)\frac{d\varphi}{\cancel{2}}$

$R = F_N \Rightarrow \underline{F_t = k_t\cdot F(\varphi)\,d\varphi}$



$\Rightarrow dF = F(\varphi)\sin d\varphi$

(3) nal 6.

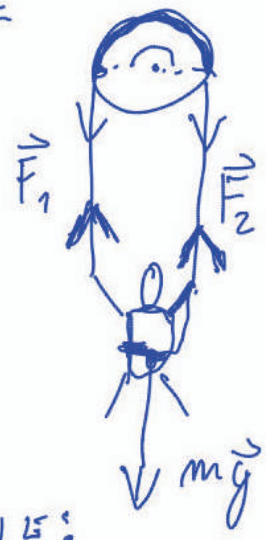
$m, k_t = k_e$

a) $F_{min} = ?$

$F_{max} = ?$

b) $a_{dviga} = ?$

$a_{spust} = ?$



IZ PREJŠNJE NALOGE:

$$\boxed{F_1 = F_2 \cdot e^{-k \cdot \varphi}}$$

$$\underline{F_1 = F_2\, e^{-k \cdot \pi}}$$

a) MIRUJE

$$\underline{F_1 + F_2 = mg}$$

$$F_2 (e^{-k\pi} + 1) = mg$$

$$\underline{F_2 = mg/(1 + e^{-k\pi}) > F_1}$$

$$F_2 = F_{20}$$

$$\boxed{\begin{aligned} F_2 &= F_{max} \\ F_1 &= F_{min} \end{aligned}}$$

b) SE GIBLJE:

$$F_1 + F_2 - mg = ma$$

$$F_2 (e^{-k\pi} + 1) - mg = ma$$

$$a = \frac{F_2}{m} (e^{-k\pi} + 1) - g$$

ZA GIBANJE GOR:

$$F_2 = F_{20} + F_2'$$

F_{20}: ZA RAVNOVESJE

F_2': PRESEŽEK ZA GIBANJE NAVZGOR

$$a = \Big(\overbrace{\frac{mg}{(1 + e^{-k\pi})}}^{F_{20}} + F_2'\Big) \cdot \frac{(1 + e^{-k\pi})}{m} - g$$

$$a = \cancel{g} + \frac{F_2' (1 + e^{-k\pi})}{m} - \cancel{g}$$

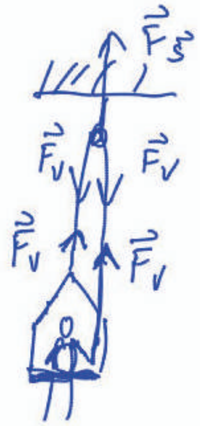
$$\underline{a = F_2' \frac{(1 + e^{-k\pi})}{m}}$$

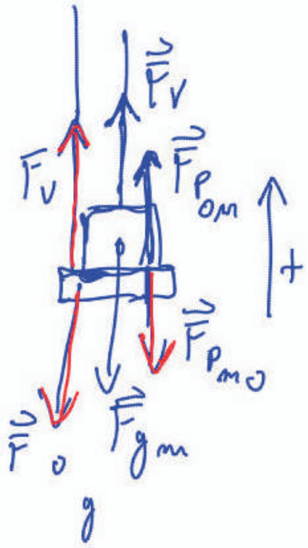
ZA GIBANJE DOL → UPORAB F_1 → $F_1 = F_{10} - F_1'$

90/91 1. kol, 2. mol

$F_p = 500 N$
$m_o = 30 kg$
$m_m = 90 kg$

$a = ?$
$F_{\check{S}} = ?$





SEŠTEJEMO

$$F_{\check{S}} = 2F_V = (m_m + m_o)(a+g) = \underline{\underline{2000 N}}$$

SILE NA MOŽA:

$$F_V + F_P - m_m g = m_m a$$

SILE NA ODER

$$F_V - m_o g - F_P = m_o a$$

ODŠTEJEMO:

$$F_P - m_m g + m_o g + F_P = (m_m - m_o) a$$

$$a = \frac{2F_P - (m_m - m_o) g}{(m_m - m_o)}$$

$$a = \frac{2F_P}{(m_m - m_o)} - g$$

$$\underline{\underline{a = 6{,}67 m/s^2}}$$

92/93 1.kol/nal 2.

$m_1 = 100\,g$

$d = 0.2\,m$

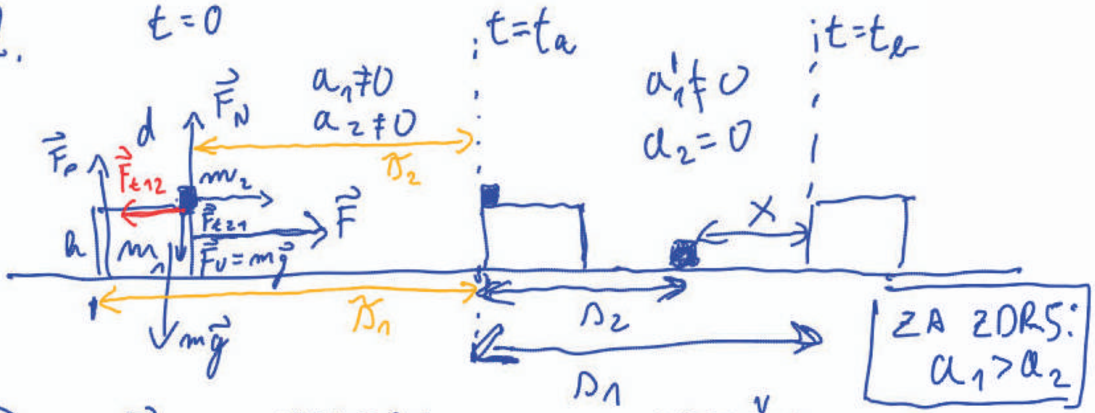
$h = 0.2\,m$

$m_2 = 100\,g$

$k_t = 0.5$

$F = 2\,N$

$x = ?$



$\vec{F}_{t12} = -\vec{F}_{t21}$

$F_{t12} = F_{t21} = F_t$

$F_t = k_t \cdot m_2 g$

KLADA:

$F - F_t = m_1 a_1$

$F - k_t m_2 g = m_1 a_1$

$a_1 = \frac{F}{m_1} - k_t g$

$a_1 = 20\frac{m}{s^2} - 5\frac{m}{s^2}$

$a_1 = 15\,m/s^2$

UTEŽ:

$F_t = m_2 a_2$

$k_t m_2 g = m_2 a_2$

$a_2 = k_t \cdot g = 5\frac{m}{s^2}$

$t_a \to t_b$:

UTEŽ → POSPEŠENO PADA

→ VODORAVNO $v_2 = konst$

$v_2 = a_2 \cdot t_a = 1\,m/s$

$s_2 = v_2 (t_b - t_a)$

KLADA:

$s_1 = v_1 (t_b - t_a) + \frac{a_1' (t_b - t_a)^2}{2}$

$v_1 = a_1 \cdot t_a = 3\,m/s$ | $F = m a_1'$

t_a: $\tilde{s}_1 - \tilde{s}_2 = d$ $\quad \tilde{s}_1 = a_1 \frac{t_a^2}{2}$ $\quad \tilde{s}_2 = a_2 \frac{t_a^2}{2}$

$\to t_a = \frac{2d}{(a_1 - a_2)} = 0.2\,s$

$t_a \to t_b$ ⇒ UTEŽ PROSTO PADA: $h = g\frac{(t_b - t_a)^2}{2} \Rightarrow (t_b - t_a) = 0.2\,s$

$s_2 = 0.2\,m$, $s_1 = 1\,m \Rightarrow s_1 - s_2 = 0.8\,m$

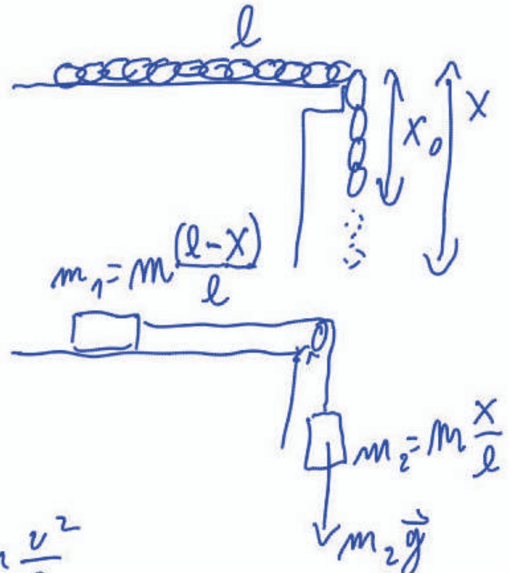
③ nal. 8

$$\frac{l, x_0}{}$$

$v(x) = ?$

$x(t) = ?$

$t_{\text{KONČNI}} = ?$



$$\cancel{m}\, a = \cancel{m}\frac{x}{l}\cdot g$$

$$a = \frac{x}{l}g \qquad a = \frac{dv}{dt}\frac{dx}{dx} \;\; \text{,,}v\text{''}$$

$$v\frac{dv}{dx} = \frac{x}{l}\cdot g$$

$$\int_0^v v\,dv = \frac{g}{l}\int_{x_0}^{x} x\,dx$$

$$\frac{v^2}{\cancel{2}} = \frac{g}{l}\left(\frac{x^2}{\cancel{2}} - \frac{x_0^2}{\cancel{2}}\right)$$

$$\underline{v = \sqrt{\frac{g}{l}\left(x^2 - x_0^2\right)}}$$

$$\Delta W_k = m\frac{v^2}{2}$$

$$\Delta W_p = m_2 g h_k - m_2 g h_z$$

---

x(t):

$$v = \frac{dx}{dt}$$

$$\sqrt{\frac{g}{l}}\sqrt{x^2 - x_0^2} = \frac{dx}{dt}$$

$$\int_0^t dt = \sqrt{\frac{l}{g}}\int_{x_0}^{x}\frac{dx}{\sqrt{x^2 - x_0^2}}$$

$$\int_0^t dt = \sqrt{\frac{l}{g}}\int_{\operatorname{arch} 1}^{\operatorname{arch}\frac{x}{x_0}}\frac{x_0\,\mathrm{sh}\,u\,du}{\sqrt{x_0^2\,\mathrm{ch}^2 u - x_0^2}}$$

$$t = \sqrt{\frac{l}{g}}\int_{\operatorname{arch}1}^{\operatorname{arch}\frac{x}{x_0}}\frac{\cancel{\mathrm{sh}\,u}\cdot du}{\cancel{\sqrt{\mathrm{ch}^2 u - 1}}}$$

$$\mathrm{ch}\,x = \tfrac{1}{2}\left(e^{x} + e^{-x}\right) \leftarrow \text{VERIŽNICA}$$

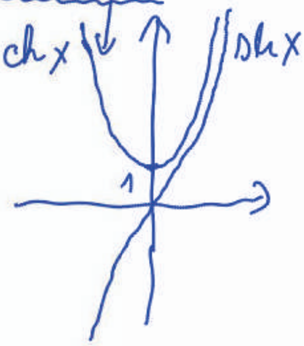
$$\mathrm{sh}\,x = \tfrac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\mathrm{ch}'x = \mathrm{sh}\,x$$

$$\mathrm{sh}'x = \mathrm{ch}\,x$$

$$\underline{\mathrm{ch}^2 x - \mathrm{sh}^2 x = 1}$$

$$\hookrightarrow \underline{\mathrm{sh}^2 x = \mathrm{ch}^2 x - 1}$$



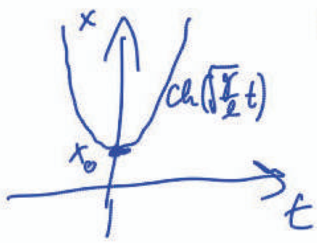
$$\begin{cases} x = x_0\,\mathrm{ch}\,u \Rightarrow u = \operatorname{arch}\frac{x}{x_0} \\ dx = x_0\,\mathrm{sh}\,u\,du \end{cases}$$

$$t = \sqrt{\frac{l}{g}}\int_{\operatorname{arch}1}^{\operatorname{arch}\frac{x}{x_0}} du = \sqrt{\frac{l}{g}}\left(\operatorname{arch}\frac{x}{x_0} - \underbrace{\operatorname{arch}1}_{=0}\right) \Rightarrow \underline{t = \sqrt{\frac{l}{g}}\operatorname{arch}\frac{x}{x_0}}$$

$$\sqrt{\frac{g}{l}}\cdot t = \operatorname{arch}\frac{x}{x_0} \;\Big/\; \text{INVERZ}$$

$$\mathrm{ch}\left(\sqrt{\frac{g}{l}}\,t\right) = \frac{x}{x_0}$$

$$\underline{x = x_0\,\mathrm{ch}\left(\sqrt{\frac{g}{l}}\cdot t\right)}$$



$$\boxed{\begin{matrix}\text{KONČNI}\\ \text{ČAS}\end{matrix}} \quad \underline{t_k = \sqrt{\frac{l}{g}}\operatorname{arch}\frac{l}{x_0}}$$

DIF. ENAČBA: $a = \frac{g}{l} \cdot x$

$$\ddot{x} = \frac{g}{l} x$$

REŠUJEMO Z NASTAVKOM

$$x = A e^{\sqrt{\frac{g}{l}} \cdot t} + B e^{-\sqrt{\frac{g}{l}} t}$$

$$\dot{x} = A \sqrt{\frac{g}{l}} e^{\sqrt{\frac{g}{l}} t} - B \sqrt{\frac{g}{l}} e^{-\sqrt{\frac{g}{l}} t}$$

$$\ddot{x} = A \frac{g}{l} e^{\sqrt{\frac{g}{l}} t} + B \frac{g}{l} e^{-\sqrt{\frac{g}{l}} t} = \frac{g}{l} \cdot x$$

$A, B = \text{konst.}$

ZAČETNI POGOJI:

$t = 0$: • $x = x_0$

$$\to x_0 = A + B \to x_0 = 2A \Rightarrow A = B = \frac{x_0}{2}$$

• $\dot{x} = 0 \to 0 = A\sqrt{\frac{g}{l}} - B\sqrt{\frac{g}{l}} \Rightarrow A = B$

$$x = \frac{x_0}{2}\left(e^{\sqrt{\frac{g}{l}} t} + e^{-\sqrt{\frac{g}{l}} t}\right) = x_0 \operatorname{ch}\left(\sqrt{\frac{g}{l}} t\right)$$

(3) nal 11

$m_1 = 3\,kg$

$m_2 = 2\,kg$

$\alpha = 30°$

$k_{t1} = 0.2$

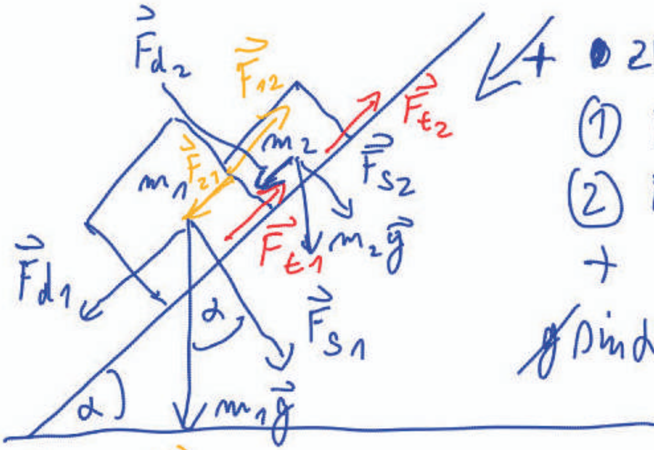
$k_{t2} = 0.05$

$k_{l1} = 0.25$

$k_{l2} = 0.1$

$\alpha = ?$

$F_{12} = ?$



$\vec{F}_{21} = -\vec{F}_{12}$

$F_{d1} = m_1 g \sin\alpha$

$F_{l1} = k_{l1} \cdot F_{s1} = k_{l1} m_1 g \cdot \cos\alpha$

$F_{d2} = m_2 g \sin\alpha$

$F_{l2} = k_{l2}\, m_2\, g \cos\alpha$

\+ • ZDRS → GLEDAMO LEPENJE

(1) $F_{d1} + F_{21} - F_{l1} = 0$

(2) $F_{d2} - F_{12} - F_{l2} = 0$

$+ \quad F_{d1} + F_{d2} = F_{l1} + F_{l2}$

$\cancel{g}\sin\alpha\,(m_1 + m_2) = \cancel{g}\cos\alpha\,(k_{l1} \cdot m_1 + k_{l2} \cdot m_2)$

$$\underbrace{\operatorname{tg}\alpha = \frac{k_{l1} m_1 + k_{l2} m_2}{m_1 + m_2}}$$

$$\underbrace{\alpha = 10.8°}$$

• KO DRSI → GLEDAMO TRENJE:

(1) $F_{d1} + F_{21} - F_{t1} = m_1 a \quad / \frac{1}{m_1}$

(2) $F_{d2} - F_{12} - F_{t2} = m_2 a \quad / \frac{1}{m_2}$

$$\frac{\cancel{m_1} g \sin\alpha}{\cancel{m_1}} + \frac{F_{21}}{m_1} - \frac{\cancel{m_1} g \cos\alpha \cdot k_{t1}}{\cancel{m_1}} = a$$

$$\frac{\cancel{m_2} g \sin\alpha}{\cancel{m_2}} - \frac{F_{12}}{m_2} - \frac{\cancel{m_2}\, g \cos\alpha\, k_{t2}}{\cancel{m_2}} = a$$

$F_{12} = F_{21}$

ODŠTEJEMO: $F_{12}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - g\cos\alpha\,(k_{t1} - k_{t2}) = 0$

$$\underbrace{F_{12} = g\cos(k_{t1} - k_{t2})\,\frac{1}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}}$$

$$\underline{\underline{F_{12} = 1{,}6\,N}}$$

ČETRTEK 5.11. OB 8:30 – 10:00

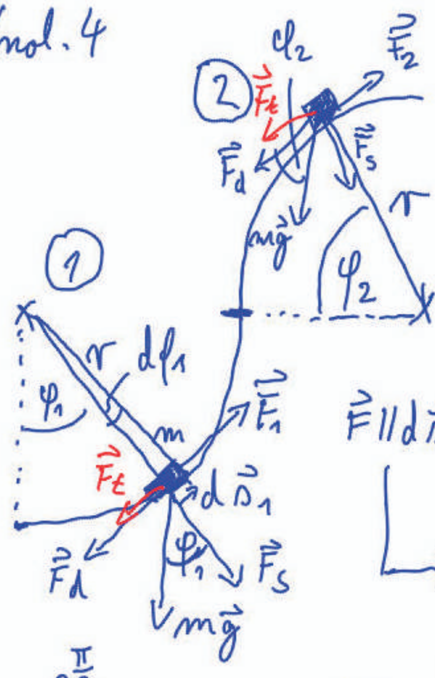
09/10 kol 1./nal. 4

$m = 2\ kg$

$r = 20\ m$

$k_t = 0{,}3$

$A = ?$



POČASNO = ENAKOMERNO

① $F - F_t - F_d = 0$

$\boxed{F_1 = mg(k_t \cdot \cos\varphi_1 + \sin\varphi_1)}$

② $\boxed{F_2 = mg(k_t \cdot \sin\varphi_2 + \cos\varphi_2)}$

$\vec{F} \parallel d\vec{s}$

$ds_1 = r\, d\varphi_1$

$A = \int \vec{F} d\vec{s}$

$$A_1 = \int F_1 ds_1 = mgr\int_0^{\frac{\pi}{2}} (k_t \cdot \cos\varphi_1 + \sin\varphi_1)\, d\varphi_1$$

$$= mgr\left(k_t \sin\varphi_1\Big|_0^{\pi/2} - \cos\varphi_1\Big|_0^{\pi/2}\right)$$

$$\boxed{A_1 = mgr(k_t + 1)}$$

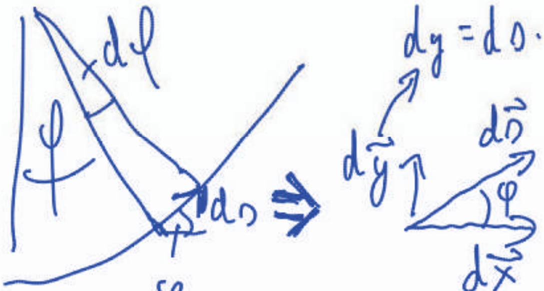
$$\boxed{A_2} = \int F_2 ds_2 = mgr\int_0^{\frac{\pi}{2}} (k_t \sin\varphi_2 + \cos\varphi_2)\, d\varphi_2$$

$$= mgr\left(-k_t \cos\varphi_2\Big|_0^{\pi/2} + \sin\varphi_2\Big|_0^{\pi/2}\right)$$

$$\boxed{A_2 = mgr(k_t + 1)}$$

$$\underline{A} = A_1 + A_2 = \underline{2mgr(k_t + 1)}$$

$$= \underline{1{,}04\ kJ}$$

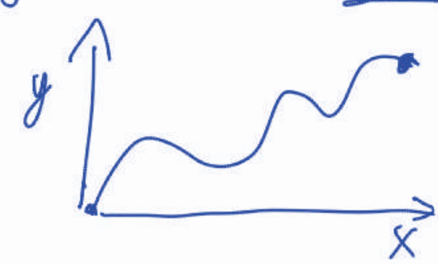


$dy = ds \cdot \sin\varphi$

$dx = ds \cos\varphi$

$$A = \int mg(k_t \cos\varphi_1 + \sin\varphi_1)\, ds$$

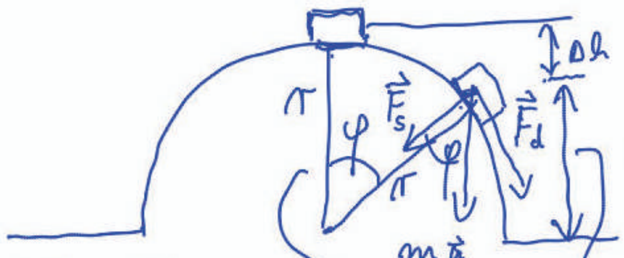
$$= mg\int_0^{2r} (k_t \cdot dx + dy) = \underline{2mgr(k_t + 1)}$$



3 nal 13

$\varphi = ?$

$h_f = 0$



KJE SE ODLEPI ?

$F_s = m a_r$; $a_r = \frac{v^2}{r}$

$\omega^2 r$

$\cancel{m} g \cos\varphi = \cancel{m} \frac{v^2}{r}$

$\boxed{g \cos\varphi = \frac{v^2}{r}}$

ENERGIJE:

$\cancel{m} g \Delta h = \cancel{m} \frac{v^2}{2}$; $\Delta h = r - r\cos\varphi$

$g r(1-\cos\varphi) = \frac{v^2}{2} \longleftarrow \Delta h = r(1-\cos\varphi)$

$\boxed{v^2 = 2 g r (1-\cos\varphi)}$

$\cancel{g} \cos\varphi = \frac{2 \cancel{g} \cancel{r} (1-\cos\varphi)}{\cancel{r}}$

$3\cos\varphi = 2$

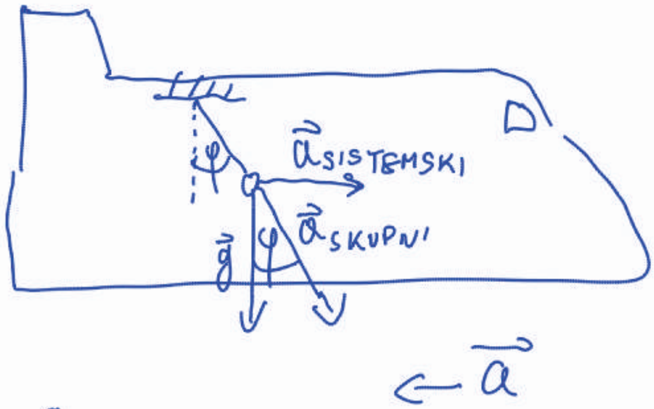
$\cos\varphi = \frac{2}{3} \Rightarrow \varphi = 48^\circ$

ZBIRKA 9 zad. 12/st. 13

$a = -0{,}7\,m/s^2$

$l = 1\,m$

$\varphi = ?$



$$tg\,\varphi = \frac{a_{SISTEMSKI}}{g} = \frac{0{,}7}{10} \rightarrow \varphi = 4{,}1^\circ$$

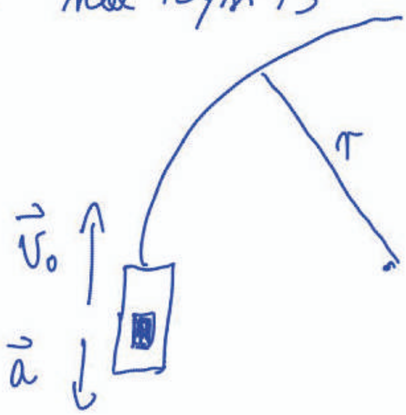
ZBIRKA 9 nal 15/st 13

$a = -g/2$

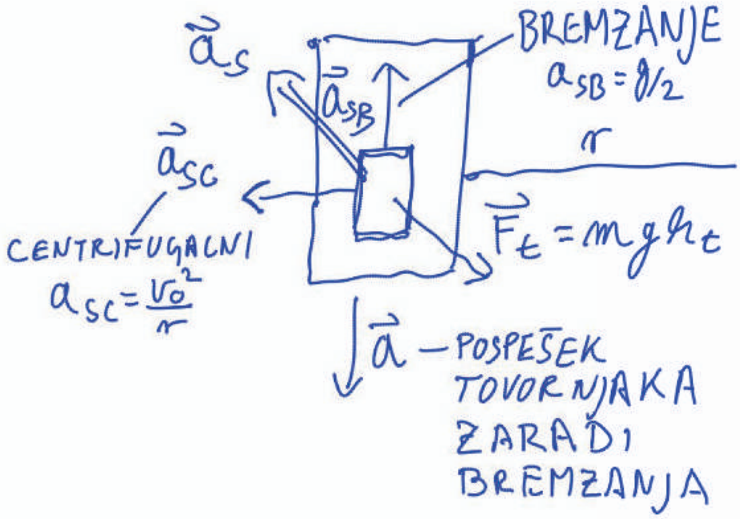
$r = 20\,m$

$k_t = 0.6$

$v_0 = ?$



V SISTEMU TOVORNJAKA:



POSPEŠEK, KI GA ČUTI KLADA:

$$\vec{a}_s = \vec{a}_{SB} + \vec{a}_{SC}$$

$$a_s = \sqrt{a_{SB}^2 + a_{SC}^2}$$

$$a_s = \sqrt{\frac{g^2}{4} + \frac{v_0^4}{r^2}}$$

$$a_s^2 - \frac{g^2}{4} = \frac{v_0^4}{r^2}$$

$$r^2 g^2 \left(k_t^2 - \frac{1}{4}\right) = v_0^4 \Rightarrow v_0 = \sqrt[4]{r^2 g^2 \left(k_t^2 - \frac{1}{4}\right)}$$

$$v_0 = 8{,}14\ m/s$$

TRENJE:

$$\cancel{m} a_s = \cancel{m} g k_t$$

$$a_s = g k_t$$

③ nal 15

h

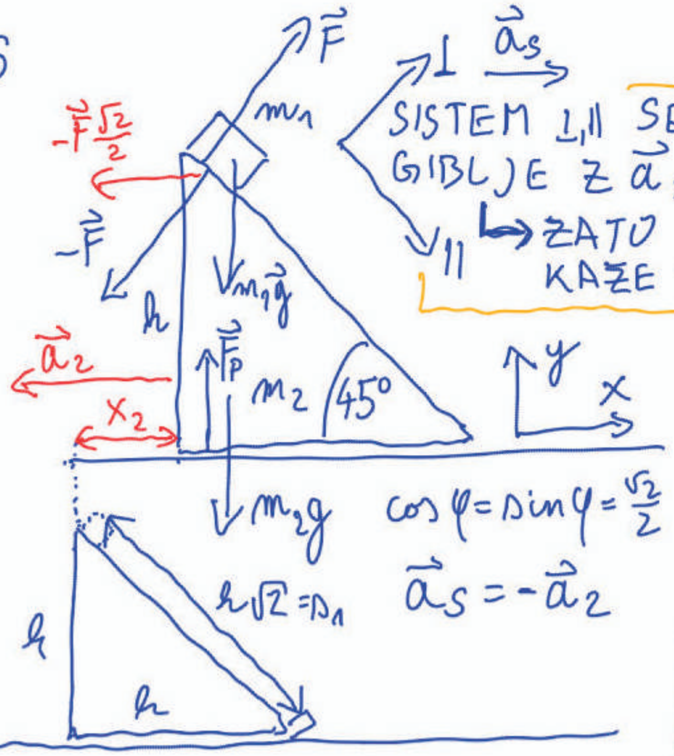
$h_f = 0$

$t = ?$

$x_2 = ?$

$m_2 \gg m_1$

$m_2 \ll m_1$



$\cos\varphi = \sin\varphi = \frac{\sqrt{2}}{2}$

$\vec{a}_S = -\vec{a}_2$

$\perp$ $\vec{a}_S \rightarrow$

SISTEM $\perp,\parallel$ SE GIBLJE Z $\vec{a}_2$ LEVO

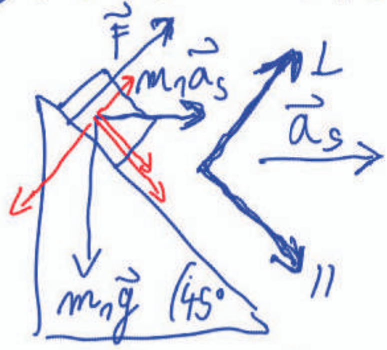
$\parallel$ ↳ ZATO $\vec{a}_S$ KAŽE DESNO

$\Rightarrow \boxed{a_S = -a_2 = \frac{\sqrt{2}}{2}\frac{F}{m_2}}$

② $X: -F\cdot\frac{\sqrt{2}}{2} = m_2 a_2$

$\boxed{F = -\sqrt{2}\, m_2 a_2}$

① V SISTEMU $\perp,\parallel$:



$\parallel: m_1 a_S\frac{\sqrt{2}}{2} + m_1 g\frac{\sqrt{2}}{2} = m_1 a_{1\parallel}$

$\perp: F + m_1 a_S\frac{\sqrt{2}}{2} - m_1 g\frac{\sqrt{2}}{2} = 0$

$F_{②}, \perp_{①}: \cancel{\sqrt{2}}\, m_2 a_S + m_1 a_S\frac{\cancel{\sqrt{2}}}{2} - m_1 g\frac{\cancel{\sqrt{2}}}{2} = 0$

$a_S\left(m_2 + \frac{m_1}{2}\right) = m_1 g\frac{1}{2}$

$$\boxed{a_S = g\frac{m_1}{(2m_2 + m_1)} = g\frac{1}{(1+2\gamma)};\ \gamma = \frac{m_2}{m_1}}$$

$a_S, \parallel: \cancel{m_1} g\frac{\sqrt{2}}{2}\frac{1}{(1+2\gamma)} + \cancel{m_1} g\frac{\sqrt{2}}{2} = \cancel{m_1} a_{1\parallel}$

$g\frac{\sqrt{2}}{2}\,\frac{1+1+2\gamma}{1+2\gamma} = a_{1\parallel}$

$$\boxed{g\sqrt{2}\,\frac{1+\gamma}{1+2\gamma} = a_{1\parallel}}$$

POT KLADE:

$s_1 = h\sqrt{2} = a_{1\parallel}\frac{t^2}{2}$

ČAS: ↓

$t = \sqrt{\frac{2\sqrt{2}\,h}{a_{1\parallel}}}$

$t = \sqrt{\frac{2\cancel{\sqrt{2}}\,h(1+2\gamma)}{g\cancel{\sqrt{2}}(1+\gamma)}}$

$$\boxed{t = \sqrt{\frac{2h}{g}\left(1 + \frac{\gamma}{1+\gamma}\right)};\ \gamma = \frac{m_2}{m_1}}$$

$m_2 \gg m_1: \gamma \to \infty: \underline{t = \sqrt{\frac{4h}{g}}}$

$m_2 \ll m_1: \gamma \to 0: \underline{t = \sqrt{\frac{2h}{g}}}$

POT KLANCA: x_2

$$\boxed{x_2 = a_2\frac{t^2}{2} = \cancel{g}\frac{1}{\cancel{(1+2\gamma)}\cancel{2}}\,\frac{\cancel{2}h}{\cancel{g}}\,\frac{\cancel{(1+2\gamma)}}{(1+\gamma)} = \boxed{\frac{h}{(1+\gamma)}}}$$

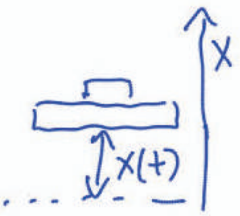
$\gamma \to \infty: \underline{x_2 = 0}$

$\gamma \to 0: \underline{x_2 = h}$

ZBIRKA 9 nal 10/st. 12

$\nu = 4 s^{-1}$

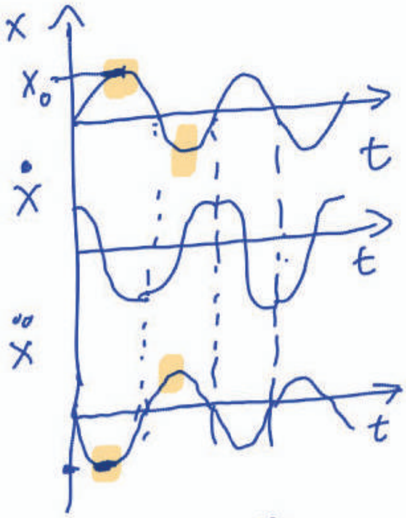
$x = X_0 \sin(\omega t)$



$x = x_0 \sin(\omega t)$

$\dot{x} = \omega x_0 \cos(\omega t)$

$\ddot{x} = -\omega^2 x_0 \sin(\omega t) = -\omega^2 x$

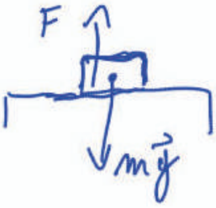


$X_0 = ?$

↑

DA SE UTEŽ ODLEP

ODLEPI SE KO $|\ddot{x}| = max$



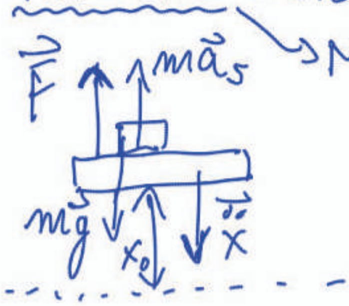
- GREMO V SISTEM UTEŽI: $\vec{a}_s = -\ddot{\vec{x}}$

V NASPROTNI SMERI OD $\ddot{\vec{x}}$

NA VRHU KAŽE $\ddot{\vec{x}}$ NAVZDOL $(\ddot{x} = -\omega^2 x)$

→ MAKSIMUM $|\ddot{x}| = \omega^2 x_0$

$a_s = \omega^2 x_0$



$\Rightarrow F + ma_s - mg = 0$

DA SE ODLEPI:

$F = 0 \Rightarrow$ NE ČUTI PODLAGE

$a_s = g$

$\omega = 2\pi\nu$

$\omega^2 x_0 = g$

$x_0 = \frac{g}{\omega^2} = \frac{g}{4\pi^2\nu^2} = \underline{\underline{1{,}6\ cm}}$

ZBIRKA 9 nal 24/st 14

VODA ČUTI CENTRIFUGALNO SILO
SISTEMSKO

SISTEMSKI POSPEŠEK:

$$a_s = \omega^2 r$$

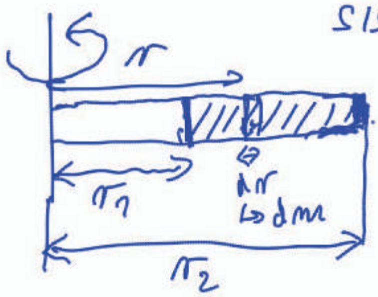
$m = 100\,g$

$\nu = 120\,s^{-1}$

$r_1 = 8\,cm$

$r_2 = 15\,cm$

$F = ?$



$$dm = m\frac{dr}{(r_2 - r_1)} \Rightarrow dF = a_s \cdot dm$$

$$\int_0^F dF = \frac{\omega^2 m}{(r_2 - r_1)}\int_{r_1}^{r_2} r\,dr$$

$$F = \frac{\omega^2 m}{(r_2 - r_1)}\left(\frac{r_2^2 - r_1^2}{2}\right)$$

$r_2^2 - r_1^2 = (r_2 - r_1)(r_2 + r_1)$

$\omega = 2\pi\nu$

$$F = \frac{\omega^2 m}{2}(r_2 + r_1)$$

$$F = 2\pi^2\nu^2 m(r_2 + r_1) = \underline{\underline{6{,}5 \cdot 10^3\,N}}$$

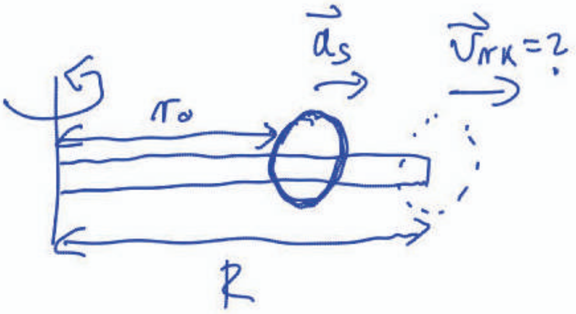
ZBIRKA 9 nal 23/st 14

$R = 1\,m$

$r_0 = 70\,cm$

$\omega = 0.7\,s^{-1}$

$v_{rK} = ?$



• $\vec{a}_s = \omega^2 \vec{r} \parallel v_r$

• $a_s = \frac{dv_r}{dt}\frac{dr}{dr} = v_r \frac{dv_r}{dr}$

$$v_r \frac{dv_r}{dr} = \omega^2 r$$

$$\int_0^{v_{rK}} v_r \, dv_r = \omega^2 \int_{r_0}^{R} r \, dr$$

$$\frac{v_{rK}^2}{\cancel{2}} = \omega^2 \frac{R^2 - r_0^2}{\cancel{2}}$$

$$v_{rK} = \omega \sqrt{R^2 - r_0^2}$$

$$\boxed{v_{rK} = 0{,}5\,m/s}$$

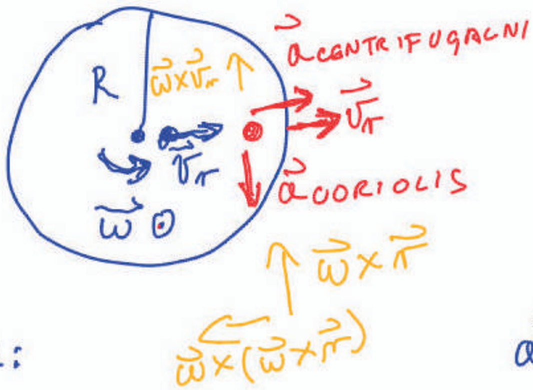
3 nal 19

$R = 2\,m$

$\omega = 0.5\,s^{-1}$

$v_r = 1\,m/s$

$k_l = ?$



POSPEŠEK V ROTIRAJOČEM S.

$$\vec{a} = \vec{a}_i - \underbrace{2\vec{\omega}\times\vec{v}}_{\text{CORIOLIS}} - \underbrace{\vec{\omega}\times(\vec{\omega}\times\vec{r})}_{\substack{\text{CENTRIFUGALNI} \\ \omega^2 r}}$$

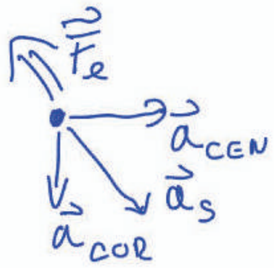
↳ MEJNI PRIMER: $r = R$

• LEPENJE:

$F_l = m a_s$

$\cancel{m} g k_l = \cancel{m} a_s$

$\boxed{a_s = k_l g}$

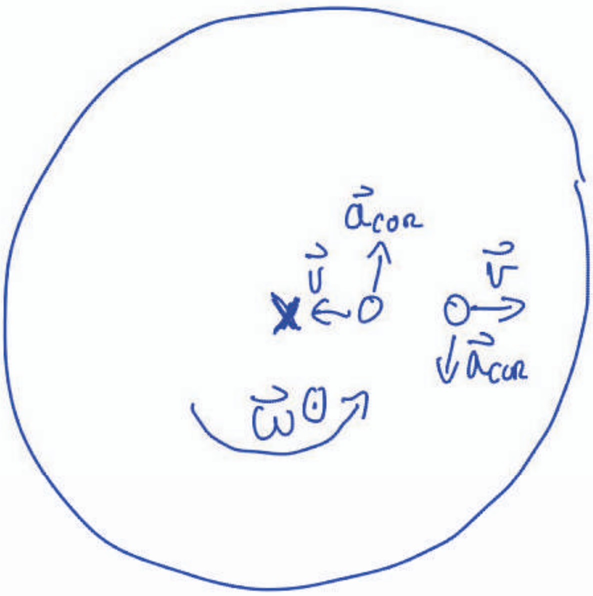


$$a^2_{SKUPNI} = a^2_{CORIOLIS} + a^2_{CENTRI}$$

$$a_s = \sqrt{(2\omega v_r)^2 + (\omega^2 R)^2}$$

$$k_l = \frac{\omega}{g}\sqrt{4 v_r^2 + \omega^2 R^2}$$

$$\boxed{k_l = 0{,}11}$$



ZBIRKA 9: nal 19/st 13

$\vartheta = 30°$

$h = 125\,m$

$x = ?$

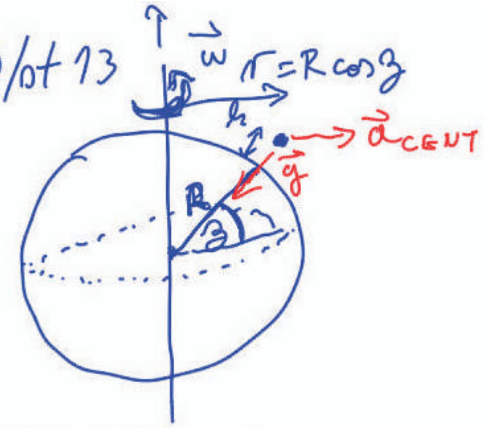
$\omega = 2\pi\nu = \frac{2\pi}{t_0}$

$t_0 = 1\,dan$

$= 24 \cdot 3600\,s$

$R = 6400\,km$

$\cos\vartheta = \frac{\sqrt{3}}{2}$



VRTENJE ZEMLJE:
OD ZAHODA PROTI VZHODU!

- $\vec{a}_{CENT} = -\vec{\omega} \times (\vec{\omega} \times \vec{r})$

$a_{CENT} = \omega^2 \cdot R\cos\vartheta$

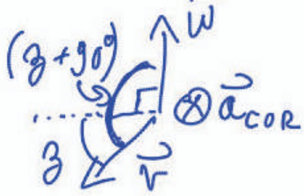
$= \frac{4\pi^2 \; 6400\,000\,m}{(24 \cdot 3600)^2 \cdot s^2} \cdot \frac{\sqrt{3}}{2}$

$= 0{,}03\,m/s^2$

$\Downarrow$

$\{ a_{CENT} \ll g \Rightarrow \vec{v} \parallel \vec{g} \}$

$\hookrightarrow \vec{v} = \vec{g} \cdot t$



- $\vec{a}_{COR} = -2\vec{\omega} \times \vec{v}$

$a_{COR} = 2\omega v \sin(\vartheta + 90°)$

$= 2\omega v \cos\vartheta$

$= 2\omega g t \cos\vartheta$

- ČAS: $h = \frac{g t^2}{2}$

$t = \sqrt{\frac{2h}{g}}$

- HITROST:

$v_t = \int_0^t a_{COR}\, dt = \cancel{2}\omega g \cos\vartheta \frac{t^2}{\cancel{2}}$

- POT:

$x = \int_0^t v_t\, dt = \omega g \cos\vartheta \frac{t^3}{3}$

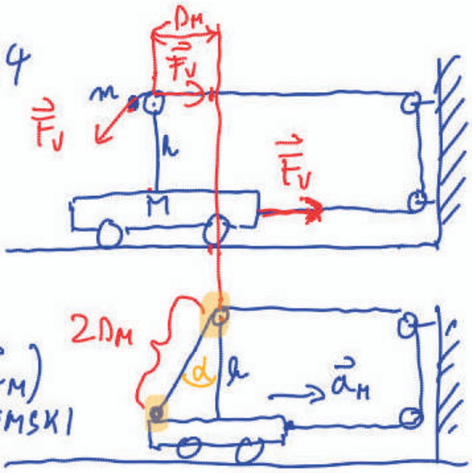
$x = \frac{\omega g \cos\vartheta}{3} \left(\frac{2h}{g}\right)^{3/2} = 2{,}62\,cm$

94/95 1.kol/nal 4

$M = 300\,g$
$m = 150\,g$
$h = 20\,cm$

$t = ?$

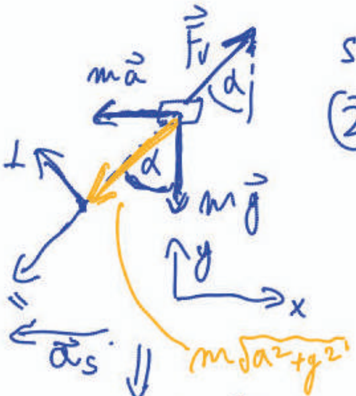
• POSPEŠENI SISTEM VOZIČKA ($\vec a = -\vec a_M$) SISTEMSKI



SILE NA VOZIČEK:

X: $F_v + F_v - F_v \sin\alpha = M a_M$

(1) $F_v(2-\sin\alpha) = M a_M$



SILE NA UTEŽ V S. VOZIČKA

(2) ||: $m\sqrt{a^2+g^2} - F_v = m\,2a$

$\to F_v = m\left(\sqrt{a^2+g^2}-2a\right)$

• POT VOZIČKA: $s_M \propto a_M$
• POT UTEŽI V SISTEMU VOZIČKA: $2s_M \propto 2a_M$

$tg\alpha = \frac{a}{g}$

$\sin\alpha = \frac{a}{\sqrt{a^2+g^2}}$ (3)

(1)+(3) $F_v\left(2-\frac{a}{\sqrt{a^2+g^2}}\right) = Ma$

(2) $\to m\left(\sqrt{a^2+g^2}-2a\right)\left(2-\frac{a}{\sqrt{a^2+g^2}}\right) = Ma$

$2\sqrt{a^2+g^2} - a - 4a + \frac{2a^2}{\sqrt{a^2+g^2}} = \frac{M}{m}a$

$2\sqrt{a^2+g^2}\left(1+\frac{a^2}{(a^2+g^2)}\right) = a\left(\frac{M}{m}+5\right) \;/^2$

$4(a^2+g^2)\left(1+\frac{2a^2}{(a^2+g^2)}+\frac{a^4}{(a^2+g^2)^2}\right) = a^2\left(\frac{M}{m}+5\right)^2$

$4\left(a^2+g^2+2a^2+\frac{a^4}{(a^2+g^2)}\right) = a^2\left(\frac{M}{m}+5\right)^2 \;/\cdot(a^2+g^2)$

$4\left(a^4+2a^2g^2+g^4+2a^4+2a^2g^2+a^4\right) = \left(a^4+a^2g^2\right)\left(\frac{M}{m}+5\right)^2$

$a^4\underbrace{\left[16-\left(\frac{M}{m}+5\right)^2\right]}_{C}+a^2\left[16-\left(\frac{M}{m}+5\right)^2\right]g^2+4g^4=0$

$C = 16-\left(\frac{M}{m}+5\right)^2 = -33 \longrightarrow a^2 = \frac{-Cg^2 \pm \sqrt{C^2g^4-16Cg^4}}{2C} = -\frac{g^2}{2}\left(1\pm\sqrt{1-\frac{16}{C}}\right)$

$a = 0{,}33\,g$ $\to$ $tg\alpha = \frac{a}{g} = 0{,}33 \Rightarrow \alpha = 18{,}3^\circ$

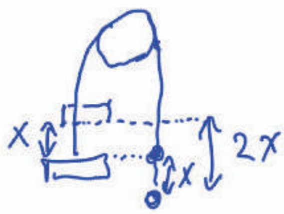
POT UTEŽI:

• $2s_M = \frac{h}{\cos\alpha}$

• $2s_M = \underbrace{2a}_{\text{POSPEŠEK UTEŽI}}\cdot\frac{t^2}{2}$

$\Rightarrow \frac{h}{\cos\alpha} = at^2 \Rightarrow t = \sqrt{\frac{h}{a\cos\alpha}} = \sqrt{\frac{h}{g\,tg\alpha\cdot\cos\alpha}} = \sqrt{\frac{h}{g\sin\alpha}}$

$t = 0{,}25\,s$



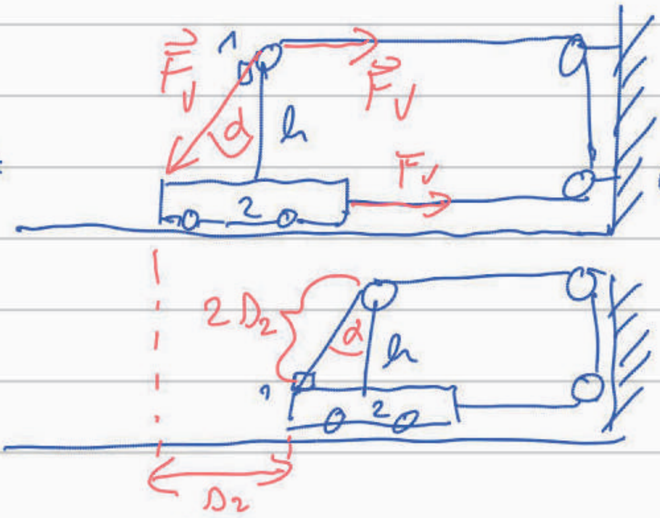
94/95 1.kol/nal 4

$m_1 = 150\,g$

$m_2 = 300\,g$

$h = 20\,cm$

$t = ?$



• SILE VOZIČEK:

X: $F_V + F_V - F_V \sin\alpha = m_2 a_2$

$\underline{F_V = m_2 a_2/(2-\sin\alpha)}$ (1)

• SILE UTEŽ V SISTEMU VOZIČKA: $(\vec{a}_s = -\vec{a}_2)$

X: $\underline{F_V \cdot \sin\alpha - m_1 a_s = m_1 \tilde{a}_{1x}}$ (2)

Y: $\underline{F_V \cdot \cos\alpha - m_1 g = m_1 \tilde{a}_{1y}}$ (3)

• POT: $s_2 = \frac{1}{2}\frac{h}{\cos\alpha}$, $s_2 = a_2 \cdot t^2/2$ $\Rightarrow \underline{a_2 = \frac{h}{\cos\alpha \cdot t^2}} = \underline{a_s}$ (5)

$h = -\tilde{a}_{1y}\, t^2/2 \Rightarrow \underline{\tilde{a}_{1y} = -\frac{2h}{t^2}}$ (4)

• KOT: $\underline{\operatorname{tg}\alpha = \frac{\tilde{a}_{1x}}{\tilde{a}_{1y}}}$ (6)

$\Rightarrow$ (1)+(2): $\sin\alpha \frac{m_2 a_s}{(2-\sin\alpha)} - m_1 a_s = m_1 \tilde{a}_{1x}$ $/:m_1$

$a_s\left(\frac{m_2}{m_1}\frac{\sin\alpha}{2-\sin\alpha} - 1\right) = \tilde{a}_{1x}$

(4),(5),(6) $\Rightarrow$ $\frac{\cancel{h}}{\cancel{\cos\alpha}\,\cancel{t^2}}\left(\frac{m_2}{m_1}\frac{\sin\alpha}{2-\sin\alpha} - 1\right) = -\frac{2\cancel{h}}{\cancel{t^2}}\frac{\sin\alpha}{\cancel{\cos\alpha}}$ $/\cdot(2-\sin\alpha)$

$\frac{m_2}{m_1}\sin\alpha - 2 + \sin\alpha + 4\sin\alpha - 2\sin^2\alpha = 0$

$-2\sin^2\alpha + \left(5 + \frac{m_2}{m_1}\right)\sin\alpha - 2 = 0$ $/\cdot(-1)$

$\sin\alpha = \frac{1}{4}\left(5 + \frac{m_2}{m_1} \pm \underbrace{\sqrt{\left(5+\frac{m_2}{m_1}\right)^2 - 4\cdot 4}}_{33}\right)$

$\sin\alpha = \frac{1}{4}\left(7 \pm \sqrt{33}\right) = 0.31$

$\underline{\alpha = 18{,}3^\circ}$

(1)+(3)+(4): $\frac{m_2 a_2}{(2-\sin\alpha)}\cdot\cos\alpha - m_1 g = -m_1\frac{2h}{t^2}$

(5) $\Rightarrow$ $\frac{m_2 \cdot h}{(2-\sin\alpha)}\frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha}\cdot t^2} - m_1 g = -m_1\frac{2h}{t^2}$ $/:m_1$

$g = \frac{h}{t^2}\left(\frac{m_2}{m_1}\frac{1}{(2-\sin\alpha)} + 2\right)$

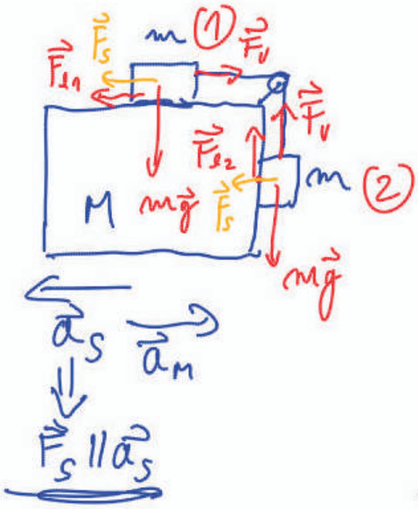
$t = \sqrt{\frac{h}{g}\left(\frac{m_2}{m_1}\frac{1}{(2-\sin\alpha)} + 2\right)}$

$\underline{\underline{t = 0{,}267\,s}}$

(3) nal 22

$k_e \rightarrow$ PODATEK

$a_M = ?$



ČIM MANJŠI $\vec{a}_M$

V SISTEMU VELIKE KLADE $(\vec{a}_s = -\vec{a}_M)$

$F_{e2} = m a_s \cdot k_e \rightarrow$ GOR

$F_{e1} = m g k_e \rightarrow$ LEVO

$(1): F_v - F_{e1} - m a_s = 0$

$(2): F_v + F_{e2} - mg = 0$

$- : -F_{e1} - m a_s - F_{e2} + mg = 0$

$-\cancel{m} g k_e - \cancel{m} a_s - \cancel{m} a_s k_e + \cancel{m} g = 0$

$a_s(1 + k_e) = g(1 - k_e)$

$a_s = g \dfrac{1 - k_e}{1 + k_e} < g$

③ nal 23

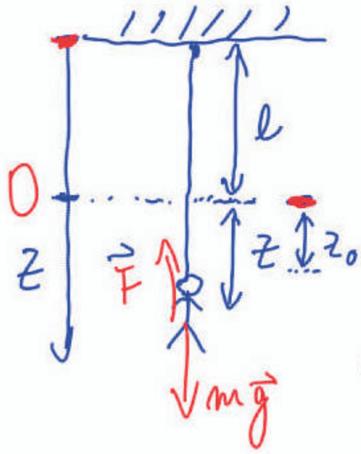
$l = 10\,m$

$k = 80\,N/m$

$m = 80\,kg$

- $z(v_{max}) = ?$

$z_{max} = ?$



- V RAVNOVESJU (TOČKA OKOLI KATERE NIHA)

$F = mg \quad ; \quad F = kz$

$k z_0 = mg$

$\boxed{z_0 = \frac{mg}{k}} = 10\,m$

NAJVEČJA HITROST

- NAJVEČJI RAZTEZEK (MIRUJE)

$|\Delta W_p| = |\Delta W_{pr}|$

$\hookrightarrow W_k = 0$

$mg(l + z_{max}) = \frac{1}{2} k z_{max}^2$

$\frac{1}{2} k z_{max}^2 - mg z_{max} - mgl = 0$

$z_{max} = \frac{mg \pm \sqrt{m^2g^2 + 2kmgl}}{k}$

$= \frac{mg}{k}\left(1 \pm \sqrt{1 + \frac{2kl}{mg}}\right)$

$= z_0 (1 \pm \sqrt{3}) = \begin{cases} 27\,m \\ -7\,m \end{cases}$

TO BI VELJALO ČE BI BILI PRIPETI NA VZMET PRI DVIGANJU GOR

NAŠA REŠITEV

NAJNIŽJA LEGA

- SPLOŠNO Z W:

$\Delta W = \Delta W_p + \Delta W_k + \Delta W_{pr} = 0$

$-mg(l+z) + \frac{mv^2}{2} + \frac{kz^2}{2} = 0$

$\hookrightarrow \boxed{v^2 = 2g(l+z) - \frac{k}{m} z^2}$

$v_{max} \Rightarrow \frac{dv}{dz} = 0 \Rightarrow z_0$

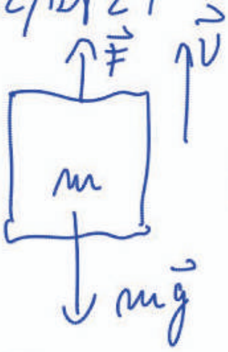
ZBIRKA 9 nal 2/str 21

$m = 500\,kg$

$F = 5200\,N$

$t = 10\,s$

$P = ?$



$$P = \frac{dA}{dt}$$

$$\underline{P = F \cdot v}$$

• SILE: $F - mg = ma$

$$\underline{a = \frac{F}{m} - g}$$

$\hookrightarrow a \neq a(t)$

• HITROST: $\underline{v = a \cdot t}$

$$P = F \cdot a t = F\left(\frac{F}{m} - g\right) \cdot t = 5200\,N\left(\frac{5200\,N}{500\,kg} - 10\,\frac{m}{s^2}\right)$$

$$\underline{P = 30{,}7\,kW}$$

$$[W] = \frac{Nm}{s} = \frac{J}{s}$$

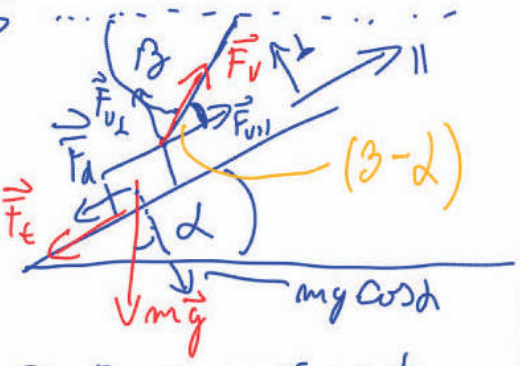
$$P = \frac{dA}{dt} = \frac{d(\int F\,dx)}{dt\,\frac{dx}{dx}} = v \cdot F$$

(3) nal 25

$N = 50$
$a = 1\,m/s^2$
$t_1 = 1\,s$
$t_2 = 2\,s$
$k_t = 0.1$
$\alpha = 30^\circ$
$\beta = 70^\circ$
$m = 80\,kg$

$P(t_1) = ?$
$P(t_2) = ?$

$P = \vec{F} \cdot \vec{v}$



• SILE:

$\parallel: \; \underline{F_V \cdot \cos(\beta-\alpha) - F_t - mg\sin\alpha = ma}$

• TRENJE

$\underline{F_t} = F_N \cdot k_t = (mg\cos\alpha - F_{V\perp})\, k_t$

$= \underline{(mg\cos\alpha - F_V \sin(\beta-\alpha))\, k_t}$

$\underline{P = F_{V\parallel}\, v_\parallel \cdot N} \; ; \; \underline{v_\parallel = at}$

$\rightarrow F_V\cos(\beta-\alpha) - [mg\cos\alpha - F_V\sin(\beta-\alpha)]\,k_t - mg\sin\alpha = ma$

$F_V[\cos(\beta-\alpha) + k_t\sin(\beta-\alpha)] = m[a + g\sin\alpha + g k_t\cos\alpha]$

$$F_V = \frac{m[a + g(\sin\alpha + k_t\cos\alpha)]}{[\cos(\beta-\alpha) + k_t\sin(\beta-\alpha)]} \neq F_V(t)$$

$\underline{F_V = 578{,}8\,N}$?

$P = F_V\cos(\beta-\alpha)\cdot at \cdot N \Rightarrow P(t_1) = 443{,}4 \cdot 50\,W$

$(t_2) = 2 \cdot 443{,}4 \cdot 50\,W$

(3) nal. 27.

m

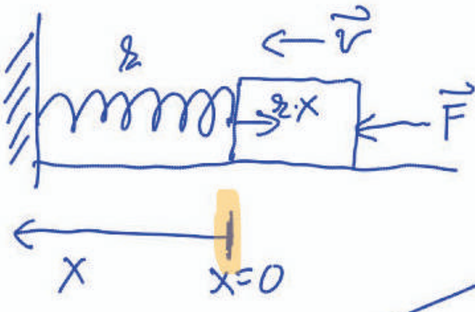
v_0

k

$v = v_0 - b x$

b

$x(P = P_{max}) = ?$



• SILE:

$$-kx + F = ma$$

$$-kx + F = -b(v_0 - bx)m$$

$$\boxed{F = x(k + b^2 m) - v_0 b m}$$

$$a = \frac{dv}{dt}\frac{dx}{dx} = v\frac{dv}{dx}$$

$$a = (v_0 - bx)(-b)$$

$$P = F \cdot v$$

$$P = [x(k + b^2 m) - v_0 b m](v_0 - bx)$$

$$P = -x^2 b(k + b^2 m) + x[v_0(k + b^2 m) + v_0 b^2 m] - v_0^2 b m$$

ZA MAKSIMUM

$$\frac{dP}{dx} = 0 = -2xb(k + b^2 m) + v_0[(k + b^2 m) + b^2 m]$$

$$x = \frac{v_0[(k + b^2 m) + b^2 m]}{2b(k + b^2 m)} = \boxed{\frac{v_0(k + 2b^2 m)}{2b(k + b^2 m)}}$$

4 Gibalna količina, energija

ZBIRKA 9 nal. 35/str. 15

$M = 150\,kg$

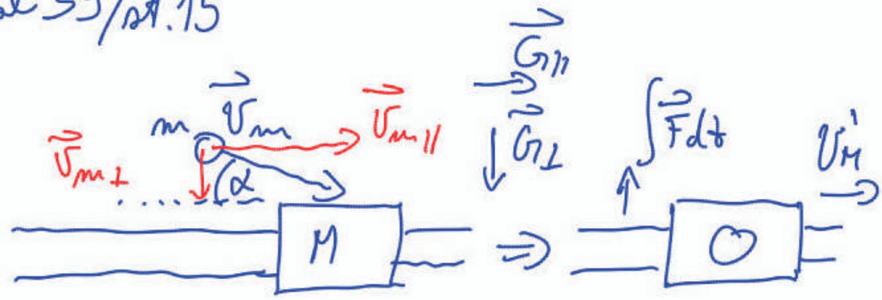
$m = 70\,kg$

$v_m = 5\,m/s$

$\alpha = 30°$

$v_M' = ?$

$\int F_\perp dt = ?$



• GIBALNA KOLIČINA: $\vec{G} = m\vec{v}$; $\Delta G = \int F dt$ ↳ SUNEK SILE

$\parallel$: $m v_{m\parallel} = (m+M) v_M'$

$$v_M' = \frac{m}{m+M} v_m \cdot \cos\alpha = 1{,}4\,m/s$$

$\perp$: $m v_{m\perp} = \int F dt = m v_m \cdot \sin\alpha = 175\,Ns$

ZBIRKA 9 nal 33/st 15

$2r = 10\,cm$

$M = 1\,kg$

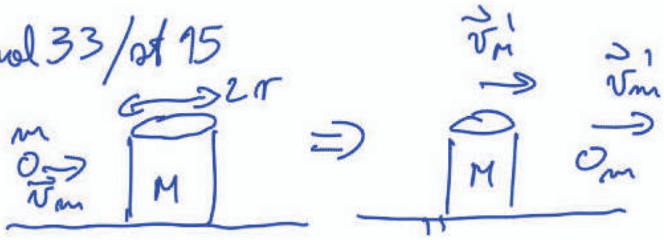
$m = 2\,g$

$v_m = 300\,m/s$

$F = 500\,N$

$v_m' = ?$

$v_M' = ?$



• DELO, KI GA OPRAVI IZSTRELEK: $\underline{A = F \cdot 2r = 50\,J}$

• ENERGIJE ZA IZSTRELEK:

$$\frac{m v_m^2}{2} = \frac{m v_m'^2}{2} + A \quad / \cdot 2$$

$$m v_m^2 = m v_m'^2 + 2F \cdot 2r \quad / : m$$

$$v_m'^2 = v_m^2 - \frac{4Fr}{m}$$

$$\underline{v_m' = \sqrt{v_m^2 - \frac{4Fr}{m}}} = \underline{200\,m/s}$$

• GIBALNA K.:

$$m v_m = m v_m' + M v_M'$$

$$\hookrightarrow \underline{v_M' = \frac{m}{M}(v_m - v_m')}$$

$$\underline{v_M' = 0{,}2\,m/s}$$

$$W_{k,M}' = \frac{M v_M'^2}{2} = \frac{1 \cdot 0{,}04}{2}\,J \ll \underline{50\,J}$$

ZBIRKA 9 nal 28/24.

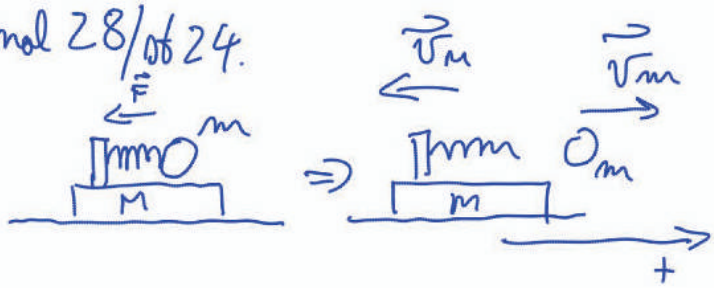
$k = 2\,N/cm$

$M = 0{,}4\,kg$

$F = 100\,N$

$m = 0.1\,kg$

$\tilde{v} = ?$



$$\vec{\tilde{v}} = \vec{v}_m - \vec{v}_M$$

$$\tilde{v} = |v_m| + |v_M|$$

HITROST KROGLICE GLEDE NA PIŠTOLO

• GIBALNA KOLIČINA:

$$0 = m v_m + M v_M \Rightarrow \underbrace{v_M = -\frac{m}{M} v_m} = \underline{-5\,m/s}$$

• ENERGIJE:

$$W_{pr} = W_{k\,m} + W_{k\,M}$$

$$\underbrace{\frac{k x^2}{2} = \frac{m v_m^2}{2} + \frac{M v_M^2}{2}}$$

• SILE:

$$F = k x \Rightarrow \underbrace{x = \frac{F}{k}}$$

$$\cancel{k}\frac{F^2}{k^{\cancel{2}}} = m v_m^2 + \frac{M m^2}{M^{\cancel{2}}} \cdot v_m^2 = v_m^2 m\left(1 + \frac{m}{M}\right)$$

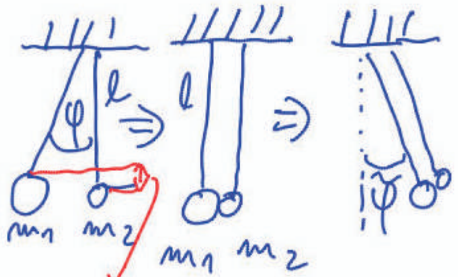
$$v_m = F \frac{1}{\sqrt{k m \left(1 + \frac{m}{M}\right)}} = 100\,N \frac{1}{\sqrt{200\,\frac{N}{m} \cdot 0{,}1\,kg \left(1 + \frac{1}{4}\right)}} = \underline{20\,m/s}$$

$$\underline{\tilde{v} = 25\,m/s} \qquad \tilde{v} = v_m - v_M = (20 + 5)\,m/s$$

ZBIRKA 9 zad 26/str 23

$m_1 = 50\,g$
$m_2 = 10\,g$
$l = 50\,cm$
$\varphi = 5^\circ$

$\tilde{\varphi} = ?$
$\Delta W_{IZGUBA} = ?$



$\Delta h = l - l\cos\varphi$
$= l(1-\cos\varphi)$

NEPROŽNI TRK

• PRED TRKOM:

$\Delta W_{p1} = \Delta W_{k1}$

$\cancel{m_1} g l(1-\cos\varphi) = \frac{\cancel{m_1} v_1^2}{2} \Rightarrow v_1^2$

• TRK:

$m_1 v_1 = (m_1 + m_2)\cdot v'$

• PO TRKU:

$\Delta W_{k1+2} = \Delta W_{p1+2}$

$\frac{\cancel{m_1+m_2}}{2} v'^2 = \cancel{(m_1+m_2)} g l(1-\cos\tilde{\varphi})$

KVADRIRAMO

$m_1^2\, 2\cancel{gl}(1-\cos\varphi) = (m_1+m_2)^2\, 2\cancel{gl}(1-\cos\tilde{\varphi})$

$\frac{m_1^2}{(m_1+m_2)^2}(1-\cos\varphi) = 1-\cos\tilde{\varphi}$

$\cos\tilde{\varphi} = 1-(1-\cos\varphi)\frac{m_1^2}{(m_1+m_2)^2} \Rightarrow \tilde{\varphi} = 4{,}16^\circ$

• $\Delta W_{IZGUBE} = \Delta W_{p1} - \Delta W_{p1+2} = m_1 g l(1-\cos\varphi) - (m_1+m_2) g\, l(1-\cos\tilde{\varphi})$

$= gl\left[m_1(1-\cos\varphi) - (m_1+\cancel{m_2})(1-\cos\varphi)\frac{m_1^2}{(m_1+m_2)^{\cancel{2}}}\right]$

$= gl\,m(1-\cos\varphi)\left[1-\frac{m_1}{m_1+m_2}\right]$

$= 1{,}55\cdot 10^{-4}\,J$

$\parallel$ Nm

(4) nal 5

$v_D = 15\,m/s$

$\alpha = 20°$

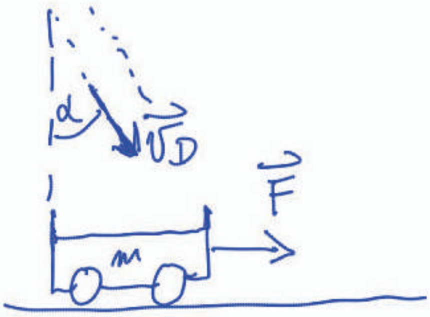
$\phi_m = 0.5\,kg/min$

$F = 0.02\,N$

$m_0 = 30\,kg$

$v(t) = ?$

$v(10\,min) = ?$



$m_D = \phi_m \cdot t$

$v_{D\parallel} = v_D \cdot \sin\alpha$

$m = m_0 + m_D = m_0 + \phi_m t$

• GIBALNA KOLIČINA:

$$\int_0^t F\,dt + m_D v_{D\parallel} = m v$$

$$F \cdot t + \phi_m t\, v_D \sin\alpha = (m_0 + \phi_m t) v$$

$$v = \frac{(F + \phi_m v_D \sin\alpha)\, t}{m_0 + \phi_m t}$$

$$v(10\,min) = 1\,m/s$$

(4) nal 6

m

v_0, k_t

$v_k = ?$

$\Delta W_k = ?$

$\Delta W_k = A_t$

G: $m v_0 = 2 m v_k \Rightarrow v_k = \frac{v_0}{2}$

W: $\frac{m v_0^2}{2} = \frac{2 m v_k^2}{2} + A_t \Rightarrow A_t = \frac{m}{2} v_0^2 \left(1 - \frac{1}{2}\right) = \underline{\underline{\frac{1}{2} \overset{\frac{m v_0^2}{2}}{\overset{\|}{W_{k_0}}}}}$

DOLOČI A_t IZ DEFINICIJE ZA DELO $A_t = \int F_t \cdot ds$

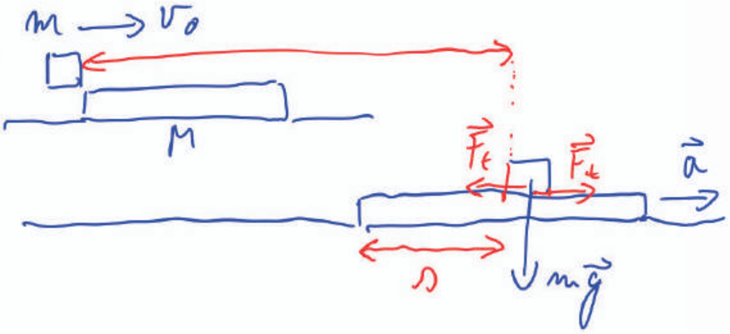
(4) nal 6

m

v_0

k_t

A_t



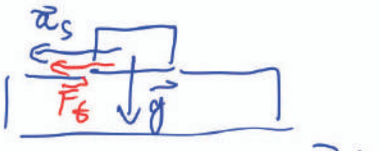
• DELO:

$$\underline{A_t = \int F_t \, ds = F_t \cdot s}$$

• TRENJE:

$$\underline{F_t = m g k_t}$$

• SISTEM SPLAVA:



$$\underline{\vec{a}_s = -\vec{a}}$$

$\longrightarrow +$

$\Longrightarrow$

• SILE NA UTEŽ V SISTEMU SPLAVA:

$$-m a_s - F_t = m a_u$$

$$-\cancel{m} g k_t - \cancel{m} g k_t = \cancel{m} a_u$$

$$\underline{a_u = -2 g k_t}$$

↳ KONST.

• POSPEŠEK SPLAVA:

$$F_t = m a$$

$$\cancel{m} g k_t = \cancel{m} a \Rightarrow \underline{a = g k_t}$$

$|a_s| = a$

• POT V SISTEMU SPLAVA

$$v_K^2 = v_Z^2 + 2 a_u s \leftarrow \begin{cases} v_{ZAČ} = v_0 \\ v_{KONČ} = 0 \end{cases}$$

$$0 = v_0^2 - 4 g k_t s$$

$$\underline{s = \frac{v_0^2}{4 g k_t}}$$

$$\underline{A_t} = F_t \cdot s = m g \cancel{k_t} \cdot \frac{v_0^2}{4 \cancel{g k_t}}$$

$$= \frac{m v_0^2}{4} = \underline{\underline{\frac{1}{2} W_{k0}}}$$

ZBIRKA 9 nal 37/st 15

$m_1 = 0{,}6\ kg$

$v_1 = 8\ m/s$

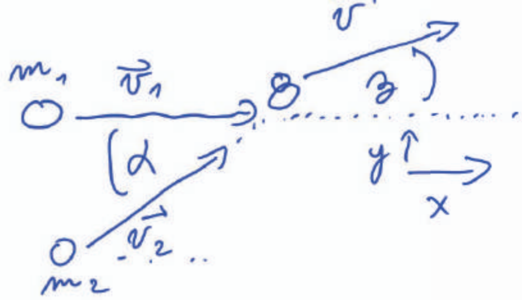
$m_2 = 0{,}4\ kg$

$v_2 = 10\ m/s$

$\alpha = 30°$

$v' = ?$

$\beta = ?$



• GIBALNA KOLIČINA

$$\begin{cases} X: m_1 v_1 + m_2 v_2 \cdot \cos\alpha = (m_1 + m_2) \cdot v' \cdot \cos\beta \\ Y: m_2 v_2 \cdot \sin\alpha = (m_1 + m_2) v' \sin\beta \end{cases}$$

$$\Rightarrow \operatorname{tg}\beta = \frac{G_Y}{G_X} = \frac{m_2 v_2 \sin\alpha}{m_1 v_1 + m_2 v_2 \cos\alpha} \Rightarrow \underline{\underline{\beta = 13{,}6°}}$$

$$\text{Iz } Y: v' = \frac{m_2 v_2 \sin\alpha}{(m_1 + m_2) \sin\beta} = \underline{\underline{8{,}5\ m/s}}$$

ZBIRKA 9 nal 41/st 16

$M = 500\,kg$

$m = 80\,kg$

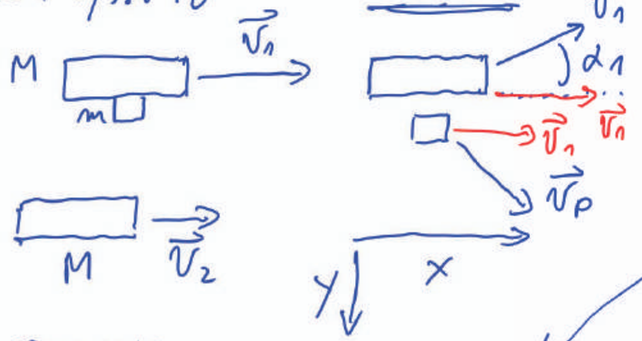
$v_1 = 100\,m/s$

$v_2 = 120\,m/s$

$v_\perp = 25\,m/s$

$\alpha = ?$

ODRIV:



• KER V X-SMERI NE DELUJE NOBENA SILA

↳ $v_1 = v_1' \cdot \cos\alpha_1$

ODRIV:

$$X: \ (\cancel{m+}M)v_1 = (\cancel{m+}M)v_1'\cos\alpha_1$$

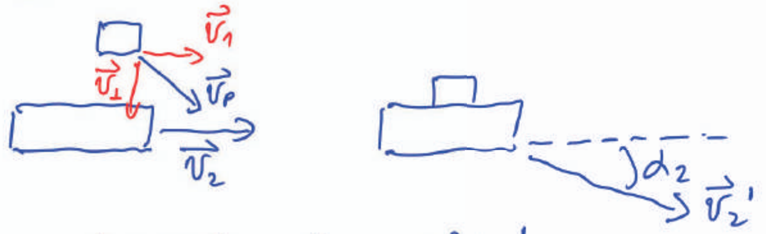
$$Y: \ 0 = m v_\perp + M v_1' \cdot \sin\alpha_1$$

$$\Rightarrow X: \ v_1'\cos\alpha_1 = v_1$$

$$Y: \ v_1'\sin\alpha_1 = -\frac{m}{M} v_\perp$$

$$\Rightarrow \operatorname{tg}\alpha_1 = -\frac{m}{M}\frac{v_\perp}{v_1} \Rightarrow \alpha_1 = -2{,}29^\circ$$

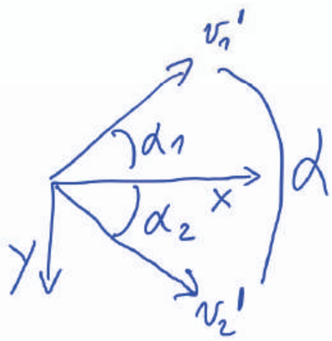
PRISTANEK:



$$X: \ m v_1 + M v_2 = (m+M) v_2' \cdot \cos\alpha_2$$

$$Y: \ m v_\perp = (m+M) v_2' \sin\alpha_2$$

$$\rightarrow \operatorname{tg}\alpha_2 = \frac{m v_\perp}{m v_1 + M v_2} \Rightarrow \alpha_2 = 1{,}68^\circ$$

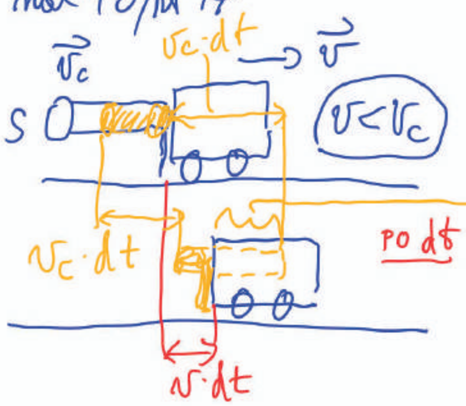


$$\Rightarrow \alpha = |\alpha_1| + |\alpha_2| = 4^\circ$$

ZBIRKA 9 nal 48/st 17

$m = 50\,g$
$S = 6\,cm^2$
$\Phi_V = 1\,dm^3/s$
$t = 10\,s$

$v(t) = ?$



$(v < v_c)$

$\Rightarrow$ VODA SE UPOČASNI Z v_c NA v

$\Rightarrow$ VOLUMEN VODE, KI SE V dt ZALETI V VOZIČEK:

$$\underbrace{(v_c - v)\,dt \cdot S}_{V'} \Rightarrow \boxed{\phi_V' = \frac{V'}{dt} = S(v_c - v)}$$

$$\Phi_V = S \cdot v_c$$

$$\Downarrow$$

$$\boxed{v_c = \frac{\phi_V}{S}}$$

- SUNEK SILE NA VOZIČEK:

$$dG: \quad F\,dt = dm\,(v_c - v) \quad /:dt$$

$$F = \underbrace{\frac{dm}{dt}}_{\phi_m}(v_c - v)$$

$$\phi_m = \rho \cdot \phi_V' = \rho \cdot S(v_c - v)$$

$$\Rightarrow \boxed{F = \rho \cdot S\,(v_c - v)^2}$$

- SILE: $F = ma$

$$\rho S (v_c - v)^2 = m\frac{dv}{dt}$$

$$\int_0^t dt = \frac{m}{\rho S}\int_0^v \frac{dv}{(v_c - v)^2}$$

$$t = \frac{m}{\rho S}\int_{v_c}^{v_c - v} \frac{-du}{u^2} =$$

NOVA SPREMENLJIVKA:

$$\boxed{\begin{aligned} v_c - v &= u \\ -dv &= du \end{aligned}}$$

$$t = \frac{m}{\rho S}\,\frac{1}{u}\Big|_{v_c}^{v_c - v}$$

$$\frac{\rho S \cdot t}{m} = \frac{1}{v_c - v} - \frac{1}{v_c}$$

$$\frac{1}{v_c} + \frac{\rho S t}{m} = \frac{1}{v_c - v}$$

$$v_c - v = \frac{1}{\frac{1}{v_c} + \frac{\rho S t}{m}}$$

$$v = v_c - \frac{1}{\frac{1}{v_c} + \frac{\rho S t}{m}} \quad \begin{matrix} /\cdot v_c \\ /\cdot v_c \end{matrix}$$

$$v = v_c\left(1 - \frac{1}{1 + \frac{\rho S t v_c}{m}}\right)$$

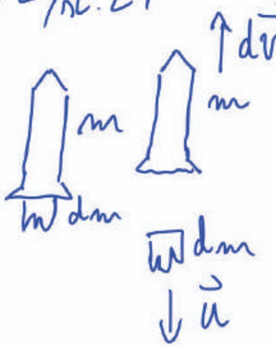
$$v = v_c \frac{\frac{\rho S t v_c}{m}}{1 + \frac{\rho S t v_c}{m}}$$

$$\boxed{v = v_c \frac{1}{1 + \frac{m}{\rho S v_c t}}}$$

ZBIRKA 9 nal 32/st. 24

$m_0 = 3000\,t$
$t = 150\,s$
$\phi_m = 14\,t/s$
$u = 2{,}5\,km/s$

$v(t) = ?$



• BREZ ZUNANJIH SIL:

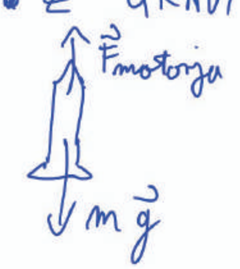
$$\Delta G = 0 = m\,dv - dm\,u$$

$$m\,dv = dm\,u \quad /:dt$$

$$m\frac{dv}{dt} = \frac{dm}{dt}u$$

$$\boxed{F = ma = \phi_m u} \leftarrow \text{SILA IZHODNIH PLINOV NA RAKETO}$$

• Z GRAVITACIJO (g = konst.):



$$F - F_g = ma$$

$$\phi_m u - mg = m\frac{dv}{dt} \quad /:m$$

MASA RAKETE:

$$m = m_0 - \phi_m \cdot t$$

$$\frac{\phi_m u}{(m_0 - \phi_m t)} - g = \frac{dv}{dt}$$

$$\int_0^v dv = \int_0^t \left(\frac{u}{\frac{m_0}{\phi_m} - t} - g\right) dt$$

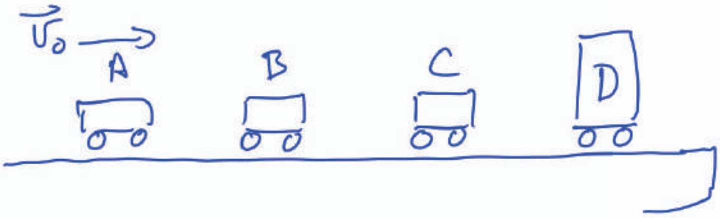
$$u\int_0^t \frac{dt}{\frac{m_0}{\phi_m} - t} = u\int_{\frac{m_0}{\phi_m}}^{\frac{m_0}{\phi_m}-t} \frac{-dz}{z} = u \ln \frac{\frac{m_0}{\phi_m}}{\frac{m_0}{\phi_m} - t}$$

$$\frac{m_0}{\phi_m} - t = z$$
$$-dt = dz$$

$$v = u \ln \frac{\frac{m_0}{\phi_m}}{\frac{m_0}{\phi_m} - t} - gt$$

$$\underline{v = u \ln \frac{1}{1 - \underbrace{\frac{\phi_m t}{m_0}}_{\text{MASA GORIVA}}} - gt} = 1{,}54\,km/s$$

VOZIČKI



$m_A = m_B = m_C = m$

$m = 2\,kg$

$m_D = 3\,kg$

$v_0 = 5\,m/s$

$v_D = ?$

a) KO SE ZLEPIJO

b) KO SE PROŽNO ODBIJEJO

a) G: $m_A v_0 = (3m_A + m_D) v_D$

$$v_D = \frac{m_A}{(3m_A + m_D)} v_0 = \underline{\underline{1{,}11\,m/s}}$$

b) • TRK A, B:

G: $\cancel{m_A} v_0 = \cancel{m_A}(v_A' + v_B') \Rightarrow v_A' = v_0 - v_B' = 0$

W: $\frac{\cancel{m_A} v_0^2}{\cancel{2}} = \frac{\cancel{m_A}}{\cancel{2}}(v_A'^2 + v_B'^2)$

$$v_0^2 = (v_0 - v_B')^2 + v_B'^2$$

$$\cancel{v_0^2} = \cancel{v_0^2} - 2 v_0 \cancel{v_B'} + v_B'^{\cancel{2}} + v_B'^{\cancel{2}}$$

$$2v_0 = 2v_B' \Rightarrow \underline{v_B' = v_0}$$

$\rightarrow v_C = v_0$

• TRK C,D:

G: $m_C v_0 = m_C v_C' + m_D v_D' \Rightarrow v_C' = \frac{m_C v_0 - m_D v_D'}{m_C} = \frac{2\cdot 5 - 3\cdot 4}{2}\,m/s = \underline{\underline{-1\,m/s}}$

W: $\frac{m_C v_0^2}{\cancel{2}} = \frac{m_C v_C'^2}{\cancel{2}} + \frac{m_D v_D'^2}{\cancel{2}}$

$$m_C v_0^2 = \cancel{m_C} \frac{(m_C v_0 - m_D v_D')^2}{m_C^{\cancel{2}}} + m_D v_D'^2$$

$$\cancel{m_C v_0^2} = \frac{\cancel{m_C^2 v_0^2} - 2 m_C m_D v_0 v_D' + m_D^2 v_D'^2}{m_C} + m_D v_D'^2$$

$$0 = -2 m_D v_0 \cancel{v_D'} + \left(\frac{m_D^2}{m_C} + m_D\right) v_D'^{\cancel{2}} \quad /: m_D$$

$$2 v_0 = \left(1 + \frac{m_D}{m_C}\right) v_D'$$

$$v_D' = v_0 \frac{2}{\left(1 + \frac{m_D}{m_C}\right)} = \frac{4}{5} v_0 = \underline{\underline{4\,m/s}}$$

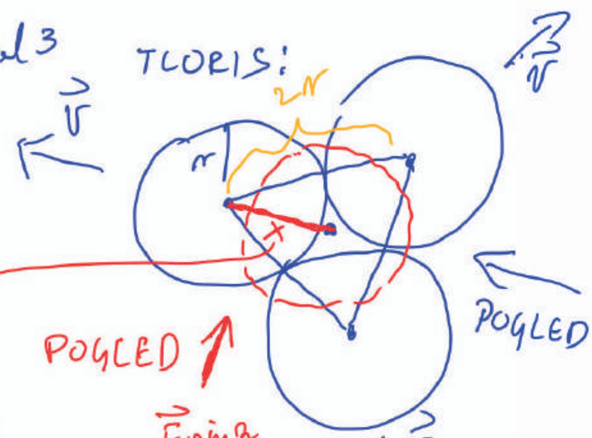
94/95 kol 1./nal 3

$v = 2\,m/s$

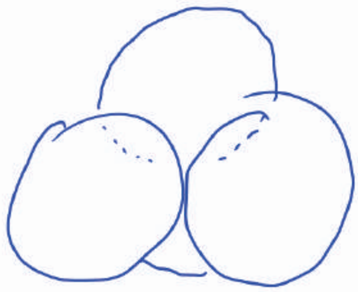
$v_z = ?$

$h = ?$

TLORIS:



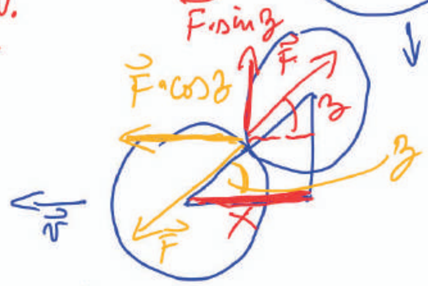
OB TRKU OD STRANI:



$x = \frac{2}{3}\cdot$ VIŠINA ENAKOSTRN. TRIKOTNIKA

$$\frac{2}{3}\cdot a\cdot\frac{\sqrt{3}}{2}$$

$$\boxed{x = 2r\cdot\frac{\sqrt{3}}{3}}$$



$$2r\cdot\cos\vartheta = x$$

$$\cos\vartheta = \frac{x}{2r}$$

$$\boxed{\cos\vartheta = \frac{\sqrt{3}}{3}}$$

$$tg\,\vartheta = \frac{\sin\vartheta}{\cos\vartheta} = \frac{\sqrt{1-\cos^2\vartheta}}{\cos\vartheta} = \sqrt{2}$$

• SUNEK SILE V SMER GIBANJA KROGEL:

$$G:\ \int F\,dt\cdot\cos\vartheta = mv$$

$$\boxed{\int F\,dt = \frac{mv}{\cos\vartheta}}$$

• SUNEK SILE SPODNJIH KROGEL NA ZGORNJO KROGLO

↳ PRISPEVKI V RAVNINI SE BODO ZARADI SIMETRIJE IZNIČILI

• OSTANEJO LE NAVPIČNI PRISPEVKI

TRI KROGLE

SPREMEMBA HITROSTI ZGORNJE KROGLE

$$G:\ 3\int F\,dt\cdot\sin\vartheta = m(v_K - v_Z)$$

$$3\cdot \cancel{m} v\, tg\,\vartheta = \cancel{m}(v_K - v_Z)$$

$$\boxed{v_K = v_Z + 3v\,tg\,\vartheta}$$

ENERGIJE:

$$W:\ \frac{mv_z^2}{2} = 3\,\frac{mv^2}{2} + \frac{mv_k^2}{2}$$

$$v_z^2 = 3v^2 + (v_z + 3v\,tg\,\vartheta)^2$$

$$\cancel{v_z^2} = \cancel{3}v^{\cancel{2}} + \cancel{v_z^2} + \cancel{6}^{\,2}\cancel{v}v_z\,tg\,\vartheta + \cancel{9}^{\,3}v^{\cancel{2}}tg^2\vartheta$$

$$-2v_z\,tg\,\vartheta = v(1 + 3tg^2\vartheta)$$

$$v_z = -v\,\frac{(1+3tg^2\vartheta)}{2\,tg\,\vartheta} = -5\,m/s$$

HITROST JE POZITIVNA ZA GIBANJE GOR

IN NEGATIVNA ZA GIBANJE DOL → TOREJ NA ZAČETKU

$$\boxed{v_K = 3{,}53\,m/s}$$

KAKO VISOKO SE ODBIJE:

$$W:\ \cancel{m}gh = \frac{\cancel{m}v_K^2}{2}$$

$$\boxed{h = \frac{v_K^2}{2g} = 0{,}64\,m}$$

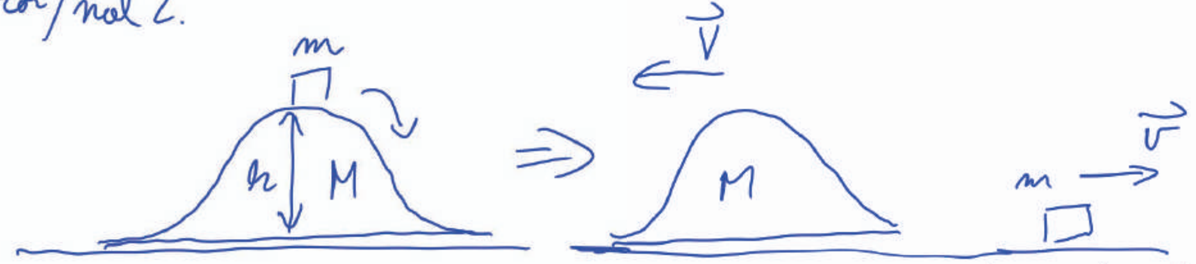
93/94 1.kol/nal 2.

$M = 3\,kg$

$m = 1\,kg$

$\Delta v = 4\,m/s$

$h = ?$



$|\Delta v| = |v - V| = |v| + |V|$

$G: \quad 0 = mv + MV$

$W: \quad mgh = \frac{mv^2}{2} + \frac{MV^2}{2}$

$V = -\frac{m}{M} v$

$\Delta v = v\left(1 + \frac{m}{M}\right) = v\left(\frac{m+M}{M}\right)$

$v = \Delta v \frac{M}{m+M}$

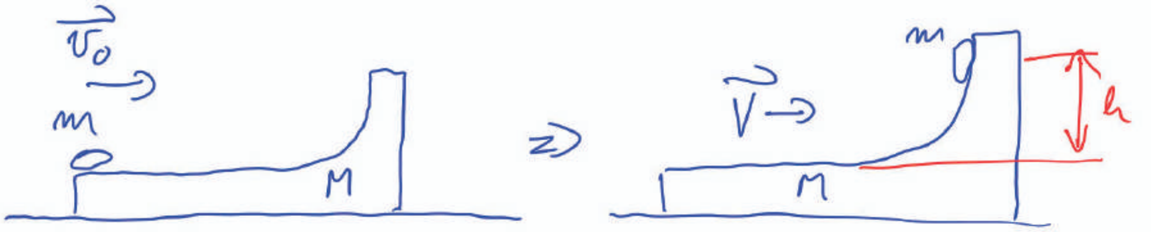
$$mgh = \frac{1}{2}\left(m \Delta v^2 \frac{M^2}{(m+M)^2} + M\left(\frac{m}{M}\right)^2 \Delta v^2 \frac{M^2}{(m+M)^2}\right) \quad / \cdot \frac{2}{m}$$

$$2gh = \Delta v^2 \frac{M^2}{(m+M)^2}\left(1 + \frac{\cancel{M}}{\cancel{m}} \frac{m^{\cancel{2}}}{M^{\cancel{2}}}\right)$$

$$h = \frac{\Delta v^2}{2g} \frac{M^{\cancel{2}}}{(m+M)^{\cancel{2}}} \frac{\cancel{(M+m)}}{\cancel{M}}$$

$$h = \frac{\Delta v^2}{2g} \frac{M}{m+M} = \underline{\underline{0,6\,m}}$$

(4) nal 15



M

$m = \frac{M}{4}$

$v_0 = 2\, m/s$

$h = ?$

a) RAMPA PRITRJENA NA PODLAGO.

b) RAMPA DRSI BREZ TRENJA.

→ V NAJVIŠJI TOČKI: RAMPA IN DISK IMATA ENAKO HITROST IN TO VODORAVNO

a) W: $\frac{\cancel{m} v_0^2}{2} = \cancel{m} g h \Rightarrow h = \frac{v_0^2}{2g} = 0.2\, m$

b) W: $\frac{m v_0^2}{2} = m g h + \underbrace{\frac{m v^2}{2} + \frac{M V^2}{2}} \Big/ \cdot 2$

→ V NAJVIŠJI TOČKI $v = V$!

$m v_0^2 = 2 m g h + (m+M) v^2$

G: $m v_0 = (m+M) v$

→ $v = \frac{m}{m+M} v_0$ → HITROST GIBANJA TEŽIŠČA

$\cancel{m} v_0^2 = 2 \cancel{m} g h + \cancel{(m+M)} \frac{m^{\cancel{2}}}{(m+M)^{\cancel{2}}} v_0^2$

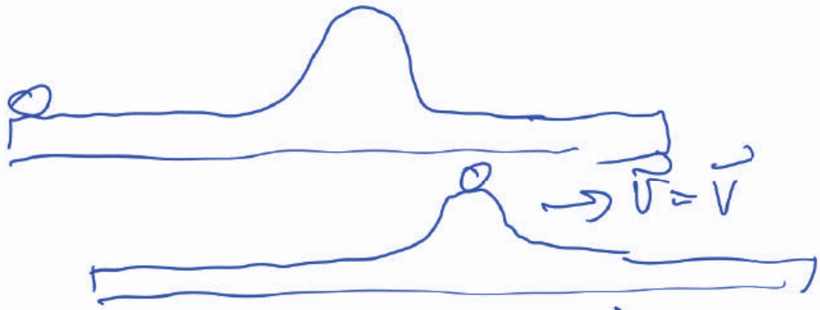
$2 g h = v_0^2 \left(1 - \frac{m}{m+M}\right)$

$h = \frac{v_0^2}{2g} \left(1 - \frac{\frac{1}{4}}{\frac{5}{4}}\right) = \frac{v_0^2}{2g} \frac{4}{5} = \frac{2}{5} \frac{v_0^2}{g}$

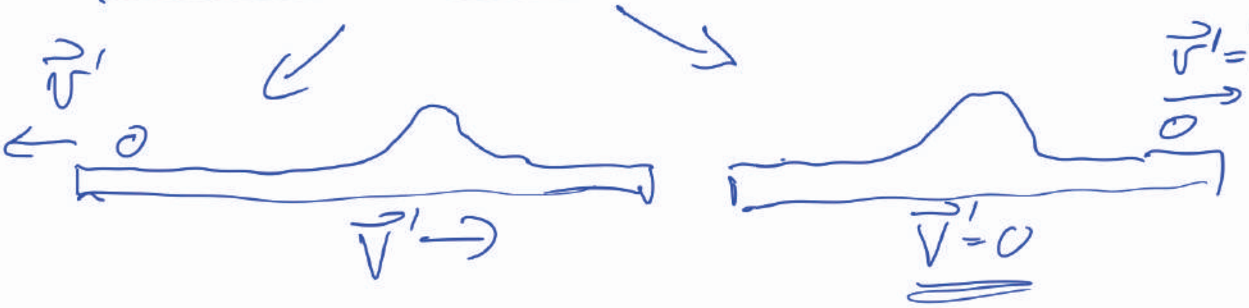
$m = \frac{M}{4}$

$h = 0{,}16\, m$

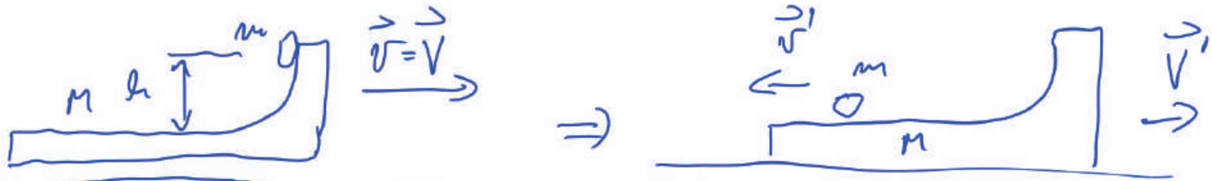
b)



⇒ m IN M IMATA ENAKO HITROST



c)



$\underline{v' = ?}$

$$W:\ mgh + \frac{m+M}{2}v^2 = \frac{mv'^2}{2} + \frac{MV'^2}{2}$$

$$G:\ (m+M)v = mv' + MV' \Rightarrow V' = \frac{(m+M)v - mv'}{M}$$

ZAČETNA $W_k = \frac{mv_0^2}{2}$

$$W:\ \frac{mv_0^2}{2} = \frac{mv'^2}{2} + \frac{M}{2}\frac{1}{M^2}\left[(m+M)v - mv'\right]^2 \quad /\cdot\frac{2}{m}$$

$$v_0^2 = v'^2 + \frac{1}{mM}\left[(m+M)^2v^2 - 2m(m+M)vv' + m^2v'^2\right]$$

TEŽIŠČE

$$0 = v'^2\left(1+\frac{m}{M}\right) - v'\left(1+\frac{m}{M}\right)2v + \frac{(m+M)^2}{mM}v^2 - v_0^2$$

$\underline{v = \frac{m}{m+M}v_0}$ $\longrightarrow$

$$0 = v'^2\left(1+\frac{m}{M}\right) - v'2\frac{\cancel{m+M}}{M}\cdot\frac{m}{\cancel{m+M}}v_0 + \frac{\cancel{(m+M)^2}}{\cancel{m}M}\cdot\frac{m^{\cancel{2}}v_0^2}{\cancel{(m+M)^2}} - v_0^2$$

$$0 = v'^2\frac{m+M}{M} - 2\frac{m}{M}v_0v' + v_0^2\left(\frac{m}{M} - 1\right)$$

$$v' = \frac{2\frac{m}{M}v_0 \pm\sqrt{4\frac{m^2}{M^2}v_0^2 - 4\frac{m+M}{M}v_0^2\frac{(m-M)}{M}}}{2\frac{m+M}{M}} \qquad m^2-M^2$$

$$= \frac{\cancel{2}\frac{m}{M}v_0 \pm\sqrt{\cancel{4}v_0^2\frac{M^2}{M^2}}}{\cancel{2}\frac{m+M}{M}} = v_0\frac{M}{m+M}\left(\frac{m}{M}\pm 1\right)$$

$$= v_0\frac{\cancel{M}}{m+M}\frac{m\pm M}{\cancel{M}}$$

$$= \begin{cases} v_0 & (+) \rightarrow \\ \frac{m-M}{m+M} & (-) \leftarrow \end{cases}$$

90/91 1.kol/1.nal

$v = 1{,}5\ m/s$

$m_1 = 150\ kg$

$v_1 = -v_2 = v$

$m_2 = 50\ kg$

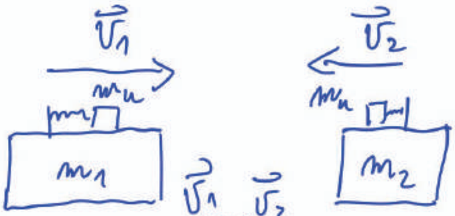
$m_u = 10\ g$

$k = 0{,}4\ N/m$

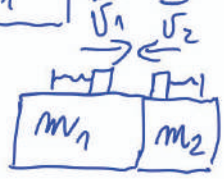
$x_1 = ?$

$x_2 = ?$

A



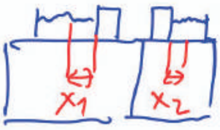
B



$\longrightarrow v'$

← OB TRKU IMATA VOZIČKA ŽE HITROST v'
UTEŽI PA IMATA ŠE VEDNO HITROSTI v_1 IN v_2

C



$\longrightarrow v'$

← KO STA VZMETI NAJBOLJ RAZTEGNJENI TAKRAT SE UTEŽI GIBLJETA ENAKO KOT VOZIČKA

A→B: NEPROŽNI TRK

$$G:\ m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$v' = \frac{m_1 - m_2}{m_1 + m_2} \cdot v = \frac{1}{2} v$$

B→C: GLEDAMO ENERGIJO V SISTEMU ZLEPLJENIH VOZIČKOV

1. VZMET: $\Delta W_{k1} = \Delta W_{pr1}$

$$\frac{m_u}{2} v_{1R}^2 = \frac{k x_1^2}{2}$$

$$x_1 = v_{1R} \sqrt{\frac{m_u}{k}}$$

$$\underline{\underline{x_1 = 0{,}119\ m}}$$

ZAČETNA HITROST:

$v_{1R} = v_1 - v' = \frac{1}{2} v$

KONČNA:

$v_k = 0$

2. VZMET: $x_2 = v_{2R} \sqrt{\frac{m_u}{k}}$

$v_{2R} = |v_2| + v' = \frac{3}{2} v = 3 v_{1R}$

$$\underline{\underline{x_2}} = 3 x_1 = \underline{\underline{0{,}356\ m}}$$

6 Gravitacija

⑥ nal 1.

M = MASA ZEMLJE

$G = 6{,}67 \cdot 10^{-11} \frac{Nm^2}{kg^2}$

$R_Z = 6400\,km$

$$\cancel{m} g = G \frac{\cancel{m} M}{r^2}$$

$$g = G \frac{M}{r^2}$$

$$g = \frac{GM}{(R_Z+h)^2}$$

$$g = g_0 \frac{R_Z^2}{(R_Z+h)^2}$$

• $r = R_Z + h$

• NA POVRŠINI $g_0 = G \frac{M}{R_Z^2}$

$\hookrightarrow GM = g_0 \cdot R_Z^2$

$$g = g_0 \frac{1}{(1+\frac{h}{R_Z})^2} = g_0 \frac{1}{1+2\frac{h}{R_Z}+\left(\frac{h}{R_Z}\right)^2} = g_0\left(1-\frac{2h}{R_Z}\right)$$

$h \ll R$

$\hookrightarrow \frac{1}{1+2\frac{h}{R_Z}} \sim 1 - \frac{2h}{R_Z}$

$\mathcal{H} = G$

⑥ nal 2 KAKO DOBIMO $W_p = mgz$ IZ

POTENCIAL:

$\hookrightarrow U = -\frac{m_1 m_2 \mathcal{H}}{r}$

$= -\frac{m M \mathcal{H}}{r}$

$$W_p = -\frac{mM\mathcal{H}}{r} = -\frac{m g_0 R_Z^2}{r} = -\frac{m g_0 R_Z^2}{R_Z+h} =$$

$M\mathcal{H} = g_0 R_Z^2$

$r = R_Z + h$

$$= -\frac{m g_0 R_Z}{1+\frac{h}{R_Z}} \sim -m g_0 R_Z\left(1-\frac{h}{R_Z}\right) = -m g_0 R_Z + m g_0 h$$

$\hookrightarrow$ KONSTANTA — KI SE TU ODŠTEJE

$U_K - U_Z$

$$W_p = \Delta U = m g_0 R_Z^2\left(-\frac{1}{R_Z+r_K}+\frac{1}{R_Z+r_Z}\right) = m g_0 R_Z^2\left(\frac{\cancel{-R_Z}-r_Z+\cancel{R_Z}+r_K}{(R_Z+r_K)(R_Z+r_Z)}\right) =$$

$r_K, r_Z \ll R_Z \rightarrow R_Z \quad R_Z$

U_K, U_Z, $R_Z + r_K$, $R_Z + r_Z$

$$= m g_0 \cancel{R_Z^2} \frac{(r_K - r_Z)}{\cancel{R_Z^2}}$$

$$= m g_0 \underbrace{(r_K - r_Z)}_{h} = m g_0 h$$

GRAVITACIJSKI POTENCIAL:

$$F = \frac{m_1 m_2 \mathcal{H}}{r^2}$$

$\vec{F}$, $\vec{r}$

$$U = \underbrace{\int_{\infty}^{r} \vec{F} \cdot d\vec{r}}_{} = -\frac{m_1 m_2 \mathcal{H}}{r}$$

$$\vec{F} = -\underbrace{\vec{\nabla}}_{\text{GRADIENT}} U = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot U$$

$= (0, 0, mg)$

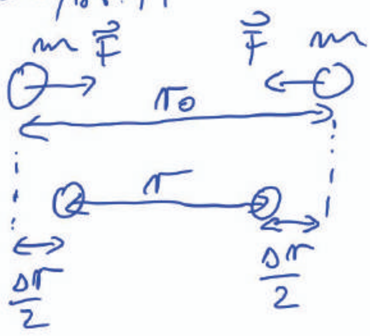
ZBIRKA 9 nal 28/st. 14

$m = 10\,g$

$r_0 = 10\,cm$

$t = 2\,min$

$r(t) = ?$



$r = r_0 - \Delta r$

$\Delta r \rightarrow$ MAJHEN

$$F = \frac{m m G}{r^2} \sim \frac{m^2 G}{r_0^2} = \text{konst.}$$

ENAKOMERNO POSPEŠENO

• SILA: $F = ma \Rightarrow a = \frac{mG}{r_0^2}$

• POT: $\frac{\Delta r}{\cancel{2}} = \frac{a t^2}{\cancel{2}}$

$$\Delta r = \frac{m G t^2}{r_0^2} = \frac{10^{-2}\,kg \cdot 6{,}67\,\frac{N m^2}{kg^2} \cdot 2^2 \cdot 60^2\,s^2\, 10^{-11}}{10^{-2}\,m^2} = \underline{\underline{9{,}6 \cdot 10^{-7}\,m}}$$

$\underline{\Delta r << r}$

• OCENA NAPAKE:

$F_0 = \text{konst.}$

$F_0 = \frac{m^2 G}{r_0^2}$

$F = \frac{m^2 G}{r^2}$

$r = r_0 - \Delta r$

$$\frac{F - F_0}{F_0} = \frac{\cancel{m^2 G}\left(\frac{1}{r^2} - \frac{1}{r_0^2}\right)}{\cancel{m^2 G}\,\frac{1}{r_0^2}} = \frac{r_0^2}{r^2} - 1 =$$

$$= \frac{r_0^2}{r_0^2 - 2\Delta r r_0 + \Delta r^2} - 1 = \frac{\cancel{r_0^2} - \cancel{r_0^2} + 2\Delta r r_0 - \underbrace{\Delta r^2}_{0}}{r_0^2 - \underbrace{2\Delta r r_0 + \Delta r^2}_{0}}$$

$$= \underline{\frac{2\Delta r}{r_0}} = \frac{2 \cdot 10^{-6}}{10^{-1}} = \underline{\underline{2 \cdot 10^{-5}}}$$

ZBIRKA 9 nal 31/st 15

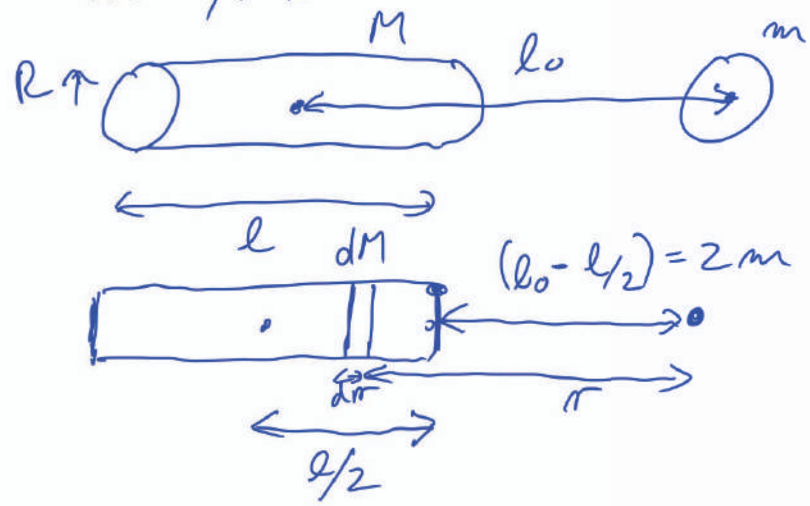
$M = 10\,t$

$l = 10\,m$

$m = 1000\,kg$

$l_0 = 12\,m$

$a = ?$



$R << l$

DEL SILE:

$$dF = \frac{m \gamma \, dM}{r^2} \; ; \; dM = M \frac{dr}{l}$$

$$\int_0^F dF = \frac{m M \gamma}{l} \int_{l_0 - \frac{l}{2}}^{l_0 + \frac{l}{2}} \frac{dr}{r^2}$$

$$F = \frac{m M \gamma}{l} \left(\frac{1}{-r} \right) \Bigg|_{l_0 - \frac{l}{2}}^{l_0 + \frac{l}{2}} = \frac{m M \gamma}{l} \left(\frac{1}{l_0 - \frac{l}{2}} - \frac{1}{l_0 + \frac{l}{2}} \right)$$

$$a_m = \frac{F}{m} = 2.5 \cdot 10^{-9}\, m/s^2$$

$$a_M = \frac{F}{M} = 2.5 \cdot 10^{-10}\, m/s^2$$

ZBIRKA 9 nal 30/st 15

$M = 200\,t$

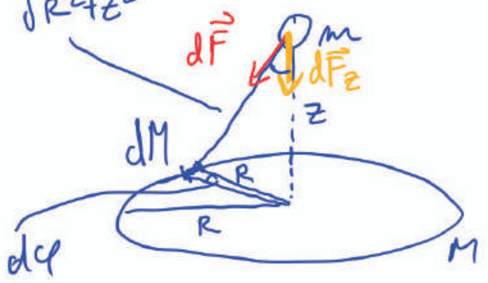
$R = 12\,m$

$z = 10\,m$

$m = 20\,t$

$a = ?$

OD PREJŠNJIČ



$$dM = M\frac{d\varphi}{2\pi}$$

PRAVOKOTNE KOMPONENTE SE ODŠTEJEJO:

GLEDAMO SAMO z KOMPONENTO:

$$dF_z = dF\frac{z}{\sqrt{R^2+z^2}} = \frac{m\,dM\,\varkappa}{(R^2+z^2)}\cdot\frac{z}{\sqrt{R^2+z^2}}$$

$$\int_0^F dF_z = \frac{mM\varkappa}{2\pi}\frac{z}{(R^2+z^2)^{3/2}}\int_0^{2\pi} d\varphi$$

$$F = \frac{mM\varkappa}{\cancel{2\pi}}\frac{z}{(R^2+z^2)^{3/2}}\cdot\cancel{2\pi}$$

$$\underline{F = \frac{mM\varkappa\, z}{(R^2+z^2)^{3/2}}} = 7\cdot10^{-3}\,N$$

$$a_m = \frac{F}{m} = 7\cdot10^{-4}\,m/s^2$$

$$a_M = \frac{F}{M}$$

DODATEK: Z $\vec{\nabla}$ VEKTOR

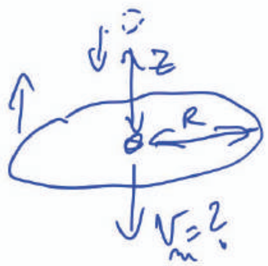
$$\vec{F} = \vec{\nabla}U = \vec{\nabla}W_p = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\left(\frac{mM\varkappa}{\sqrt{R^2+z^2}}\right) =$$

$$= \left(0, 0, mM\varkappa\left(-\frac{1}{\cancel{2}}\frac{1}{(R^2+z^2)^{3/2}}\cdot\cancel{2}z\right)\right) =$$

$$= (0, 0, 1)\cdot\left(-\frac{mM\varkappa\, z}{(R^2+z^2)^{3/2}}\right)$$

⑥ nal 6 → DOPOLNILNO VPRAŠANJE:

HITROST LADJE KO PADE SKOZI SREDIŠČE?



↳ Z ENERGIJAMI!

$$W_{pz} = W_{pk} + W_{k_m} + W_{k_M}$$

$$-\frac{mM\varkappa}{\sqrt{R^2+z^2}} = -\frac{mM\varkappa}{R} + \frac{m v_m^2}{2} + \frac{M v_M^2}{2} \quad /\cdot 2$$

GIB. KOL.

$$m v_m + M v_M = 0$$

$$\hookrightarrow v_M = -\frac{m}{M}v_m$$

$$2mM\varkappa\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+z^2}}\right) = m v_m^2 + \cancel{M}\frac{m^2}{M^{\cancel{2}}}v_m^2$$

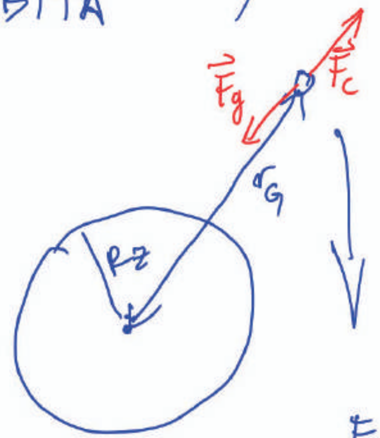
$$2\cancel{m}M\varkappa\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+z^2}}\right) = \cancel{m}v_m^2\left(1 + \frac{m}{M}\right)$$

$$\underline{v_m = \sqrt{\frac{2M\varkappa}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+z^2}}\right)}} = \ldots$$

(6) nal 4

r_G (GEOSTACIONARNA ORBITA) = ?

SATELIT JE VES ČAS NAD ISTO TOČKO $\Rightarrow$ $\omega = \omega_Z = 2\pi\nu = \frac{2\pi}{t_0}$; $\underline{t_0 = 1\,\text{dan}}$



$$F_g = \frac{mM\varkappa}{r^2} = \frac{m g_0 R_Z^2}{r^2} \; ; \; \cancel{m} g_0 = \frac{\cancel{m} M \varkappa}{R_Z^2}$$

$$\hookrightarrow M\varkappa = g_0 R_Z^2$$

$$F_c = m a_r = m\omega^2 r$$

$$R_Z = 6400\,\text{km}$$

$$F_g = F_c$$

$$\frac{\cancel{m} g_0 R_Z^2}{r^2} = \cancel{m}\omega^2 r$$

$$\underline{r_G} = \sqrt[3]{\frac{g_0 R_Z^2}{\omega^2}} = \underline{42160\,\text{km}}$$

$$r_G - R_Z \sim \underline{36000\,\text{km}}$$

(6) nal 5

DOLČI 1. IN 2. KOZMIČNO HITROST!

- 1. KOZ. v:

$$F_g = F_c \;\; (\text{PRI } r = R_Z)$$

$$\cancel{m} g_0 = \cancel{m}\frac{v^2}{R_Z} \Rightarrow \underline{v = \sqrt{g_0 R_Z} \sim 7{,}9\,\text{km/s}}$$

- 2. KOZ. v: DA POBEGNEMO GRAVITACIJI

$$\cancel{m} g_0 R_Z = \frac{\cancel{m} v^2}{2}$$

$$\underline{v = \sqrt{2 g_0 R_Z} \sim 11{,}2\,\text{km/s}}$$

5 Navor, statika, vrtenje, vrtilna količina

95/96 1. kol/nal 2

KOCKA

$m = 12\ kg$

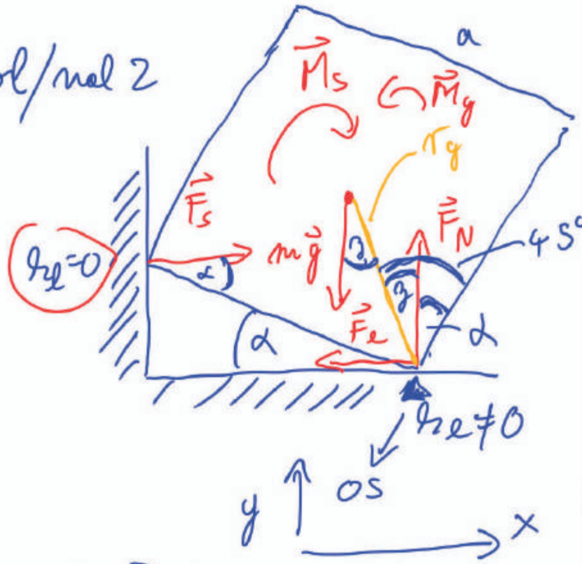
$a = 60\ cm$

$\alpha = 30^\circ$

$k_l = 0.5$

$F_{STENA} = ?$

$k_{l\,min} = ?$



SILE:

$X: F_s - F_e = 0$

$y: F_N - mg = 0$

NAVORI:

$$\underbrace{|\vec{F}_s \times \vec{r}_s|}_{\text{VRTI DESNO}} = \underbrace{|m\vec{g} \times \vec{r}_g|}_{\text{VRTI LEVO}}$$

$$F_s \cdot a \cdot \sin\alpha = mg \cdot a \frac{\sqrt{2}}{2} \sin\beta$$

$$F_s = \frac{mg\sqrt{2}}{2} \frac{\sin\beta}{\sin\alpha}$$

$$\boxed{F_s = 43\ N}$$

• ROČICE:

$r_s = a$

$r_g = a \cdot \sqrt{2}/2$

• KOT:

$\beta = 45^\circ - \alpha = 15^\circ$

ZA $k_{l\,min}$: $F_s = F_e = F_N \cdot k_{l\,min} = mg\, k_{l\,min}$

$$k_{l\,min} = \frac{F_s}{mg} = \frac{\sqrt{2}}{2} \frac{\sin\beta}{\sin\alpha} = \underline{0.366}$$

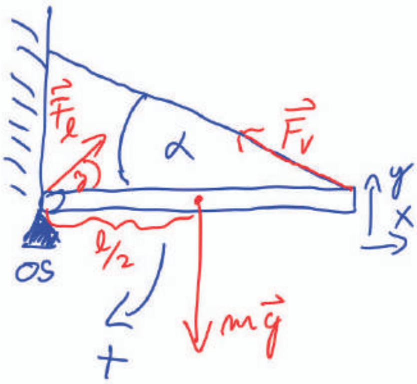
ZBIRKA 9 nal 6/st.5

$mg = 50\,N$

$\alpha = 30^\circ$

$F_V = ?$

$F_\ell = ?$



SILE:

$X:\ F_\ell \cdot \cos\beta = F_V \cdot \cos\alpha$

$Y:\ F_\ell \cdot \sin\beta + F_V \cdot \sin\alpha = mg$

NAVORI:

- $M_g = mg \cdot \frac{\ell}{2}$
- $M_V = F_V\, \ell \cdot \sin\alpha$

$M_g = M_V$

$mg\frac{\cancel{\ell}}{2} = F_V \cancel{\ell} \sin\alpha$

$\boxed{F_V = \frac{mg}{2\sin\alpha} = \underline{\underline{50\ N}}}$

$X \rightarrow F_\ell = F_V \frac{\cos\alpha}{\cos\beta}$

$Y \rightarrow F_V \cdot \frac{\cos\alpha}{\cos\beta} \cdot \sin\beta + F_V \cdot \sin\alpha = mg$

$F_V(\cos\alpha \cdot tg\,\beta + \sin\alpha) = mg$

$\frac{\cancel{mg}}{2\sin\alpha}(\cos\alpha \cdot tg\,\beta + \sin\alpha) = \cancel{mg}\ /\cdot 2$

$\frac{tg\,\beta}{tg\,\alpha} + 1 = 2$

$\frac{tg\,\beta}{tg\,\alpha} = 1 \Rightarrow tg\,\beta = tg\,\alpha \Rightarrow \boxed{\alpha = \beta}$

$X:\ \boxed{F_\ell} = F_V = \underline{\underline{50\ N}}$

ZBIRKA 9 nal 14/sA.6

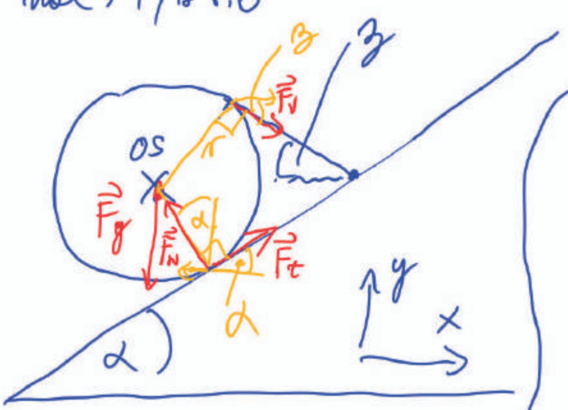
$\alpha = 30°$

$F_g = 50\,N$

$\beta = 45°$

VSE SILE = ?

$k_t = ?$



NAVORI:

$F_V \cdot \cancel{r} = F_t \cdot \cancel{r} \Rightarrow F_V = F_t$

SILE:

x: $F_t \cos\alpha + F_V \cdot \cos\beta - F_N \cdot \sin\alpha = 0$

y: $F_t \sin\alpha - F_V \sin\beta + F_N \cdot \cos\alpha - mg = 0$

$F_t = F_N \cdot k_t \longrightarrow$ x: $\cancel{F_N} k_t(\cos\alpha + \cos\beta) = \cancel{F_N} \sin\alpha$

$$\underline{k_t} = \frac{\sin\alpha}{\cos\alpha + \cos\beta} = \underline{0{,}318}$$

y: $F_N k_t(\sin\alpha - \sin\beta) + F_N \cos\alpha = mg$

$$F_N = mg \frac{1}{\cos\alpha + k_t(\sin\alpha - \sin\beta)}$$

$\underline{F_N = 62{,}5\,N}$

$\underline{F_V} = \underline{F_t} = F_N \cdot k_t = \underline{19.9\,N}$

ZBIRKA 9 nal 8./st 6

$l = 3\,m$

$F_g = 150\,N$

$x_1 = 1\,m$

$d = 1\,m$

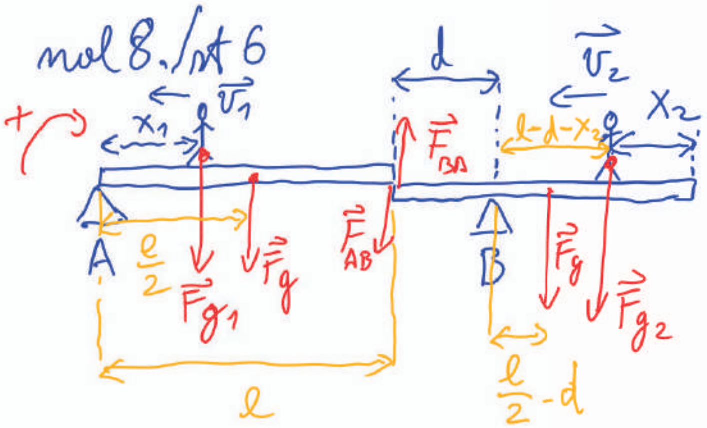
$F_{g_1} = 800\,N$

$F_{g_2} = 600\,N$

$v_2 = 10\,cm/s$

x_2 (RAVNOVESJE) = ?

v_1 (— " —) = ?



$|F_{BA}| = |F_{AB}| = F$

NAVOR:

A: $F_{g_1}x_1 + F_g\frac{l}{2} - Fl = 0$

B: $-Fd + F_g\left(\frac{l}{2}-d\right) + F_{g_2}(l-d-x_2) = 0$

IZ A: $F = F_{g_1}\frac{x_1}{l} + F_g\frac{1}{2}$

V B: $-\left(F_{g_1}\frac{x_1}{l} + F_g\frac{1}{2}\right)d + F_g\left(\frac{l}{2}-d\right) + F_{g_2}(l-d-x_2) = 0$

$-F_{g_1}\frac{x_1}{l}d + F_g\left(\frac{l}{2}-d-\frac{d}{2}\right) + F_{g_2}(l-d) = F_{g_2}x_2$

$x_2 = \frac{F_g}{F_{g_2}}\cdot\frac{1}{2}\underbrace{(l-3d)}_{0} + (l-d) - \frac{F_{g_1}}{F_{g_2}}\frac{x_1}{l}d$

$$\underline{x_2 = l - d - \frac{F_{g_1}}{F_{g_2}}\frac{d}{l}\cdot x_1}$$

$\rightarrow \underline{x_2 = 1{,}56\,m}$

DA PRIDEMO DO HITROSTI ODVAJAMO

$$v_2 = \frac{dx_2}{dt} = -\frac{F_{g_1}}{F_{g_2}}\frac{d}{l}\underbrace{\left(\frac{dx_1}{dt}\right)}_{=-v_1}$$

$v_1 = -\frac{dx_1}{dt}$

$\rightarrow$ GLEDE NA SKICO

$$\underline{v_1 = +\frac{F_{g_2}}{F_{g_1}}\cdot\frac{l}{d}v_2}$$

ZBIRKA 9 nal 12/s16

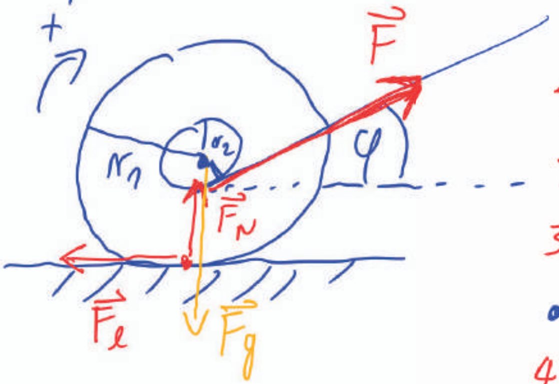
$d_1 = 2r_1 = 2,5\ cm$

$d_2 = 2r_2 = 1\ cm$

$k_l = 0.3$

φ (MIROVANJE) = ?

F(–//–) = ?



• SILE:

1 $x: -F_l + F\cdot\cos\varphi = 0$

2 $y: -F_g + F_N + F\sin\varphi = 0$

3 • $F_l = F_N \cdot k_l$

• NAVORI:

4 $F_l r_1 - F r_2 = 0$

$4: \quad F = F_l \frac{r_1}{r_2}$

$\hookrightarrow 1: \quad -\cancel{F_l} + \cancel{F_l}\frac{r_1}{r_2}\cdot\cos\varphi = 0$

$$\cos\varphi = \frac{r_2}{r_1} = \frac{1}{2,5} \Rightarrow \underline{\underline{\varphi = 66,4^\circ}}$$

$2: \quad F_N = F_g - F\sin\varphi$

$\frac{F_l}{k_l} = F_g - F\sin\varphi$

$F_l = (F_g - F\sin\varphi)k_l$

$F = \frac{r_1}{r_2}(F_g - F\sin\varphi)k_l$

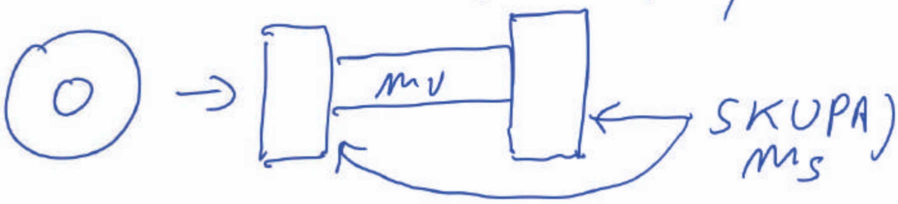
$\frac{r_2}{r_1}F = (F_g - F\sin\varphi)k_l$

$$F = \frac{F_g k_l}{\frac{r_2}{r_1} + k_l \sin\varphi} = \underline{\underline{\frac{F_g k_l}{\cos\varphi + k_l\sin\varphi}}} \sim \frac{F_g}{2}$$

5 nal 5 → DODATEK K ZBIRKA 9 nal 12/str 6

$m = 10\,g$ (MOTEK)

$m_V = m_S = \frac{m}{2}$



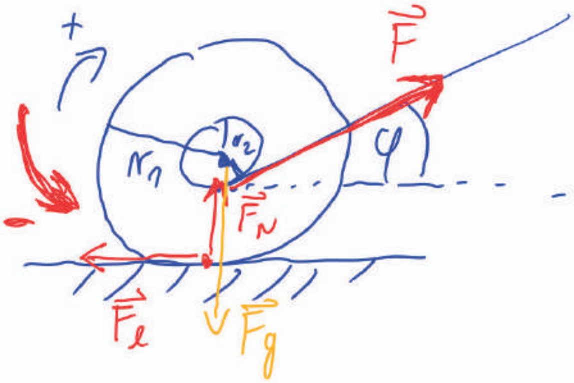
a) $a = 0,\ \alpha \neq 0$

$\varphi = ?$

$F(\varphi = 70^\circ) = ?$

$\alpha(-\text{"}-) = ?$

SPODRSUJE NA MESTU $k_l = k_t$



• SILE:

1 $x: -F_l + F\cdot\cos\varphi = 0$

2 $y: -F_g + F_N + F\sin\varphi = 0$

3 • $F_l = F_N \cdot k_l$

• NAVORI:

4 $F_l r_1 - F r_2 = -J\alpha$

$\alpha > 0$!!

$M = J\alpha$

VZTRAJNOSTNI MOMENT

$1+3+2(-k_t)$: $-\cancel{F_N k_t} + F\cos\varphi = +F_g k_t - \cancel{F_N k_t} - F\sin\varphi\, k_t$

$$F(\cos\varphi + k_t \sin\varphi) = F_g k_t$$

$$F = \frac{F_g k_t}{\cos\varphi + k_t \sin\varphi} > 0$$

$\boxed{J_{VALJ} = \frac{1}{2} m r^2}$

$1+4$: $F\cos\varphi \cdot r_1 - F r_2 = -J\alpha \quad /\cdot\left(-\frac{1}{r_1}\right)$

MORA BITI

$$\alpha = \frac{F r_1\left(\frac{r_2}{r_1} - \cos\varphi\right)}{J} > 0$$

$$J = \frac{1}{2}\left(\frac{m}{2}\right) r_2^2 + \cancel{2}\cdot\frac{1}{\cancel{2}}\left(\frac{m}{4}\right) r_1^2$$

$$J = \frac{m}{4}\left(r_2^2 + r_1^2\right)$$

$\varphi = ?$ $\quad \frac{r_2}{r_1} > \cos\varphi \Rightarrow \boxed{\cos\varphi < \frac{r_2}{r_1}}$

ZA $a = 0, \alpha \neq 0$

$$\boxed{\begin{aligned} F(70^\circ) &= 0{,}048\,N \\ \alpha(70^\circ) &= 76{,}9\,s^{-2} \end{aligned}}$$

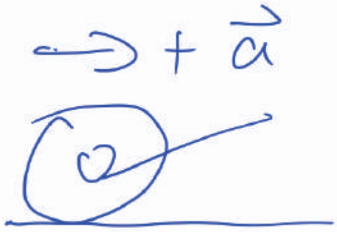
b) $a \neq 0, \alpha = 0$

$\varphi = ?$

$F(60^\circ) = ?$

$a(60^\circ) = ?$

• SILE:



1 X: $-F_\ell + F \cdot \cos\varphi = ma$

2 Y: $-F_g + F_N + F\sin\varphi = 0$

3 • $F_\ell = F_N \cdot k_\ell$

• NAVORI:

4 $F_\ell r_1 - F r_2 = 0 \Rightarrow F_N k_t r_1 = F r_2$

$$F_N = \frac{r_2}{r_1}\frac{1}{k_t}F$$

$$-F_g + \frac{r_2}{k_t r_1}F + F\sin\varphi = 0$$

$$F = \frac{F_g}{\left(\sin\varphi + \frac{r_2}{r_1}\frac{1}{k_t}\right)} > 0$$

$$-\cancel{k_t}\frac{r_2}{r_1}\frac{1}{\cancel{k_t}}F + F\cos\varphi = ma$$

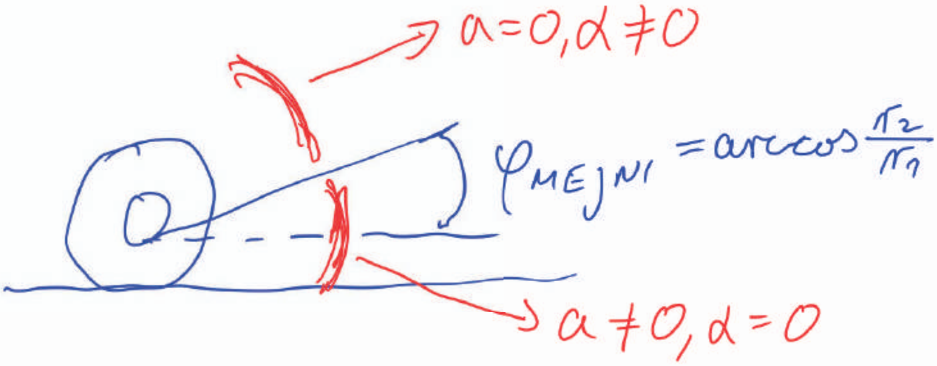
MORA BITI

$$a = \frac{F}{m}\left(\cos\varphi - \frac{r_2}{r_1}\right) > 0$$

$$\boxed{\cos\varphi > \frac{r_2}{r_1}}$$ ZA $a \neq 0$, $\alpha = 0$

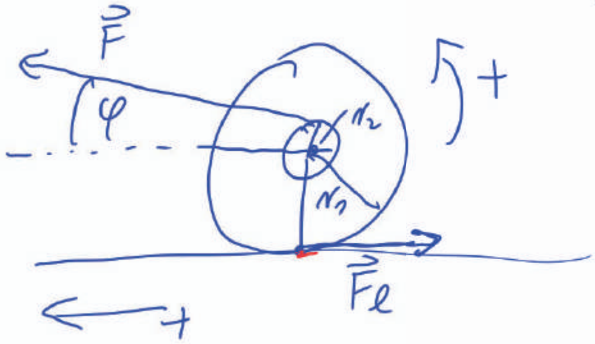
$F(60^\circ) = 0{,}045\,N$

$a(60^\circ) = 0.45\,m/s^2$



e) KOTALI BREZ SPODRSAVANJA $\Rightarrow \underline{\underline{a = \alpha r_1}}$

$a(\varphi) = ?$



SILE:

$X: \underline{F\cos\varphi - F_e = ma},$

$Y: \underline{F_N + F\sin\varphi = mg}$

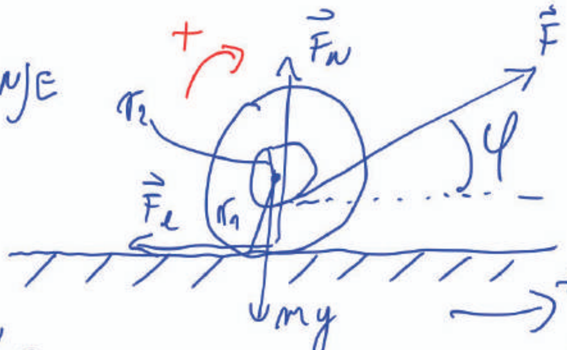
NAVORI:

$F \cdot r_2 + F_e r_1 = J\alpha$

$\underline{F r_2 + F_N \frac{r_1}{R_e} = J \frac{a}{r_1}}$

(5) nal 5

c*) KOTALJENJE



$\alpha = a / r_1$

• SILE:

$x: \; F\cos\varphi - F_e = ma$

$y: \; F_N + F\sin\varphi = mg$

• NAVOR:

$F_e \cdot r_1 - F \cdot r_2 = J \cdot \alpha$

$$(F\cos\varphi - ma) \cdot r_1 - F r_2 = J \frac{a}{r_1} \quad / \cdot \frac{1}{r_1}$$

$$F\left(\cos\varphi - \frac{r_2}{r_1}\right) = a\left(\frac{J}{r_1^2} + m\right)$$

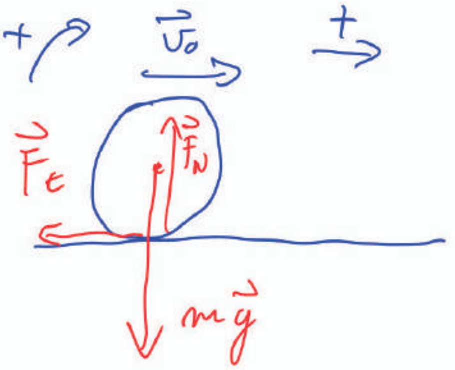
$$\rightsquigarrow \cos\varphi > \frac{r_2}{r_1}$$

(5) mol 6

$v_0 = 5\ m/s$

$k_t = 0.3$

s (DA SE ZAČNE KOTALITI) = ?

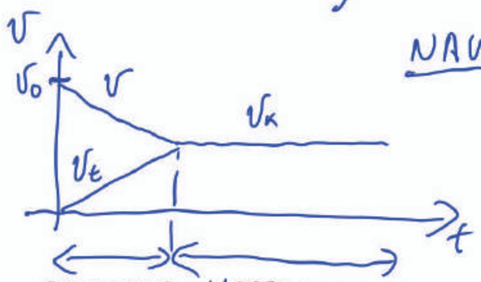


SILE: x: $-F_t = ma$

y: $F_N = mg$

$\Rightarrow -mgk_t = ma$

$\boxed{a = -gk_t}$



NAVORI: $F_t \cdot r = J\alpha$; $J_{KROGLO} = \frac{2}{5} m r^2$

$$\cancel{m} g k_t \cdot \cancel{r} = \frac{2}{5} \cancel{m} r^{\cancel{2}} \cdot \alpha$$

$$\boxed{\alpha = \frac{5}{2} \frac{g k_t}{r}}$$

HITROSTI: $v = v_0 + at$; $v_t = \underbrace{\alpha \cdot r}_{a_t} t$

$$\Rightarrow v_K = v_0 + a t_K = \alpha \cdot r t_K$$

$$v_0 - g k_t t_K = \frac{5}{2} \frac{g k_t}{\cancel{r}} \cancel{r} t_K$$

$$v_0 = t_K \left(\frac{5}{2} g k_t + g k_t\right)$$

$$\boxed{t_K = \frac{v_0}{g k_t} \frac{2}{7}}$$

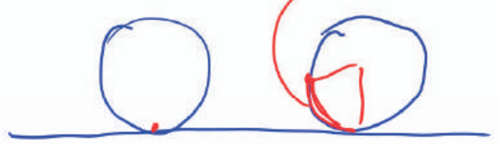
POT:

$$s = v_0 t_K + \frac{a t_K^2}{2} = \frac{v_0^2}{g k_t} \frac{2}{7} - \frac{\cancel{g} \cancel{k_t} v_0^2 \cancel{4}}{\cancel{2} g^{\cancel{2}} k_t^{\cancel{2}} 49}$$

$$= \frac{2 v_0^2}{g k_t} \left(\frac{7}{49} - \frac{1}{49}\right) = \frac{12}{49} \frac{v_0^2}{g k_t} \approx 2{,}1\ m$$

DELO TRENJA:

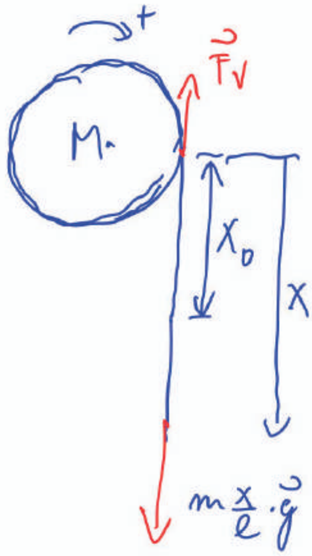
$A = F_t \cdot s'$; $s' = s - r\varphi$ ← KER VEDNO MANJ ZDRSUJE



$$W_{k_0} = W_k + W_{rot} + A \Rightarrow A$$

5) nal 8

M
l
m
x_0
$t = ?$



SILE: $m\frac{x}{l}\cdot g - F_V = m\frac{x}{l}a \Rightarrow F_V = m\frac{x}{l}(g-a)$

NAVORI: $F_V \cdot r = J\alpha \quad ; \alpha = \frac{a}{r}$

$$J = \frac{1}{2}Mr^2 + \left(1-\frac{x}{l}\right)mr^2 = \left(\frac{M}{2} + \left(1-\frac{x}{l}\right)m\right)r^2$$

$$m\frac{x}{l}(g-a)\cancel{r} = \frac{a}{\cancel{r}}\left(\frac{M}{2} + \left(1-\frac{x}{l}\right)m\right)\cancel{r^2}$$

$$m\frac{x}{l}g = a\left(\frac{M}{2} + \left(1-\cancel{\frac{x}{l}}\right)m + \cancel{\frac{x}{l}}m\right)$$

$$m\frac{x}{l}g = a\left(\frac{M}{2} + m\right) \quad /\cdot 2$$

$$\boxed{a = \frac{2m}{M+2m}\cdot\frac{x}{l}g}$$

$$a = \ddot{x} = K^2\cdot x \quad ; K = \sqrt{\frac{2m}{M+2m}\cdot\frac{g}{l}}$$

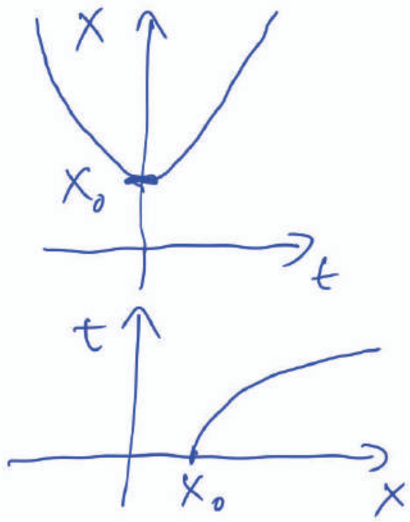
$\ddot{x} = K^2 x \Rightarrow$ REŠUJEMO Z NASTAVKOM:

$$x = Ae^{kt} + Be^{-kt}$$

$\Rightarrow$ ZAČETNI POGOJI:

$$x(t=0) = x_0 \Rightarrow x_0 = A + B = 2A \Rightarrow A = \frac{x_0}{2}$$

$$\dot{x}(t=0) = 0 \Rightarrow 0 = KA - KB \Rightarrow A = B$$



$$\Rightarrow x = x_0\frac{e^{kt}+e^{-kt}}{2} = x_0 \operatorname{ch}(Kt)$$

$$\boxed{t = \frac{1}{K}\operatorname{arcch}\frac{x}{x_0}}$$

$\Rightarrow$ REŠEVANJE Z INTEGRALOM

$\hookrightarrow$ GLEJ VAJE 3.11.2020

ZBIRKA 9 nal 4/st 17

$m_1 = 50\,g$

$m_2 = 150\,g$

$r_3 = 5\,cm$

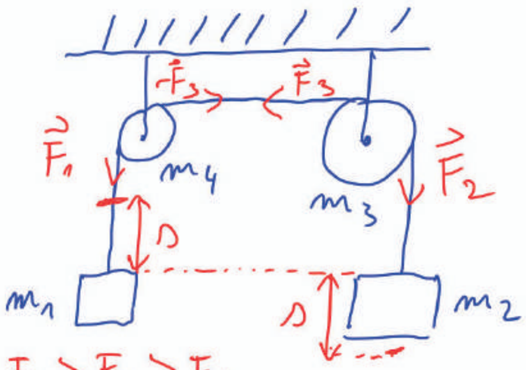
$m_3 = 200\,g$

$r_4 = 4\,cm$

$m_4 = 150\,g$

$s = 10\,cm$

$t = ?$



$F_2 > F_3 > F_1$

$\Rightarrow$ ENAKOMERNO POSPEŠENO

$s = a\frac{t^2}{2}$; $v = at = \underline{\underline{\sqrt{2as}}}$

$\Rightarrow J_4 = \frac{1}{2} m_4 r_4^2$; $J_3 = \frac{1}{2} m_3 r_3^2$

$\omega_4 = \frac{v}{r_4}$; $\omega_3 = \frac{v}{r_3}$

ENERGIJE:

$$0 = m_1 g s + \frac{m_1 v^2}{2} + \frac{J_4 \omega_4^2}{2} + \frac{J_3 \omega_3^2}{2} - m_2 g s + \frac{m_2 v^2}{2} \quad / \cdot 2$$

$$2(m_2 - m_1) g s = m_1 v^2 + \frac{1}{2} m_4 \cancel{r_4^2} \frac{v^2}{\cancel{r_4^2}} + \frac{1}{2} m_3 \cancel{r_3^2} \frac{v^2}{\cancel{r_3^2}} + m_2 v^2 \quad / \cdot 2$$

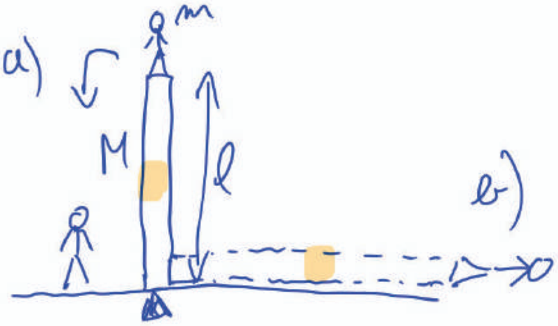
$$4(m_2 - m_1) g s = v^2 (2m_1 + m_4 + m_3 + 2m_2)$$

$$\cancel{4}(m_2 - m_1) g \cancel{a} \frac{t^2}{\cancel{2}} = \cancel{2} \cancel{a} s (2m_1 + m_4 + m_3 + 2m_2)$$

$$t = \sqrt{\frac{s(2m_1 + m_4 + m_3 + 2m_2)}{g(m_2 - m_1)}}$$

$$\underline{\underline{t = 0{,}28\,s}}$$

5 nal 13



HITROST?

$$J = \frac{1}{3} M l^2$$
$$\omega = \frac{v}{l}$$

a) $$\cancel{m} g l = \frac{\cancel{m} v^2}{2}$$
$$\boxed{v = \sqrt{2gh}}$$

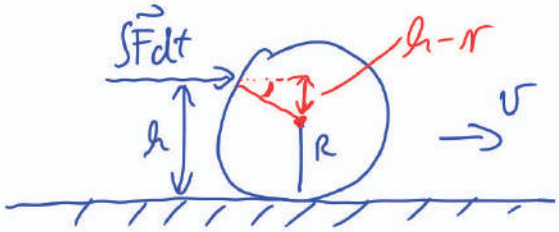
b) $$mgl + Mg\frac{l}{2} = \frac{mv^2}{2} + \frac{J\omega^2}{2} \quad / \cdot 2$$
$$2mgl + Mgl = mv^2 + \frac{1}{3}M\cancel{l^2}\frac{v^2}{\cancel{l^2}}$$
$$gl(2m + M) = v^2\left(m + \frac{M}{3}\right)$$
$$\boxed{v = \sqrt{2gl\underbrace{\frac{\left(m + \frac{M}{2}\right)}{\left(m + \frac{M}{3}\right)}}_{1}}}$$

$$v_a < v_b$$

(5) nal 14

R

$h = ?$



GIBALNA KOLIČINA

$$\vec{G} = m\vec{v} = \int \vec{F} dt$$

VRTILNA KOLIČINA

$$\vec{\Gamma} = J\vec{\omega} = \int \vec{M} dt = \int \vec{r} \times \underbrace{\vec{F} dt}_{\vec{G}}$$

• SUNEK SILE

$$\int F dt = mv$$

• SUNEK NAVORA

$$(h-r)\int F dt = J\omega$$

$$(h-r)\cancel{m}\cancel{v} = \frac{2}{5}\cancel{m}\, r^{\cancel{2}} \frac{\cancel{v}}{\cancel{r}}$$

$$\boxed{h = \frac{7}{5} r}$$

$$J = \frac{2}{5} m r^2$$

$$\omega = \frac{v}{r} \quad \leftarrow \text{KOTALI}$$

(5) nal 15

$m = 20\,kg$

$v = 12\,m/s$

$R = 1\,m$

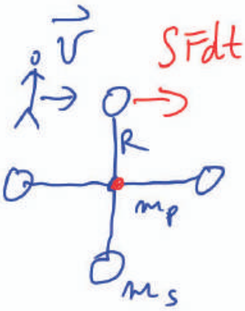
$m_p = 3\,kg$

$m_s = 2\,kg$

$J_{PALICA} = \frac{1}{12} \cdot m\,l^2$

$l = 2R$

OKOLI TEŽIŠČA



SUNEK SILE:

$$\int F dt = m \cdot v$$

UZTRAJNOSTNI MOMENT

$$J = m \cdot R^2 + 2 \cdot \left[\frac{1}{12} m_p (2R)^2\right] + 4 \cdot m_s \cdot R^2$$

$$= R^2 \left(m + \frac{2}{3} m_p + 4 m_s\right)$$

• SUNEK NAVORA: $\vec{\Gamma} = \vec{r} \times \vec{G}$, $\vec{M} = \vec{r} \times \vec{F}$

$$\int M dt = \Delta \Gamma$$

$$R \int F dt = J \omega$$

$$\cancel{R}\, m v = R^{\cancel{2}} \left(m + \frac{2}{3} m_p + 4 m_s\right) \cdot \omega$$

$$\omega = \frac{m}{\left(m + \frac{2}{3} m_p + 4 m_s\right)} \cdot \frac{v}{R}$$

$$\omega = 6{,}9\, s^{-1}$$

5) nal 16

$l = 5\,m$

$m_p = 40\,kg$

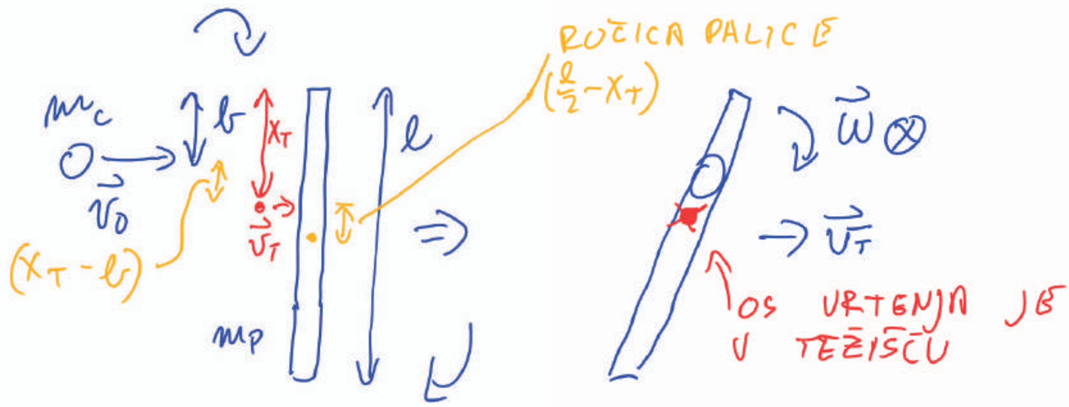
$v_0 = 5\,m/s$

$m_c = 80\,kg$

$b = l/4$

---

v, ω



GIBALNA KOL.

$$m_c v_0 = (m_c + m_p) v_T$$

$$v_T = \frac{m_c}{(m_c + m_p)} v_0$$

$$\boxed{v_T = \frac{2}{3} v_0}$$

TEŽIŠČE:

$$X_T = \frac{m_c b + m_p \cdot \frac{l}{2}}{m_c + m_p}$$

$$\boxed{X_T = \frac{1}{3} l}$$

VRTILNA KOLIČINA
(V TEŽIŠČNI SISTEM)

$$m_c (X_T - b)(v_0 - v_T) + m_p \left(\frac{l}{2} - X_T\right) v_T = J\omega$$

KER OBA PRISPEVKA VRTITA V ISTO SMER!

$$\omega = \frac{m_c (X_T - b)(v_0 - v_T) + m_p \left(\frac{l}{2} - X_T\right) v_T}{J}$$

$$\omega = \underline{\hspace{3cm}}$$

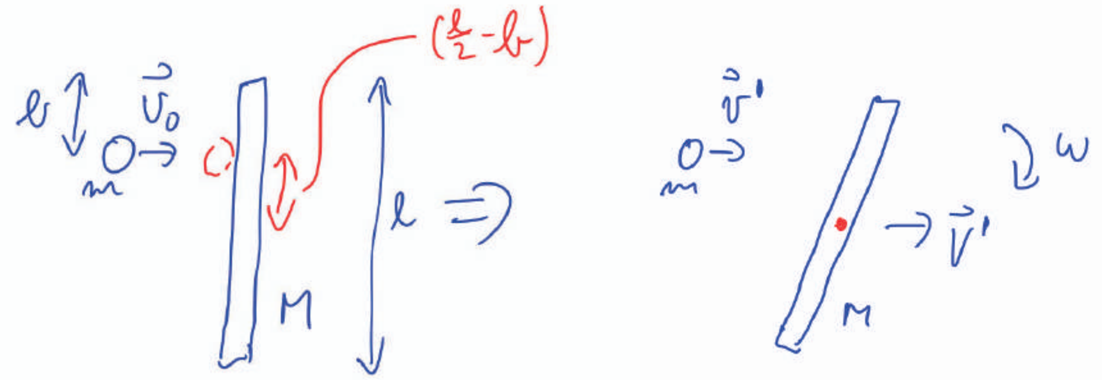
$$J = \frac{1}{12} m_p l^2 + m_p \left(\frac{l}{2} - X_T\right)^2 + m_c (X_T - b)^2$$

STEINERJEV IZREK

$$J = J_T + m (X_{osi} - X_T)^2$$

(5) nal 17

l
M
m
v_0
b



• GIBALNA KOLIČINA

$$m v_0 = m v' + M V'$$

↳ SUNEK SILE:

$$\int F\,dt = m(v_0 - v')$$

$$V' = \frac{m}{M}(v_0 - v')$$

• VRTILNA KOLIČINA (GLEDAMO SAMO DESKO)

$$\left(\frac{l}{2} - l\right)\int F\,dt = J \cdot \omega$$

$$\left(\frac{l}{2} - b\right) m (v_0 - v') = \frac{1}{12} M l^2 \omega$$

$$\omega = \frac{m\left(\frac{l}{2} - b\right)(v_0 - v')}{J}$$

• ENERGIJE:

$$\frac{m v_0^2}{2} = \frac{m v'^2}{2} + \frac{M V'^2}{2} + \frac{J \omega^2}{2}$$

$$m v_0^2 = m v'^2 + \cancel{M}\frac{m^2}{M^{\cancel{2}}}(v_0 - v')^2 + \cancel{J}\frac{m^2\left(\frac{l}{2} - l\right)^2 (v_0 - v')^2}{J^{\cancel{2}}}$$

$$\cancel{(v_0 - v')}(v_0 + v')$$

$$\cancel{m}\cancel{(v_0^2 - v'^2)} = (v_0 - v')^{\cancel{2}}\cancel{m}\left(\frac{m}{M} + \frac{m\left(\frac{l}{2} - b\right)^2}{\frac{1}{12} M l^2}\right)$$

$$v_0 + v' = (v_0 - v')\frac{m}{M}\left(1 + \frac{12\left(\frac{l}{2} - b\right)^2}{l^2}\right)$$

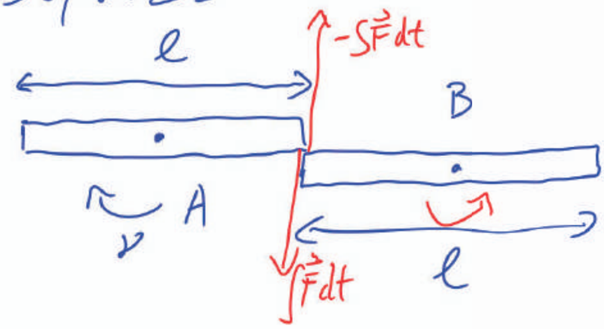
$$\boxed{v' = \frac{\frac{m}{M}\left[1 + \frac{12(l/2 + b)^2}{l^2}\right] - 1}{1 + \frac{m}{M}\left[1 + \frac{12(l/2 - b)^2}{l^2}\right]} v_0}$$

ZBIRKA 9 nal 36/st 25

$l = 1\,m$

$\nu_0 = 1\,s^{-1}$

ω_A, ω_B PO TRKU ?



• VRTILNA KOLIČINA OBRNE SMER

↳ ZARADI SIL V OSEH → ZUNANJE SILE

• Γ: A: $J\omega_0 = \underline{r\int F dt} + J\omega_A$ $\longrightarrow$ $\cancel{J}\omega_0 = \cancel{J}\omega_B + \cancel{J}\omega_A$

↳ SUNEK ZARAD TRKA

B: $r\int F dt = J\omega_B$

$\omega_B = \omega_0 - \omega_A$

• W: $\frac{J\omega_0^2}{2} = \frac{J\omega_A^2}{2} + \frac{J\omega_B^2}{2}$

$\omega_0^2 = \omega_A^2 + (\omega_0 - \omega_A)^2$

$\omega_0^2 - \omega_A^2 = (\omega_0 - \omega_A)^2$

$(\omega_0 + \omega_A)\cancel{(\omega_0 - \omega_A)} = (\omega_0 - \omega_A)^{\cancel{2}}$

$\underline{\omega_A = 0}$ $\longrightarrow$ $\underline{\omega_B = \omega_0}$

ZBIRKA 9 nal 30/st 20

$R = 25\,cm$

$m = 1\,kg$

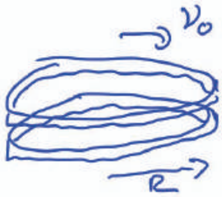
$\nu_0 = 10\,Hz$

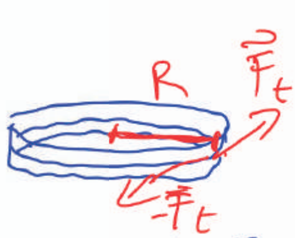
$k_t = 0.2$

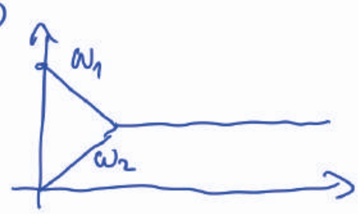
$N_{ZG} = ?$

$N_{SP} = ?$

↳ DO TAKRAT KO DOSEŽETA SKUPNO ω







Γ: $J\omega_0 = 2J\omega_K$

$\underbrace{\omega_K = \frac{\omega_0}{2}}$

M: $F_t \cdot R = J\alpha$

$\cancel{m} g k_t \cancel{R} = \cancel{m} R^{\cancel{2}} \alpha$

$\underbrace{\alpha = \frac{g k_t}{R}}$

ω: $\omega_1 = \omega_0 - \alpha t$

$\omega_2 = \alpha \cdot t$

ČAS: $\omega_2 = \omega_K = \alpha \cdot \tilde{t}$

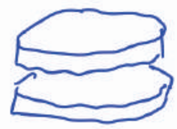
$\underbrace{\tilde{t} = \frac{\omega_0}{2}\frac{R}{g k_t}}$

KOT: $\varphi_1 = \omega_0 \tilde{t} - \frac{\alpha \tilde{t}^2}{2} = \left(\omega_0 - \frac{\alpha \tilde{t}}{2}\right)\tilde{t} = \left(\omega_0 - \frac{\cancel{g k_t}}{2\cancel{R}}\frac{\omega_0 \cancel{R}}{2\cancel{g k_t}}\right)\frac{\omega_0 R}{2 g k_t}$

$\underbrace{\varphi_1 = \frac{3}{8}\frac{\omega_0^2 R}{g k_t}} = 2\pi \cdot 30 \Rightarrow \underline{N_{ZG} = 30}$

$\varphi_2 = \frac{\alpha \tilde{t}^2}{2} = 2\pi \cdot 10 \Rightarrow \underline{N_{SP} = 10}$

KAJ ČE NAMESTO OBROČEV DISKI?



$dm = m\frac{2\cancel{\pi} r\,dr}{\cancel{\pi} R^2}$

- $J = \frac{1}{2} m R^2$
- Γ: $J\omega_0 = 2J\omega_K$

$\underbrace{\omega_K = \frac{\omega_0}{2}}$

M: $\int_0^R dF_t \cdot r = J\alpha$ $\quad dF_t = k_t \cdot dm\, g$

$m k_t g \int_0^R \frac{2 r^2 dr}{R^2} = \frac{1}{2} m R^2 \alpha$

$\frac{2\cancel{m} k_t g}{\cancel{R^2}}\frac{\cancel{R^3}}{3} = \frac{1}{2}\cancel{m} R^{\cancel{2}} \alpha$

$\underbrace{\alpha = \frac{4}{3}\frac{g k_t}{R}}$

$$\boxed{\begin{aligned}\tilde{t} &= 3\,s\\ N_{ZG} &= 22.5\\ N_{SP} &= 7.5\end{aligned}}$$

88/89 1. popr. kol. 1. rok

$m_1 = m_2 = m = 100\,g$

$r_1 = 5\,cm$

$r_2 = 10\,cm$

$\nu_0 = 5\,Hz$

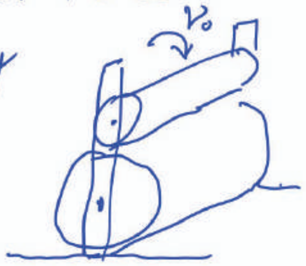
$t = 5\,s$

$k_t = 0{,}15$

a) $k_{tp} = 0$

b) $k_{tp} = 0{,}05$

$\varphi_2 = ?$



a)

• M: 1.) $F_t r_1 = \alpha_1 J_1$

$\cancel{m} g k_t \cancel{r_1} = \alpha_1 \frac{\cancel{m} r_1^{\cancel{2}}}{2}$

$\boxed{\alpha_1 = \frac{2 g k_t}{r_1}}$

2.) $F_t r_2 = \alpha_2 \cdot J_2$

$\hookrightarrow \boxed{\alpha_2 = \frac{2 g k_t}{r_2}}$

• NEHA SPODRSOVATI KO $v_{t1} = v_{t2}$

$\omega_{1K} r_1 = \omega_{2K} r_2 \Rightarrow \omega_{1K} = \omega_{2K} \frac{r_2}{r_1}$

• ČAS SPODRSOVANJA:

$\omega_1 = \omega_0 - \alpha_1 t$

$\omega_2 = \alpha_2 t$

$(\omega_0 - \alpha_1 \tilde{t}) r_1 = \alpha_2 \cdot \tilde{t}\, r_2$

$\omega_0 r_1 = \tilde{t}(\alpha_2 r_2 + \alpha_1 r_1)$

$\tilde{t} = \frac{\omega_0 r_1}{2 g k_t \left(\frac{r_2}{r_2} + \frac{r_1}{r_1}\right)} = \frac{\omega_0 r_1}{4 g k_t}$

• KOT: $\varphi_2 = \frac{\alpha_2 \tilde{t}^2}{2} + \underbrace{\omega_{2K}}_{\alpha_2 \tilde{t}}(t - \tilde{t})$

$= \frac{\cancel{2 g k_t} \cdot \omega_0^2 r_1^2}{\cancel{2} r_2 \cdot 16 g^{\cancel{2}} k_t^{\cancel{2}}} + \frac{2 \cancel{g k_t} \cdot \omega_0 r_1}{r_2 \cdot 4 \cancel{g k_t}}(t - \tilde{t})$

$\tilde{t} = \frac{\pi}{12}\,s$

$= \omega_0 \frac{r_1}{r_2}\left(\frac{\omega_0 r_1}{16 g k_t} + \frac{t - \tilde{t}}{2}\right) = 38{,}2\ \text{rad}$

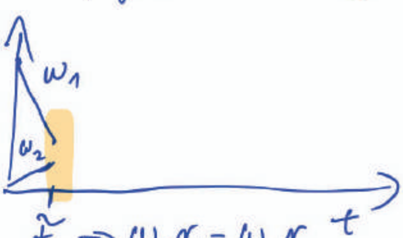
---

b) M: 1.) $F_t r_1 = \alpha_1 J_1 \Rightarrow \alpha_1 = \frac{2 g k_t}{r_1}$

2.) $r_2 (F_t - F_{tp}) = \alpha_2 J_2$

$r_2 \cancel{m} g (k_t - 2 k_{tp}) = \alpha_2 \frac{\cancel{m} r_2^2}{2} \Rightarrow \boxed{\alpha = \frac{2 g (k_t - 2 k_{tp})}{r_2}} = 10\,s^{-2}$

$F_N = 2mg$



$\tilde{t} \Rightarrow \omega_1 r_1 = \omega_2 r_2$

KO NEHA SPODRSOVATI:

$\omega_1 r_1 = \omega_2 r_2$

$(\omega_0 - \alpha_1 \tilde{t}) r_1 = \alpha_2 \cdot \tilde{t} \cdot r_2$

$\tilde{t} = \frac{\omega_0}{\alpha_1 + \alpha_2 \frac{r_2}{r_1}} = \frac{\omega_0 r_1}{2 g (k_t - k_{tp})} = \frac{\pi}{8}\,s$

• PO $\tilde{t}$: $F_t \to F_e$, $\alpha_1^* r_1 = \alpha_2^* r_2 \Rightarrow \alpha_1^* = \alpha_2^* \frac{r_2}{r_1}$

M: 1.) $F_e \cdot r_1 = \alpha_1^* J_1 \Rightarrow F_e = \alpha_2^* \frac{r_2}{\cancel{r_1}} \frac{m \cancel{r_1^2}}{2} \frac{1}{\cancel{r_1}} = \alpha_2^* \frac{m r_2}{2}$

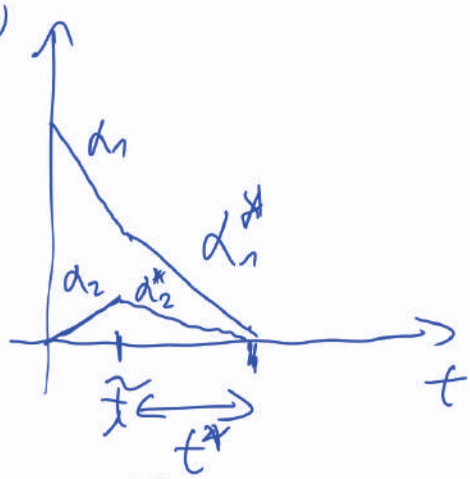
2.) KER USTAVLJA $|F_{tp}| > |F_e|$

$$r_2 (F_{tp} - F_e) = \alpha_2^* J_2$$

$$\cancel{r_2}\left(\cancel{m} g\, 2 k_{tp} - \alpha_2^* \frac{\cancel{m} r_2}{2}\right) = \alpha_2^* \frac{\cancel{m} r_2^{\cancel{2}}}{\cancel{2}}$$

$$2 g k_{tp} = \alpha_2^* r_2$$

$$\boxed{\alpha_2^* = \frac{2 g k_{tp}}{r_2}} = 10\, s^{-2}$$



$$\alpha_2^* = \alpha_2 \Rightarrow t^* = \tilde{t}$$

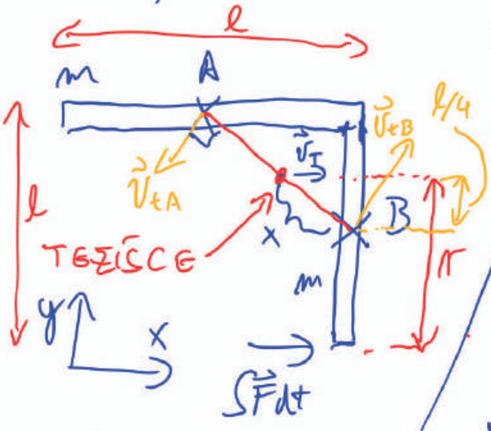
$$\Rightarrow \varphi_2 = \frac{\alpha_2 \tilde{t}^2}{2} + \frac{\alpha_2^* t^{*2}}{2} = \alpha_2 \tilde{t}^2 = 1{,}57 \text{ rad}$$

CELOTEN ČAS DO USTAVLJENJA

$$\hookrightarrow \tilde{t} + t^* = \underbrace{2\tilde{t}}_{2 \cdot \frac{\pi}{8} s} < t = 5 s$$

90/91 2. kol / 3. rok

$$\frac{v_A}{v_B} = ?$$



- $r = \frac{3}{4} l$
- $x = \frac{l}{4} \cdot \sqrt{2}$

$$G: \int F dt = 2 m v_T$$

GLEDAMO ROTACIJO OKOLI TEŽIŠČA

$$\Gamma: r \int F dt = J \omega$$

$$J = 2\left(\frac{1}{12} m l^2 + m \underbrace{\left(\frac{l}{4}\sqrt{2}\right)}_{=x}{}^2\right)$$

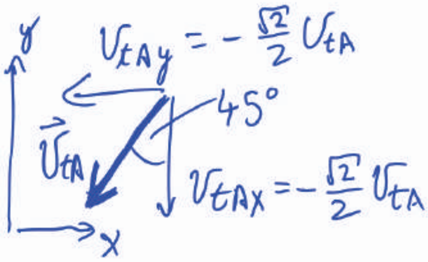
$$J = 2 m l^2 \left(\frac{1}{12} + \frac{1}{8}\right) = \frac{5}{12} m l^2$$

$$\frac{3}{4} \cancel{l} \cdot 2 \cancel{m} v_T = \frac{5}{12} \cancel{m} l^{\cancel{2}} \omega$$

$$v_T = \frac{5}{18} l \omega$$

- HITROSTI A IN B GLEDE NA TEŽIŠČE

$$v_{tA} = \omega \cdot x = v_{tB} = \frac{\sqrt{2} \cancel{l} \cdot 18}{4 \cdot 5 \cdot \cancel{l}} v_T = \frac{\sqrt{2}\, 9}{10} v_T$$



- DEJANSKE HITROSTI:

$$\vec{v}_A = (v_T, 0) + v_{tA}\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = \left(v_T - \frac{\sqrt{2}}{2} v_{tA}, -\frac{\sqrt{2}}{2} v_{tA}\right)$$

$$\vec{v}_B = (v_T, 0) + v_{tB}\left(+\frac{\sqrt{2}}{2}, +\frac{\sqrt{2}}{2}\right) = \left(v_T + \frac{\sqrt{2}}{2} v_{tB}, +\frac{\sqrt{2}}{2} v_{tB}\right)$$

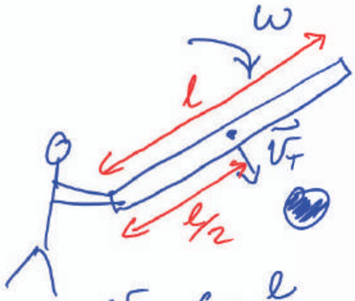
$$\frac{v_A}{v_B} = \frac{\sqrt{v_{Ax}^2 + v_{Ay}^2}}{\sqrt{v_{Bx}^2 + v_{By}^2}} = \sqrt{\frac{41}{221}}$$

(5) nal 22

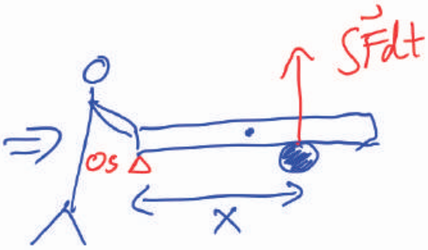
a) BREZ SUNKA V ROKI

b) SUNEK SILE V ROKI

VEDNO KIJ OBMIRUJE



$v_T = \omega \cdot \frac{l}{2}$



a) G: $m v_T - \int F dt = 0$

$$\boxed{m\omega\frac{l}{2} = \int F dt}$$

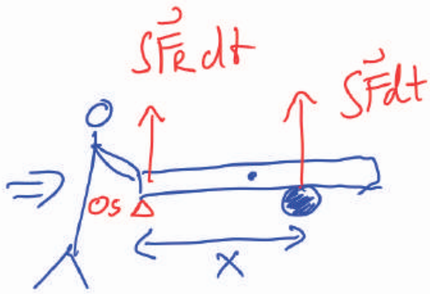
$J = \frac{1}{3} m l^2$

Γ: $J\omega - X\int F dt = 0$

$$\frac{1}{3}\cancel{m} l^{\cancel{2}}\cancel{\omega} = X \cancel{m}\cancel{\omega}\frac{\cancel{l}}{2}$$

$$\boxed{X = \frac{2}{3} l}$$

→ DA NI SUNKA V ROKI



b) G: $m\omega\frac{l}{2} - \int F_R dt - \int F dt = 0$

Γ: $J\omega - X\int F dt = 0$

$$m\omega\frac{l}{2} - \int F_R dt - \frac{1}{3}\frac{m l^2 \omega}{X} = 0$$

$$\boxed{\int F_R dt = m\omega l\left(\frac{1}{2} - \frac{l}{3X}\right)}$$

$X > \frac{2}{3} l \Rightarrow \int F_R dt > 0$ ↑

$X < \frac{2}{3} l \Rightarrow \int F_R dt < 0$ ↓

⑤ nal 24

$l_1 = 1m$

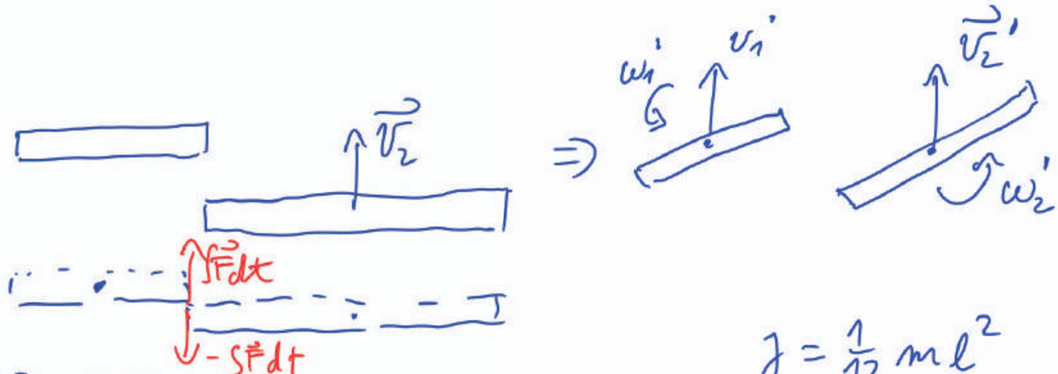
$l_2 = 2m$

$m_1 = m_2 = m$

$v_2 = 3 m/s$

$v_1' = ?$

$\omega_1' = ?$



OB TRKU:

$J = \frac{1}{12} m l^2$

G: 1.) $\int F dt = m v_1'$

2.) $m v_2 - \int F dt = m v_2' \longrightarrow \cancel{m} v_2 - \cancel{m} v_1' = \cancel{m} v_2'$

$\boxed{v_2' = v_2 - v_1'}$

T: 1.) $\frac{l_1}{2} \int F dt = J_1 \omega_1'$

$\frac{\cancel{l_1}}{2} \cancel{m} v_1' = \frac{1}{12} \cancel{m} l_1^{\cancel{2}} \omega_1' \Rightarrow \boxed{\omega_1' = 6 \cdot \frac{v_1'}{l_1}}$

2.) $\frac{l_2}{2} \int F dt = J_2 \omega_2'$

$\frac{\cancel{l_2}}{2} \cancel{m} v_1' = \frac{1}{12} \cancel{m} l_2^{\cancel{2}} \omega_2' \Rightarrow \boxed{\omega_2' = 6 \frac{v_1'}{l_2}}$

W: $\frac{1}{2} m v_2^2 = \frac{1}{2} m v_1'^2 + \frac{1}{2} J_1 \omega_1'^2 + \frac{1}{2} m v_2'^2 + \frac{1}{2} J_2 \omega_2'^2 \quad / \cdot 2$

$\cancel{m} v_2^2 = \cancel{m} v_1'^2 + \frac{1}{12} \cancel{m} \cancel{l_1^2} 36 \frac{v_1'^2}{\cancel{l_1^2}} + \cancel{m} (v_2 - v_1')^2 + \frac{1}{12} \cancel{m} \cancel{l_2^2} 36 \frac{v_1'^2}{\cancel{l_2^2}}$

$\cancel{v_2^2} = v_1'^{\cancel{2}} + 3 v_1'^{\cancel{2}} + \cancel{v_2^2} - 2 v_2 \cancel{v_1'} + v_1'^{\cancel{2}} + 3 v_1'^{\cancel{2}}$

$2 v_2 = 8 v_1'$

$\boxed{v_1' = \frac{1}{4} v_2} = 0{,}75 \, m/s$

$\underline{\omega_1'} = 6 \cdot \frac{1}{4} \frac{v_2}{l_1} = \underline{\frac{3}{2} \frac{v_2}{l_1}} = \underline{\underline{4{,}5 \, s^{-1}}}$

⑤ nal 2.5

$R = 5\,cm$

$v_1 = 10\,m/s$

$\omega_1 = 20\,s^{-1}$

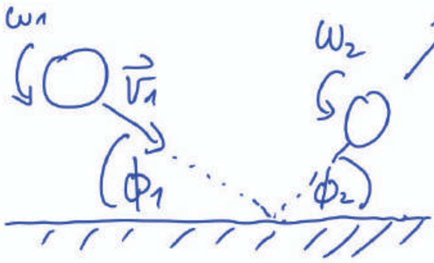
$\phi_1 = 45°$

$\omega_2 = 10\,s^{-1}$

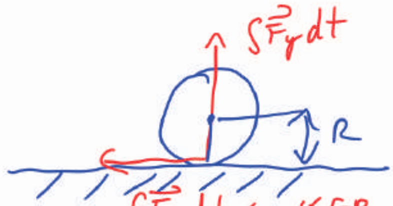
$v_2 = ?$

$\phi_2 = ?$

$|v_{1y}| = |v_{2y}|$



OB TRKU:



$\leftarrow$ KER $\omega_1 \neq \omega_2$

G: x: $m v_1 \frac{\sqrt{2}}{2} - \int F_x dt = m v_2 \cos\phi_2$

Γ: $\frac{2}{5} m R^{\cancel{2}} \omega_1 - \cancel{R} \int F_x dt = \frac{2}{5} m R^{\cancel{2}} \omega_2$

$\frac{2}{5} \cancel{m} R (\omega_1 - \omega_2) = \cancel{m} \left(v_1 \frac{\sqrt{2}}{2} - v_2 \cos\phi_2\right)$

$v_{2y} = v_2 \cdot \sin\phi_2$

$v_2 = \frac{v_{2y}}{\sin\phi_2}$

$\boxed{v_2 = \frac{v_1 \frac{\sqrt{2}}{2}}{\sin\phi_2}}$

$\frac{2}{5} R (\omega_1 - \omega_2) = v_1 \frac{\sqrt{2}}{2} \left(1 - \mathrm{ctg}\,\phi_2\right)$

$\frac{4\, R(\omega_1 - \omega_2)}{5\sqrt{2}\; v_1} = 1 - \mathrm{ctg}\,\phi_2$

$\mathrm{ctg}\,\phi_2 = 1 - \frac{4 R (\omega_1 - \omega_2)}{5\sqrt{2}\; v_1}$

$\hookrightarrow \underline{\underline{\phi_2 = 45{,}8°}}$

5 nal 23

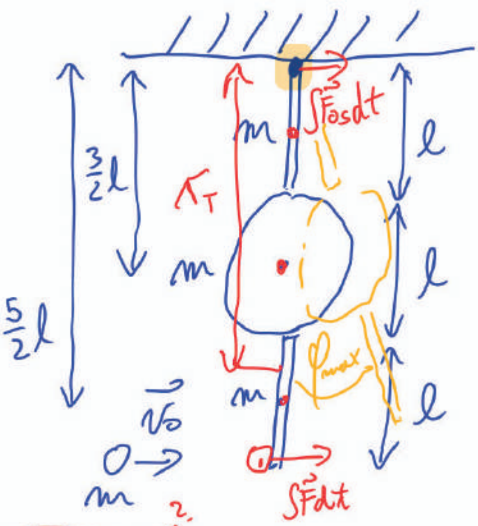
$m = 1\,kg$

$v_0 = 0{,}6\,m/s$

$l = 10\,cm$

$\varphi_{max} = ?$

$\int \vec{F}_{os}\,dt = ?$



• SUNEK SILE: $\int F dt = m v_0$

• SUNEK NAVORA: $3l \int F dt = J\omega$

$\hookrightarrow \underline{3 l m v_0 = J \omega}$

$$\cancel{3}\cancel{l}\cancel{m} v_0 = \overset{6}{\cancel{18}}\cancel{m} l^{\cancel{2}} \omega$$

$$\boxed{\omega = \frac{1}{6}\frac{v_0}{l}}$$

$$J = \underbrace{\frac{1}{3} m l^2}_{1.} + \underbrace{\frac{2}{5} m \left(\frac{l}{2}\right)^2 + m\left(\frac{3}{2}l\right)^2}_{2.} + \underbrace{\frac{1}{12} m l^2 + m\left(\frac{5}{2}l\right)^2}_{3.} + \underbrace{m(3l)^2}_{\text{KROGLICA}}$$

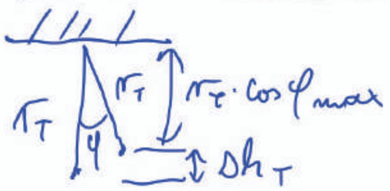
$$J = m l^2 \left(\frac{1}{3} + \frac{1}{10} + \frac{9}{4} + \frac{1}{12} + \frac{25}{4} + 9\right)$$

$$\boxed{J \doteq 18 m l^2}$$

TEŽIŠČE: $r_T = \dfrac{l\left(\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + 3\right)\cancel{m}}{4\cancel{m}}$

$$\boxed{r_T = \frac{15}{8} l}$$

$$\boxed{\Delta h_T = r_T (1 - \cos\varphi_{max})}$$



• ENERGIJE: (PO TRKU)

$$\frac{J\omega^2}{2} = 4 m g \Delta h_T$$

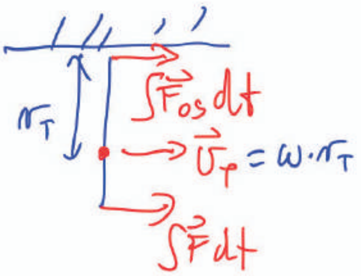
$$\frac{\cancel{18}\cancel{m}\cancel{l^2} v_0^2}{\cancel{2}\cdot \cancel{36} \cancel{l^2}} = \cancel{4}\cancel{m} g \frac{15}{\cancel{8}} l (1 - \cos\varphi_{max})$$

$$\frac{v_0^2}{30 g l} = (1 - \cos\varphi_{max})$$

$$\cos\varphi_{max} = 1 - \frac{v_0^2}{30 g l}$$

$$\boxed{\varphi_{max} = 8{,}9^\circ}$$

SUNEK SILE U OSI: $\rightarrow$ GIBANJE TEŽIŠČA:



$$4 m v_T = \int F_{os} dt + \underbrace{\int F dt}_{= m v_0}$$

$$\int F_{os} dt = 4 m v_T - m v_0$$

$$= m(4 \omega r_T - v_0)$$

$$= m\left(4 \frac{1}{6} \frac{v_0}{\cancel{l}} \frac{15}{8} \cancel{l} - v_0\right)$$

$$\boxed{\int F_{os} dt = \frac{m v_0}{4}} \rightarrow \int \vec{F}_{os} dt \text{ KAŽE U DESNO}$$

(6) 8

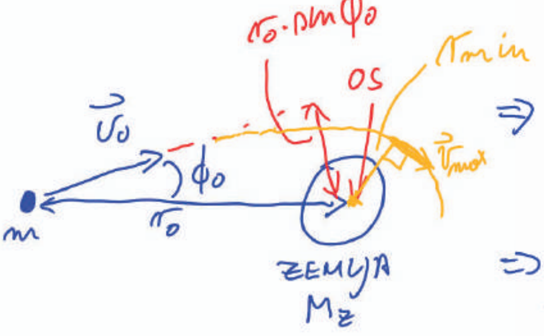
$r_0 = 100 R_Z$

$R_Z = 6400\,km$

$v_0 = 1\,km/s$

$\phi_0 = 10^\circ$

$d_{min} = ?$



⇒ KER $m << M_Z$ → TEŽIŠČE POSTAVIMO V SREDIŠČE ZEMLJE IN OS TUDI

⇒ KER NI ZUNANJIH SIL SE Γ OHRANJA

$$\Gamma: \cancel{m} v_0 r_0 \sin\phi_0 = \cancel{m} v_{max} r_{min} \Rightarrow \underline{v_{max} = v_0 \frac{r_0 \sin\phi_0}{r_{min}}}$$

$$W: \frac{m v_0^2}{2} - \frac{\kappa M_Z m}{r_0} = \frac{m v_{max}^2}{2} - \frac{\kappa M_Z m}{r_{min}}$$

$$\frac{v_0^2}{2} - \frac{\kappa M_Z}{r_0} = \frac{v_0^2 r_0^2 \sin^2\phi_0}{2 r_{min}^2} - \frac{\kappa M_Z}{r_{min}} \quad \Big/ \cdot 2 r_{min}^2$$

NA POVRŠINI:

$$\cancel{m} g_0 = \frac{\cancel{m} M_Z \kappa}{R_Z^2}$$

$$\hookrightarrow M_Z \kappa = g_0 R_Z^2$$

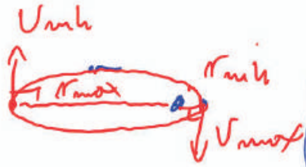
$$\underbrace{2\left(\frac{v_0^2}{2} - \frac{g_0 R_Z^2}{r_0}\right)}_{A} r_{min}^2 + \underbrace{2 g_0 R_Z^2}_{B} r_{min} \underbrace{- v_0^2 r_0^2 \sin^2\phi_0}_{C} = 0$$

$$A = 2\left(\frac{10^6\,m^2}{2\,s^2} - \frac{10\,m \cdot 6{,}4^{\cancel{2}} \cdot 10^{\cancel{12}\,6}\,m^{\cancel{2}}}{s^2 \cdot 100 \cdot \cancel{6{,}4} \cdot \cancel{10^6}\,\cancel{m}}\right) = \left(10^6 - 12{,}8 \cdot 10^5\right)\frac{m^2}{s^2} =$$

$$= -2{,}8 \cdot 10^5\,\frac{m^2}{s^2}$$

$$B = 8{,}2 \cdot 10^{14}\,m^3/s^2$$

$$C = -2{,}27 \cdot 10^{22}\,m^4/s^2$$



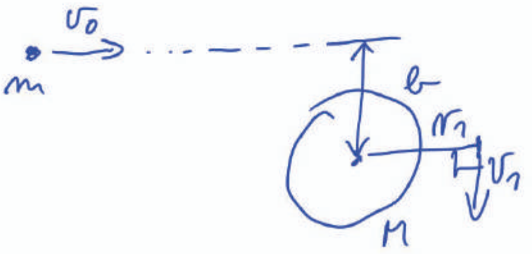
$$\Rightarrow r_{min} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \begin{cases} 2{,}9 \cdot 10^9\,m \leftarrow r_{max}(v_{min}) \\ \underline{7{,}5 \cdot 10^6\,m} \leftarrow r_{min}(v_{max}) \end{cases}$$

$$d_{min} = r_{min} - R_Z$$

$$= \underline{900\,km}$$

$$\left(\frac{v_0^2}{2} - \frac{\kappa M}{r_0}\right) = \begin{cases} < 0 \rightarrow \text{ELIPSA} \\ = 0 \rightarrow \text{PARABOLA} \\ > 0 \rightarrow \text{HIPERBOLA} \end{cases}$$

6) nal 9



$v_0 = 20\,\mathrm{km/s}$

$b = ct$

$t = 60\,\mathrm{s}$

$M = 2\cdot 10^{30}\,\mathrm{kg}$

$R = 7\cdot 10^{8}\,\mathrm{m}$

$d_{min} = ?$

$\Gamma:\ m v_0 b = m v_1 r_1 \Rightarrow \underline{v_1 = v_0 \frac{b}{r_1}}$

$W:\ \frac{m v_0^2}{2} - \frac{\kappa M m}{r_0} = \frac{m v_1^2}{2} - \frac{\kappa M m}{r_1}$

0 → KER DALEČ OD SONCA

$$\Rightarrow \frac{v_0^2}{2} = \frac{v_0^2 b^2}{2 r_1^2} - \frac{\kappa M}{r_1} \quad / \cdot 2 r_1^2$$

$$v_0^2 r_1^2 + 2\kappa M r_1 - v_0^2 b^2 = 0$$

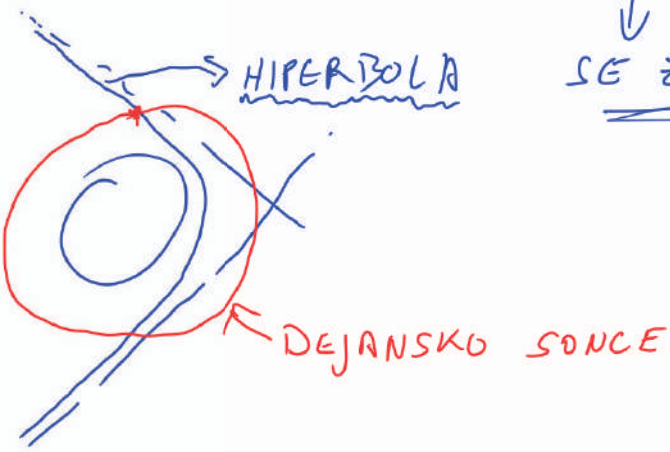
$$r_1 = \frac{-2\kappa M \pm \sqrt{4\kappa^2 M^2 + 4 v_0^4 b^2}}{2 v_0^2}$$

$$r_1 = -\frac{\kappa M}{v_0^2} \pm \sqrt{\frac{\kappa^2 M^2}{v_0^4} + b^2}$$

$$r_1 = \sqrt{\frac{\kappa^2 M^2}{v_0^4} + b^2} - \frac{\kappa M}{v_0^2} = \underline{4{,}85\cdot 10^{8}\,\mathrm{m}} < R$$

↓

SE ZALETI



(6) nal 10

$M = 5 \cdot 10^{31}\,\text{kg}$

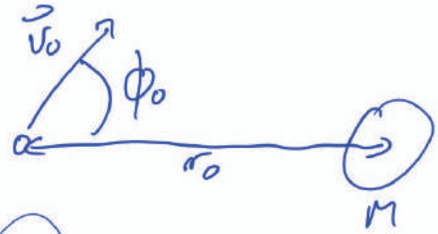
$r_0 = 6{,}3 \cdot 10^{9}\,\text{m}$

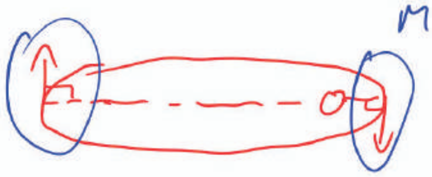
$v_0 = 20\,\text{km/s}$

$\phi_0 = 60^\circ$

$r_{min} = ?$

$r_{max} = ?$





Γ: $m v_0 r_0 \sin\phi_0 = m v_1 r_1$

$\hookrightarrow \underbrace{v_1 = v_0 \frac{r_0 \sin\phi_0}{r_1}}$

W: $\frac{m v_0^2}{2} - \frac{GMm}{r_0} = \frac{m v_1^2}{2} - \frac{GMm}{r_1}$

$$\Rightarrow \frac{v_0^2}{2} - \frac{GM}{r_0} = \frac{v_0^2 r_0^2 \sin^2\phi_0}{2 r_1^2} - \frac{GM}{r_1} \quad / \cdot 2 r_1^2$$

$$r_1 = \begin{cases} 3{,}57 \cdot 10^{7}\,\text{m} \\ 6{,}3 \cdot 10^{10}\,\text{m} \end{cases}$$

7 Nihanje

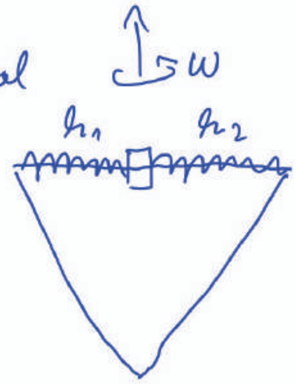
86/87 2. kol/2. nal

$\omega = 10\,s^{-1}$

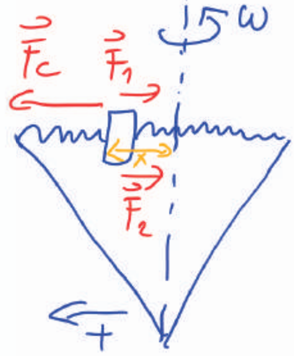
$m = 20\,g$

$k_1 = 1\,N/m$

$k_2 = 3\,N/m$



$\Rightarrow$



$F_1 = k_1 x$

$F_2 = k_2 x$

$F_c = m a_r = m\omega^2 x$

$$-F_1 - F_2 + F_c = m a$$

$$(-k_1 - k_2 + m\omega^2)x = m\ddot{x}$$

$$-\left(\frac{k_1+k_2}{m} - \omega^2\right)x = \ddot{x} \longleftrightarrow$$

ENAČBA NIHANJA:

$$\ddot{x} = -\omega_0^2 x$$

$$\ddot{x} + \omega_0^2 x = 0$$

$$\hookrightarrow \omega_0 = \sqrt{\frac{k_1+k_2}{m} - \omega^2}$$

$$\omega_0 = \sqrt{\frac{4}{0{,}02} - 100}\,s^{-1} = \underline{\underline{10\,s^{-1}}}$$

$$\underline{x = A\sin\omega_0 t + B\cos\omega_0 t},$$

$$\dot{x} = A\omega_0\cos\omega_0 t - B\omega_0\sin\omega_0 t$$

ZAČETNI POGOJI

$x(t=0) = x_1 = 1\,cm$

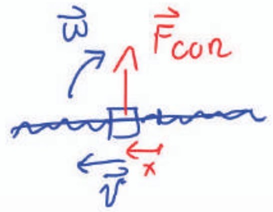
$v(t=0) = v_1 = 10\,cm/s$

$\Rightarrow x_1 = B$

$\Rightarrow v_1 = A\omega_0$

$$\Rightarrow \underline{\underline{x = \frac{v_1}{\omega_0}\sin\omega_0 t + x_1\cos\omega_0 t}}$$

TLORIS:



• CORIOLIS: $\underline{\underline{F_{COR}}} = 2\omega v_r m = 2\omega\dot{x}m$

$$= \underline{\underline{2\omega m\left(v_1\cos\omega_0 t - x_1\omega_0\sin\omega_0 t\right)}}$$

• NAVOR NA PREČKO: $M = F_{COR}\cdot x$

$$M = 2m\omega\left[x_1 v_1\cos^2\omega_0 t + \left(\frac{v_1^2}{\omega_0} - x_1^2\omega_0\right)\sin\omega_0 t\cos\omega_0 t - x_1 v_1\sin^2\omega_0 t\right]$$

$$= \underline{\underline{2m\omega x_1 v_1}}\left[\cos 2\omega_0 t + \underbrace{\left(\frac{v_1}{x_1\omega_0} - \frac{x_1\omega_0}{v_1}\right)}_{=0}\frac{1}{2}\sin 2\omega_0 t\right]$$

7 nal. 2

$k = 1\,N/cm$

$l_0 = 1\,m$

$m = 1\,kg$

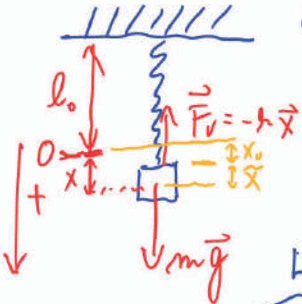
$t_0 = ?$

$l(t) = ?$

ZAČ. POGOJI:

$l(t=0) = l_0$

$|v(t=0)| = 1\,m/s$ ↓ NAVZDOL



• SILE

$$mg - F_v = ma$$

$$mg - kx = ma \quad / \cdot \frac{1}{m}$$

$$\boxed{\ddot{x} + \frac{k}{m}x = g}$$

• DOLŽINA VZMETI

$$l = l_0 + X$$

• POSPEŠEK: $a = \ddot{x}$

• RAVNOVESJE

$$F_v = mg$$

$$kX_0 = mg \Rightarrow \boxed{X_0 = \frac{mg}{k}}$$

$\Rightarrow$ GLEDAMO NIHANJE OKOLI X_0

$$X = X_0 + \tilde{X}\ ;\ X_0 = konst.$$

$$\hookrightarrow \ddot{X} = \ddot{\tilde{X}}$$

$$\ddot{\tilde{X}} + \frac{k}{m}X_0 + \frac{k}{m}\tilde{X} = g$$

$$\ddot{\tilde{X}} + \cancel{\frac{k}{m}\frac{mg}{k}} + \frac{k}{m}\tilde{X} = \cancel{g}$$

$$\boxed{\ddot{\tilde{X}} + \frac{k}{m}\tilde{X} = 0} \Rightarrow \omega_0 = \sqrt{\frac{k}{m}} \Rightarrow t_0 = \frac{2\pi}{\omega_0}$$

$$\boxed{\tilde{X} = A\sin\omega_0 t + B\cos\omega_0 t}$$

• ZAČ. POGOJI:

• $l(t=0) = l_0$

$$\boxed{l = l_0 + X = l_0 + X_0 + \tilde{X}}$$

$$\hookrightarrow \tilde{X}(t=0) = -X_0$$

$$\hookrightarrow \boxed{-X_0 = B}$$

• $v(t=0) = v_0$

$$v = \dot{\tilde{X}} = \omega_0 A\cos\omega_0 t - \omega_0 B\sin\omega_0 t$$

$$\hookrightarrow v_0 = \omega_0 A \Rightarrow \boxed{A = \frac{v_0}{\omega_0}}$$

$$\Rightarrow \boxed{l = l_0 + \underbrace{X_0 + \frac{v_0}{\omega_0}\sin\omega_0 t - X_0\cos\omega_0 t}_{X}}$$

$$\underbrace{\ddot{x} + \omega_0^2 x = C}\ ; \quad C = \text{konst.}$$

HOMOGENI DEL

$$\underbrace{\ddot{x} + \omega_0^2 x = 0}$$

$$\boxed{\begin{aligned} x_H &= A \sin \omega_0 t + B \cos \omega_0 t \\ &= x_0 \cos(\omega_0 t + \phi) \end{aligned}}$$

PARTIKULARNA REŠITEV

↳ S POSKUŠANJEM Z NASTAVKI:

E, $\sin \omega t$, ... E = konst

ZA NAŠ PRIMER

$$\underbrace{x_p = E} \rightarrow 0 + \omega_0^2 E = g$$

$$\hookrightarrow \underbrace{E = \frac{g}{\omega_0^2}} = x_0$$

CELOTNA REŠITEV: $\underbrace{x = x_H + x_p}$

ZBIRKA 9 nal 17/st 19

$R = 15\ cm$

$h = 20\ cm$

$l = 10\ cm$

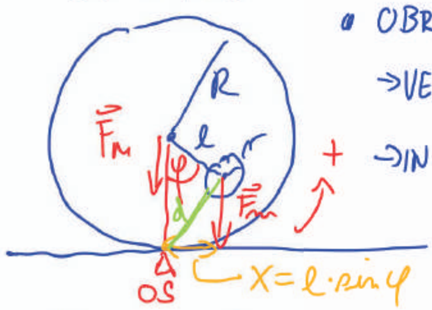
$r = 2\ cm$

$\rho_L = 0.7\ g/cm^3$

$\rho_s = 11\ g/cm^3$

$t_0 = ?$

OD STRANI:



• OBRAVNAVAMO KOT:

→ VELIK VALJ MASE: $M = \pi R^2 h \cdot \rho_L$

→ IN MALI VALJ MASE: $m = \pi r^2 h (\rho_s - \rho_L)$

• Z IZBIRO OSI V STIKU S TLEMI SE ZNEBIMO NAVORA SILE LEPENJA!

$F_m = mg$

ZA MAJHNE KOTE:

$$\boxed{\begin{array}{l} \sin\varphi \to \varphi \\ \cos\varphi \to 1 \end{array}}$$

• NAVORI:

$$-mgl \cdot \sin\varphi = J\alpha \quad ; \alpha = \ddot{\varphi}$$

$$-mgl\varphi = J\ddot{\varphi}$$

$$\ddot{\varphi} = -\underbrace{\frac{mgl}{J}}_{\omega_0^2} \cdot \varphi$$

$$J = \frac{1}{2}MR^2 + MR^2 + \frac{1}{2}mr^2 + md^2$$

$$d^2 = R^2 + l^2 - 2Rl\cos\varphi \approx R^2 + l^2 - 2Rl = (R-l)^2$$

$$\omega_0^2 = \frac{mgl}{\frac{3}{2}MR^2 + m\left(\frac{r^2}{2} + (R-l)^2\right)}$$

$$\omega_0 \neq \dot{\varphi} = \omega$$

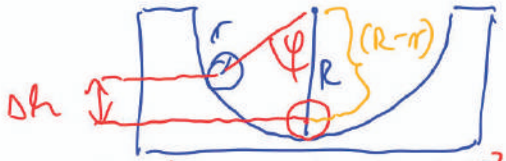
$$t_0 = \frac{2\pi}{\omega_0} = 2{,}3\ s$$

7) nal 4

R

r

t_0



$$\Delta h = (R-r)(1-\cos\varphi) \sim (R-r)\frac{\varphi^2}{2}$$

$$\cos\varphi \xrightarrow{\varphi \to 0} 1 - \frac{\varphi^2}{2}$$

KOTALJENJE: $v = \omega r \;;\; \omega \neq \dot{\varphi}$

TEŽIŠČE: $\underbrace{v = (R-r)\cdot\dot{\varphi}}$

$$\underbrace{\omega = \frac{(R-r)}{r}\dot{\varphi}}$$

ENERGIJE: $mg\Delta h + \frac{mv^2}{2} + J\frac{\omega^2}{2} = C = \text{konst}$

$$J = \frac{2}{5}mr^2$$

$$\frac{2}{m(R-r)} \Big\backslash\; mg(R-r)\frac{\varphi^2}{2} + \frac{m}{2}(R-r)^2\dot{\varphi}^2 + \frac{2m\cancel{r^2}(R-r)^2\dot{\varphi}^2}{5\cdot 2\,\cancel{r^2}} = C$$

$$g\varphi^2 + \dot{\varphi}^2(R-r)\overbrace{\left(1+\frac{2}{5}\right)}^{7/5} = \frac{2C}{m(R-r)} \;\Big/\frac{d}{dt}$$

$$g\cancel{2}\varphi\cancel{\dot{\varphi}} + \cancel{2}\cancel{\dot{\varphi}}\ddot{\varphi}(R-r)\frac{7}{5} = 0$$

$$\ddot{\varphi} = -\underbrace{\frac{5}{7}\frac{g}{(R-r)}}_{\omega_0^2}\varphi$$

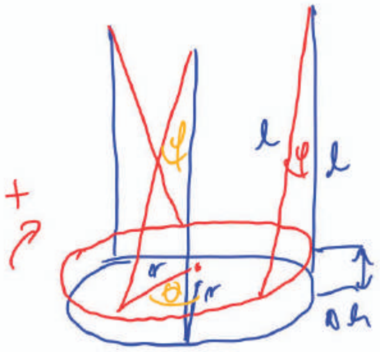
$$\omega_0^2 \Rightarrow \underline{\underline{t_0 = \frac{2\pi}{\omega_0}}}$$

⑦ nal 5

l

$\varphi(t) = ?$

$W(t) = ?$



$$\boxed{\Delta h = l(1-\cos\varphi) \overset{\varphi \to 0}{\sim} l\frac{\varphi^2}{2}}, \quad \cos\varphi \Rightarrow 1 - \frac{\varphi^2}{2}$$

$$\omega = \dot{\theta}$$

OBODNA HITROST $\dot{\theta} r = \dot{\varphi} l$

$$\boxed{\omega = \dot{\varphi}\frac{l}{r}}$$

$$W_p + W_r = C \qquad J = \frac{1}{2} m r^2$$

$$m g \Delta h + J\frac{\omega^2}{2} = C$$

$$m g l \frac{\varphi^2}{2} + \frac{1}{4} m \cancel{r^2} \dot{\varphi}^2 \frac{l^2}{\cancel{r^2}} = C \quad \Big/ \frac{d}{dt}$$

$$\frac{\cancel{m} g l}{\cancel{2}} \cancel{2} \varphi \cancel{\dot{\varphi}} + \frac{\cancel{m} l^2}{\cancel{4}_2} \cancel{2} \cancel{\dot{\varphi}} \ddot{\varphi} = 0$$

$$\ddot{\varphi} = -\underbrace{\frac{2g}{l}}_{\omega_0^2} \varphi \Rightarrow \omega_0 = \sqrt{\frac{2g}{l}}$$

$$t_0 = \frac{2\pi}{\omega_0}$$

$$\hookrightarrow \varphi = \varphi_0 \cos(\omega_0 t + \phi)$$

$$\hookrightarrow \dot{\varphi} = -\omega_0 \varphi_0 \sin(\omega_0 t + \phi)$$

$$W_p = \frac{m g l}{2} \varphi_0^2 \cos^2(\omega_0 t + \phi)$$

$$W_r = \frac{m l^2 \omega_0^2}{4} \cdot \varphi_0^2 \sin^2(\omega_0 t + \phi)$$

$$= \frac{m \cancel{l^2} 2g}{\cancel{4}_2 \cancel{l}} \cdot \varphi_0^2 \sin^2(\omega_0 t + \phi) = \frac{m g l}{2} \varphi_0^2 \sin^2(\omega_0 t + \phi)$$

$$W_{SKUPNA} = W_p + W_r = \frac{m g l}{2} \varphi_0^2 \underbrace{\left[\cos^2(\omega_0 t + \phi) + \sin^2(\omega_0 t + \phi)\right]}_{= 1}$$

$\Downarrow$

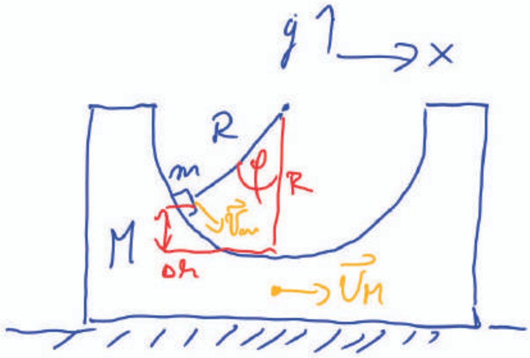
MAKSIMALNA W_p

7 nal 6

R, m, M

$\varphi_t = 0$

$t_0 = ?$



$\cos\varphi \xrightarrow{\varphi\to 0} 1 - \frac{\varphi^2}{2}$

• $\underline{\Delta h = R(1-\cos\varphi) \approx R\frac{\varphi^2}{2}}$

• $G_x = 0$

$v_{mx} m + v_M \cdot M = 0 \qquad v_{mx} \overset{\varphi\to 0}{\approx} v_m$

$\underline{v_M = -\frac{m}{M} v_m}$

W: $\frac{m v_m^2}{2} + \frac{M v_M^2}{2} + m g \Delta h = C$

$$\frac{m v_m^2}{2} + \frac{\cancel{M} m^2 v_m^2}{2 M^{\cancel{2}}} + m g R \frac{\varphi^2}{2} = C$$

• OBODNA HITROST: (V SISTEMU VELIKE KLADE)

$R\dot{\varphi} = v_{REL}$

$\varphi \to 0$

$v_{REL} = v_m - v_M$

↳ RELATIVNA v m GLEDE NA M

$\vec{v}_m$ $\vec{v}_M$

$R\dot{\varphi} = v_m - v_M = v_m\left(1 + \frac{m}{M}\right)$

$\underline{v_m = \frac{R\dot{\varphi}}{\left(1+\frac{m}{M}\right)}}$

$$\frac{d}{dt}\left|\ \frac{m}{2}\cancel{\left(1+\frac{m}{M}\right)}\frac{R^2\dot{\varphi}^2}{\left(1+\frac{m}{M}\right)^{\cancel{2}}} + m g R\frac{\varphi^2}{2} = C\right.$$

$$\frac{\cancel{m} R^{\cancel{2}}}{\cancel{2}\left(1+\frac{m}{M}\right)}\cancel{2}\cancel{\dot{\varphi}}\ddot{\varphi} + \frac{\cancel{m}\cancel{R}g}{\cancel{2}}\cancel{2}\varphi\cancel{\dot{\varphi}} = 0$$

$$\ddot{\varphi} = -\underbrace{\frac{g}{R}\left(1+\frac{m}{M}\right)}_{\omega_0^2}\varphi$$

$$\boxed{t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\frac{g}{R}\left(1+\frac{m}{M}\right)}}}$$

$$\varphi = A\sin\omega_0 t + B\cos\omega_0 t = \varphi_0\cos(\omega_0 t + \phi)$$

7 nal 7

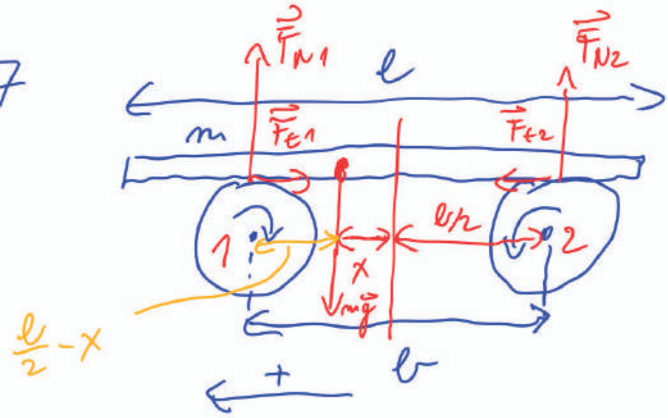
r

$b < l$

m

k

ν



SILE:

$$y:\ \underbrace{mg = F_{N1} + F_{N2}}$$

$$x:\ m\underset{\ddot{x}}{\underset{\shortparallel}{a}} = -F_{t1} + F_{t2}$$

$$F_{t1,2} = k \cdot F_{N1,2}$$

NAVORI: (OS V TEŽIŠČE)

$$F_{N1}\cdot\left(\frac{b}{2}-x\right) = F_{N2}\left(\frac{b}{2}+x\right)$$

$$\underbrace{F_{N2} = F_{N1}\frac{\left(\frac{b}{2}-x\right)}{\left(\frac{b}{2}+x\right)}}$$

$$mg = F_{N1}\left(1+\frac{\left(\frac{b}{2}-x\right)}{\left(\frac{b}{2}+x\right)}\right)$$

$$mg = F_{N1}\frac{\frac{b}{2}+\cancel{x}+\frac{b}{2}-\cancel{x}}{\frac{b}{2}+x}$$

$$\underbrace{F_{N1} = mg\frac{\frac{b}{2}+x}{b}}$$

$$F_{N2} = mg\frac{\cancel{\left(\frac{b}{2}+x\right)}}{b}\cdot\frac{\left(\frac{b}{2}-x\right)}{\cancel{\left(\frac{b}{2}+x\right)}}$$

$$\underbrace{F_{N2} = mg\frac{\frac{b}{2}-x}{b}}$$

$$x:\ \cancel{m}\ddot{x} = \frac{k\cancel{m}g}{b}\left(-\cancel{\frac{b}{2}}-x+\cancel{\frac{b}{2}}-x\right)$$

$$\ddot{x} = -\frac{2kg}{b}x$$

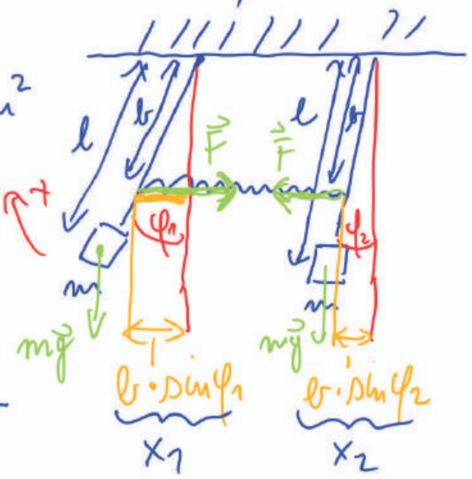
$$\underline{\underline{\omega_0 = \sqrt{\frac{2kg}{b}}}}$$

$$\underline{\underline{\nu_0 = \frac{\omega_0}{2\pi}}}$$

ZBIRKA 9 nal 18/str 19

$J = 0{,}01\ kgm^2$
$m = 1\ kg$
$l = 20\ cm$
$b = 15\ cm$
$k = 20\ N/m$

$\omega = ?$
$t_u = ?$



$$F = -kx = -k(x_1 - x_2) \overset{\varphi \to 0}{=} -kb(\sin\varphi_1 - \sin\varphi_2) \doteq -kb(\varphi_1 - \varphi_2)$$

NAVORI:

1.) $J\ddot{\varphi}_1 = -Fb\cos\underbrace{\varphi_1}_{\to 1} - mgl \cdot \underbrace{\sin\varphi_1}_{\to\varphi_1}$

$$\underline{J\ddot{\varphi}_1 = -kb^2(\varphi_1 - \varphi_2) - mgl\varphi_1}$$

2.) $J\ddot{\varphi}_2 = +Fb\cos\varphi_2 - mgl\sin\varphi_2$

$$\underline{J\ddot{\varphi}_2 = +kb^2(\varphi_1 - \varphi_2) - mgl\varphi_2}$$

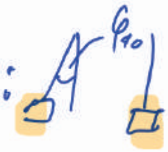
1 + 2: $J(\ddot{\varphi}_1 + \ddot{\varphi}_2) = -mgl(\varphi_1 + \varphi_2) \quad / \cdot \frac{1}{J}$

$$(\varphi_1 + \varphi_2)^{\cdot\cdot} = -\underbrace{\frac{mgl}{J}}_{\omega_+^2}(\varphi_1 + \varphi_2) \Rightarrow \underline{\omega_+ = \sqrt{\frac{mgl}{J}}}$$

1 − 2: $J(\ddot{\varphi}_1 - \ddot{\varphi}_2) = -(2kb^2 + mgl)(\varphi_1 - \varphi_2)$

$$(\varphi_1 - \varphi_2)^{\cdot\cdot} = -\underbrace{\frac{2kb^2 + mgl}{J}}_{\omega_-^2}(\varphi_1 - \varphi_2) \Rightarrow \underline{\omega_- = \sqrt{\frac{2kb^2 + mgl}{J}}}$$

⇒ SPLOŠNA REŠITEV: $\underline{(\varphi_1 \pm \varphi_2) = (\varphi_{10} \pm \varphi_{20})\cos(\omega_\pm t + \phi_\pm)}$

• ZAČETNI POGOJI ($\varphi_{20}=0, \phi_+=0, \phi_-=0$): PRI $t=0$: 

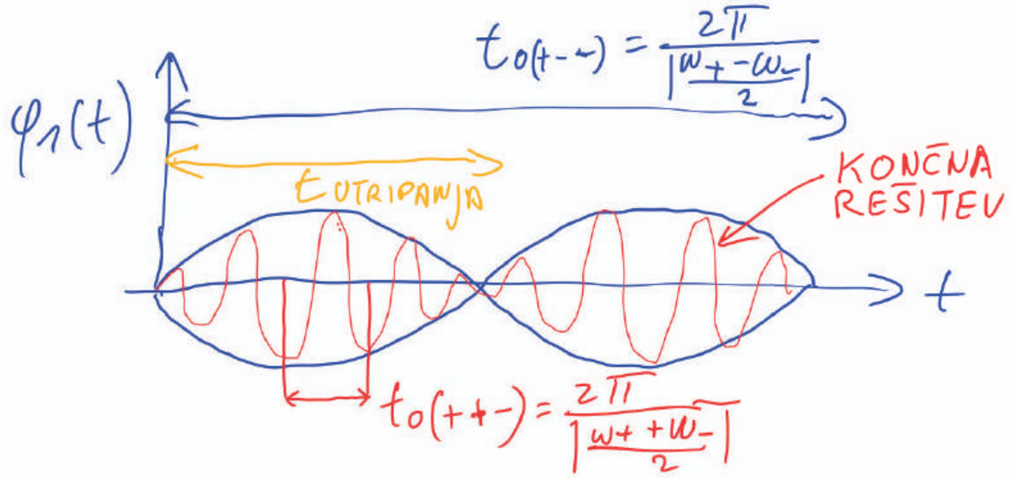
$$\begin{aligned} \varphi_1+\varphi_2 &= \varphi_{10}\cos\omega_+ t \\ \varphi_1-\varphi_2 &= \varphi_{10}\cos\omega_- t \end{aligned}$$

$(+)$: $\varphi_1 = \frac{\varphi_{10}}{2}\left[\cos\omega_+ t + \cos\omega_- t\right]$

$(-)$: $\varphi_2 = \frac{\varphi_{10}}{2}\left[\cos\omega_+ t - \cos\omega_- t\right]$

$$\begin{aligned} \cos\alpha+\cos\beta &= 2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2} \\ \cos\alpha-\cos\beta &= -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \end{aligned}$$

$$\boxed{\begin{aligned} \varphi_1 &= \varphi_{10}\cos\left(\frac{\omega_+ + \omega_-}{2}t\right)\cos\left(\frac{\omega_+ - \omega_-}{2}t\right) \\ \varphi_2 &= -\varphi_{10}\sin\left(\frac{\omega_+ + \omega_-}{2}t\right)\sin\left(\frac{\omega_+ - \omega_-}{2}t\right) \end{aligned}}$$

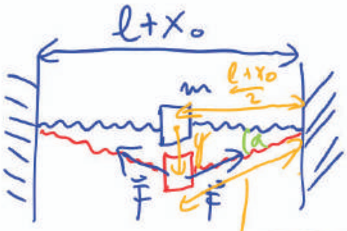


$$t_{UTRIPANJA} = \frac{1}{2}t_{0(++-)} = \frac{2\pi}{\omega_+ - \omega_-}$$

7) nal 9

$\ell \rightarrow$ NERAZTEGNJENA

x_0



$\left(\frac{d}{2}\right) = \sqrt{\left(\frac{\ell + x_0}{2}\right)^2 + y^2}$

$d = \ell + x_0 + x'$

$\hookrightarrow$ RAZTEGNJENA Z y

$d^2 = (\ell + x_0)^2 + 4y^2$

$\rightarrow y \ll (\ell + x_0)$

$\rightarrow \underline{d = \ell + x_0}$

$F = -k(d - \ell)$

$F_y = F \cdot \sin\alpha = F \frac{y}{d/2}$

$m\ddot{y} = 2 \cdot F_y$

$m\ddot{y} = -2k(d - \ell)\frac{2y}{d}$

$\ddot{y} = -\underbrace{\frac{4k}{m} \frac{x_0}{\ell + x_0}}_{\omega_0^2} \cdot y$

$\underline{t_0 = \frac{2\pi}{\sqrt{\frac{4k}{m} \frac{x_0}{\ell + x_0}}}}$

ČE $x_0 \rightarrow 0$; ČE VZMET NI PREDNAPETA NIHANJE NI VEČ HARMONIČNO

$\leftarrow$ NE HARMONIČNO

6 mal 11

$d = 384{,}400\ km$

m

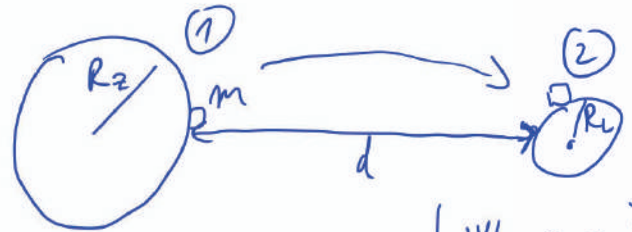
$M_L = \frac{M_z}{81}$

$R_L = \frac{R_z}{3.7}$

$R_z = 6400\ km$

$A = ?$

$W_r = ?$



$$A = \Delta W_p = W_{p2} - W_{p1}$$

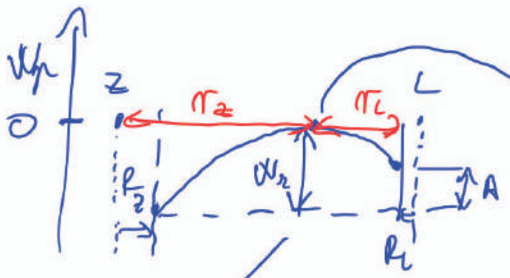
$$W_{p1} = -\frac{\varkappa M_z m}{R_z} - \frac{\varkappa M_L m}{R_L + d}$$

$$W_{p2} = -\frac{\varkappa M_z m}{R_z + d} - \frac{\varkappa M_L m}{R_L}$$

$$A = -\varkappa m\left(\frac{M_z}{\underbrace{R_z+d}_{\to d}} + \frac{M_L}{R_L} - \frac{M_z}{R_z} - \frac{M_L}{\underbrace{R_L+d}_{\to d}}\right) \leftarrow d \gg R_z, R_L$$

$$= -\frac{\varkappa m M_z}{R_z}\left(\underbrace{\frac{R_z}{d}}_{\to 0} + \frac{3{,}7}{81} - 1 - \underbrace{\frac{R_z}{81\,d}}_{\to 0}\right)$$

$$= \frac{\varkappa M_z m}{R_z}\left(1 - \frac{3{,}7}{81}\right) \sim 0{,}94\,\frac{\varkappa M_z m}{R_z}$$



$KJE? \Rightarrow F_{gL} = F_{gz}$

$$\frac{\varkappa M_L m}{r_L^2} = \frac{\varkappa M_z m}{r_z^2}$$

$$\frac{r_z^2}{r_L^2} = \frac{M_z}{M_L} = 81$$

$$r_z = 9 r_L$$

$$\begin{cases} r_z + r_L = d + R_L + R_z \\ 10 r_L = d + R_L + R_z \end{cases}$$

$$r_L = \frac{d + R_L + R_z}{10}$$

$$\to r_L \sim \frac{d}{10}$$

$$W_r = W_p(r_z, r_L) - W_p(R_z)$$

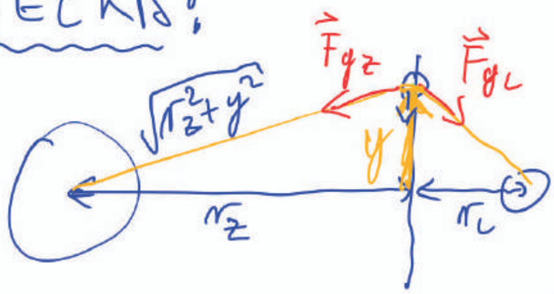
$$= -\frac{\varkappa M_z m}{r_z} - \frac{\varkappa M_L m}{r_L} + \frac{\varkappa M_z m}{R_z} + \frac{\varkappa M_L m}{R_L + d}$$

$$= -\varkappa m M_z\left[\frac{1}{9 r_L} + \frac{1}{81 r_L} - \frac{1}{R_z} - \frac{1}{81(R_L + d)}\right]$$

$$= -\frac{\varkappa M_z m}{R_z}\left(\frac{R_z}{9\,\underbrace{r_L}_{\to \frac{d}{10}}} + \frac{R_z}{81\,\underbrace{r_L}_{\to \frac{d}{10}}} - 1 - \cancel{\frac{R_z}{81(R_L + d)}}\right)$$

$$= \frac{\varkappa M_z m}{R_z}\left[1 - \frac{10 R_z}{d\,9}\left(1 + \frac{1}{9}\right)\right] \sim 0.98\cdot\frac{\varkappa M_z m}{R_z}$$

PREČKA:



$$F_{gzy} = -\frac{\varkappa M_z m}{(r_z^2+y^2)}\cdot\frac{y}{\sqrt{r_z^2+y^2}}$$

$$F_{gLy} = -\frac{\varkappa M_L m}{(r_L^2+y^2)}\cdot\frac{y}{\sqrt{r_L^2+y^2}}$$

$$m\ddot{y} = F_{gzy} + F_{gLy}$$

$$m\ddot{y} = -\varkappa m\left[\frac{M_z}{\underbrace{(r_z^2+y^2)^{3/2}}_{\to r_z^3}} + \frac{M_L}{\underbrace{(r_L^2+y^2)^{3/2}}_{\to r_L^3}}\right]\cdot y$$

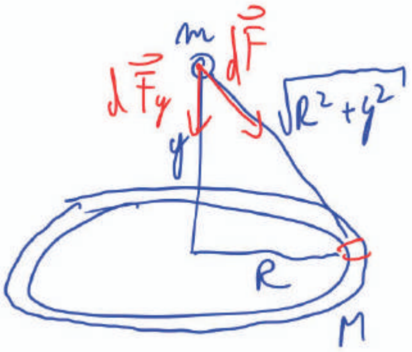
$$y < r_z, r_L \Rightarrow \quad \ddot{y} = \underbrace{-\frac{\varkappa M_z}{r_L^3}\left(\frac{1}{9^3}+\frac{1}{81}\right)}_{\omega_0^2}\cdot y$$

$r_z = 9 r_L \rightarrow$

$$\frac{1}{9^3}+\frac{1}{81} \rightarrow \frac{1}{81}\cdot\left(\frac{1}{9}+1\right)$$

$$\boxed{\omega_0 = \sqrt{\frac{\varkappa M_z}{81 r_L^3}\,\frac{10}{9}}}$$

6) nal 13



$$dF = -\frac{\mathcal{H}\,dM\,m}{R^2+y^2}$$

$$dF_y = dF\cdot\frac{y}{\sqrt{R^2+y^2}} = -\frac{\mathcal{H}\,dM\,m}{(R^2+y^2)^{3/2}}\cdot y$$

$$F_y = \int_0^M -\frac{\mathcal{H}\,m\,y}{(R^2+y^2)^{3/2}}\cdot dM$$

$$\boxed{F_y = -\frac{\mathcal{H}Mm}{(R^2+y^2)^{3/2}}\cdot y}$$

- $m\ddot{y} = F_y$

$$\cancel{m}\,\ddot{y} = -\frac{\mathcal{H}M\cancel{m}}{(R^2+\underbrace{y^2}_{\to 0})^{3/2}}\cdot y$$

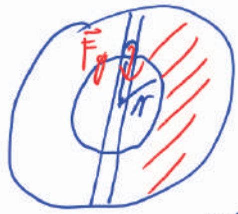
- $y << R$:

$$\hookrightarrow \ddot{y} = -\underbrace{\frac{\mathcal{H}M}{R^3}}_{\omega_0^2}\cdot y \Rightarrow \boxed{t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\frac{\mathcal{H}M}{R^3}}}}$$

(6) nal 15

K SILI PRISPEVA LE NOTRANJI DEL KROGLE!

$$m\ddot{y} = F_g$$



$$m\ddot{y} = -\frac{\mathcal{H} m\, r^{\cancel{3}} M_z}{\cancel{r^2}\, R_z^3}$$

$$F_g(r) = -\frac{\mathcal{H} M(r)\, m}{r^2}$$

$y = r \rightarrow$

$$m\ddot{r} = -\underbrace{\frac{\mathcal{H} m M_z}{R_z^3}}_{\omega_0^2} \cdot r$$

$$M(r) = \frac{4\pi r^3\, 3}{3\cdot 4\pi R_z^3}\cdot M_z$$

$$M(r) = \frac{r^3}{R_z^3}\cdot M_z$$

$$\cancel{m} g_0 = \frac{\mathcal{H} M_z \cancel{m}}{R_z^2}$$

$$\omega_0^2 = \frac{g_0}{R_z}$$

$$t_0 = 2\pi\sqrt{\frac{R_z}{g_0}}$$

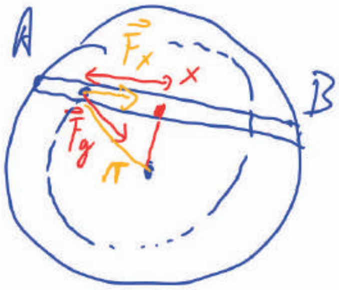
1. KOZMIČNA HITROST:

$$v = \sqrt{g_0 R_z}$$

$$\omega = \frac{v}{R_z} = \sqrt{\frac{g_0}{R_z}}$$

ENAKO $\longrightarrow$

(6) nal 15 b



$$F_g(r) = -\frac{\mathcal{G} m M(r)}{r^2} = -m g_0 \frac{r}{R_z}$$

$$; \quad M(r) = \frac{r^3}{R_z^3} \cdot M_z$$

$$\cancel{m} g_0 = \frac{\mathcal{G} M_z \cancel{m}}{R_z^2}$$

$$\hookrightarrow \frac{M_z \mathcal{G}}{R_z^2} = g_0$$

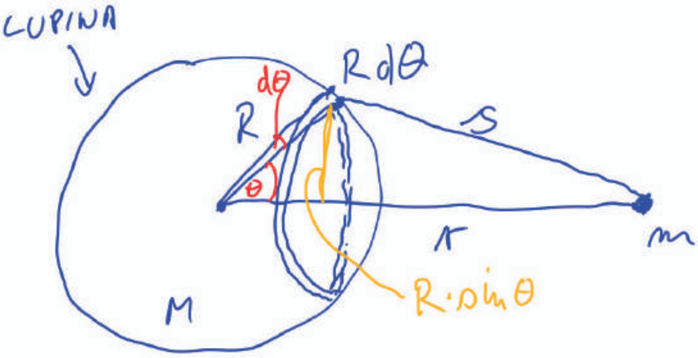
$$F_x = F_g \frac{x}{r} = -m g_0 \frac{\cancel{r}}{R_z} \cdot \frac{x}{\cancel{r}} = -\frac{m g_0}{R_z} \cdot x$$

$$\cancel{m} \ddot{x} = -\frac{\cancel{m} g_0}{R_z} \cdot x$$

$$\ddot{x} = -\frac{g_0}{R_z} \cdot x \Rightarrow \omega_0 = \sqrt{\frac{g_0}{R_z}}$$

← ISTO KOT ZA TUNEL SKOZI SREDIŠČE

SILA IN POTENCIAL MASIVNE LUPINE:



$$s^2 = r^2 + R^2 - 2Rr\cos\theta$$

$$dM = \frac{2\cancel{\pi}(\cancel{R}\sin\theta)\cdot\cancel{R}\,d\theta}{4\cancel{\pi}\cancel{R^2}}\cdot M = \frac{\sin\theta\, d\theta}{2}M$$

$$\int_0^{W_p} dW_p = -\frac{\varkappa m\, dM}{s} = -\int_0^{\pi}\frac{\varkappa m \sin\theta\, d\theta\, M}{2\sqrt{r^2+R^2-2Rr\cos\theta}}$$

$$u = r^2 + R^2 - 2Rr\cos\theta$$

$$du = +2Rr\sin\theta\, d\theta \Rightarrow \sin\theta\, d\theta = \frac{du}{2Rr}$$

$$\underline{W_p} = -\frac{\varkappa m M}{2\cdot 2Rr}\int_{(R-r)^2}^{(R+r)^2}\frac{du}{\sqrt{u}} = -\frac{\varkappa m M}{4Rr}\left(2\sqrt{u}\right)\Big|_{(R-r)^2}^{(R+r)^2} = -\frac{\varkappa m M}{2Rr}\left(R + r - \underline{|R-r|}\right)$$

!

• $r \le R$: $\quad |R-r| = R-r$

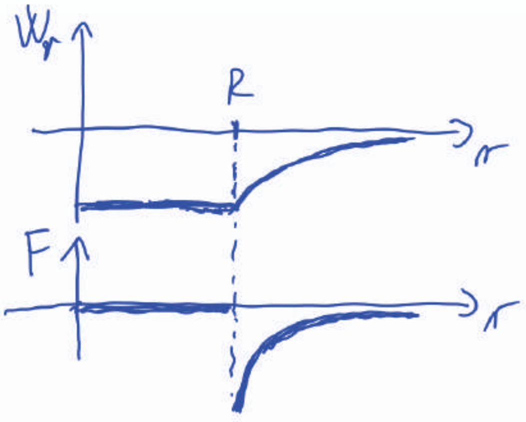
$$\underline{W_p} = -\frac{\varkappa m M}{2Rr}\left(\cancel{R} + r - \cancel{R} + r\right) = \underline{-\frac{\varkappa m M}{R}} = \text{konst.}$$

$$\underline{\vec{F}_g} = -\vec{\nabla} W_p = \underline{0}$$

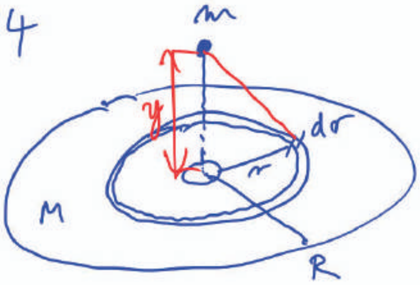
• $r > R$: $\quad |R-r| = r-R$

$$\underline{W_p} = -\frac{\varkappa m M}{2Rr}\left(R + \cancel{r} - \cancel{r} + R\right) = \underline{\underline{-\frac{\varkappa m M}{r}}}$$

$$\hookrightarrow \vec{F}_g = -\vec{\nabla} W_p = \underline{-\frac{\varkappa m M}{r^2}} \leftarrow \text{KOT TOČKASTO TELO}$$



6 nal 14



$$dM = \frac{2\pi r\,dr}{\pi R^2}\cdot M = \frac{2r\,dr}{R^2}M$$

MAJHNA LUKNJICA:

$$\int_0^{W_p} dW_p = -\frac{\kappa m\,dM}{R^2\sqrt{r^2+y^2}} = -\int_0^R \frac{\kappa m M\, 2r\,dr}{R^2\sqrt{r^2+y^2}} = -\frac{\kappa m M}{R^2}\int_{y^2}^{y^2+R^2}\frac{du}{\sqrt{u}} =$$

$$u = r^2+y^2$$
$$du = 2r\,dr$$

$$W_p = -\frac{\kappa m M}{R^2}\left(2\sqrt{u}\right)\Big|_{y^2}^{y^2+R^2} = -\frac{\kappa m M 2}{R^2}\left(\sqrt{y^2+R^2}-|y|\right)$$

$\Rightarrow$ PRIMER $y \ll R$: $W_p = -\frac{2\kappa m M}{R^2}\left(R-|y|\right)$
$$= -\frac{2\kappa m M}{R}\Big(1-\underbrace{\frac{|y|}{R}}_{\to 0}\Big) = -\frac{2\kappa m M}{R}$$

$$R\to\infty: W_p = 0$$

$$\vec{F}_y = -\vec{\nabla} W_p = +\frac{2\kappa m M}{R^2}\left(\frac{1}{\cancel{2}}\frac{\cancel{2}y}{\sqrt{y^2+R^2}}-1\right) = -\frac{2\kappa m M}{R^2}\Big(1-\underbrace{\frac{y}{\sqrt{y^2+R^2}}}_{y\ll R:\ \to 0}\Big)$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$y \ll R$: $F_y = -\frac{2\kappa m M}{R^2} = \text{konst}$

PRI OZKEM OBROČU: $F \propto -y$ TUKAJ PA NE

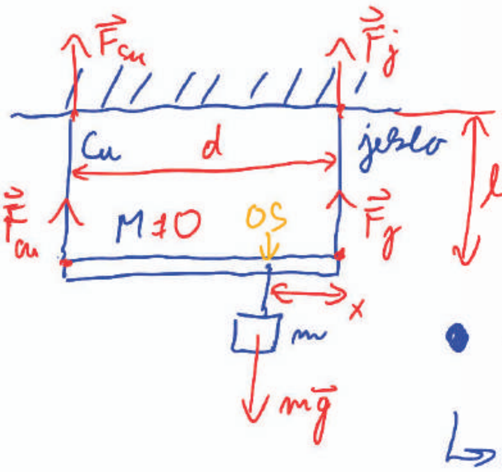
10 Elastomehanika in stisljivost

PON 11:15 - 12:50 (5 min ODMOR)
PET 8:30 - 11:00 (5 min + 10 min ODMORI)

10 nal 1.

$m = 5\,kg$
$M = 0$
$E_{Cu} = 1.2 \cdot 10^{11}\,N/m^2$
$E_j = 2 \cdot 10^{11}\,N/m^2$

$x = ?$



$$\frac{F}{S} = E\,\frac{\Delta l}{l}$$

• $$\frac{\Delta l}{l} \cdot S = \frac{F_{cu}}{E_{cu}} = \frac{F_j}{E_j}$$

↳ DA PALICA OSTANE VODORAVNA

NAVOR: $F_{cu}(d-x) = F_j\,x$

$$\cancel{F_{cu}}\,(d-x) = \cancel{F_{cu}} \cdot \frac{E_j}{E_{cu}}\,x$$

$$d = x\left(1 + \frac{E_j}{E_{cu}}\right)$$

$$\boxed{x = d\,\frac{E_{cu}}{E_{cu}+E_j}}$$

ZA DOMA
$M \neq 0$
• PRI NAVORU: $Mg\,r_M$
• SILE ....

10 nal 2

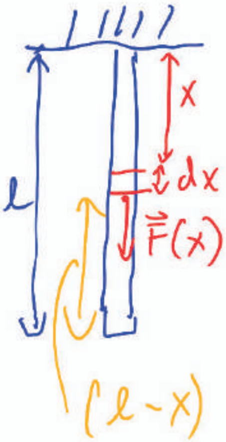
$l = 2\,m$

$S = 1\,cm^2$

$\rho_{Cu} = 8900\,kg/m^3$

$E_{Cu} = 1.2 \cdot 10^{11}\,N/m^2$

$\Delta l = ?$



• $F(x) = (l-x) \cdot S \cdot \rho_{Cu} \cdot g$

• $\frac{F}{S} = E \frac{du}{dx}$; du JE RAZTEZEK DELČKA dx

$$\frac{(l-x)\cancel{S}\rho_{Cu} \cdot g}{\cancel{S}} = E \frac{du}{dx}$$

$$\int_0^l \frac{\rho_{Cu} g (l-x)\,dx}{E} = \int_0^{\Delta l} du$$

$$\frac{\rho_{Cu} g}{E}\left(lx - \frac{x^2}{2}\right)\Big|_0^l = \Delta l$$

$$\boxed{\Delta l = \frac{\rho_{Cu} g}{E}\frac{l^2}{2}}$$

$$= 2{,}9\,\mu m$$

(10) nal 3

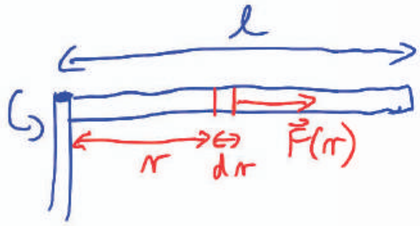
$l = 5\,m$

$\nu = 500\,min^{-1}$

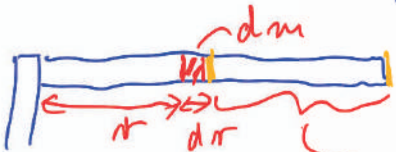
$E = 1.5 \cdot 10^{11}\,N/m^2$

$\rho = 400\,kg/m^3$

$\Delta l = ?$



DA PRIDEMO DO $F(r)$



$$dF = dm \cdot a_r = S\rho\omega^2 r\,dr$$

$$a_r = \omega^2 r$$

$$dm = S \cdot dr \cdot \rho$$

$$F(r) = S\rho\omega^2 \int_r^l r\,dr$$

$$F(r) = S\rho\omega^2 \frac{(l^2 - r^2)}{2}$$

RAZTEZEK:

$$\frac{F}{S} = E\frac{du}{dr}$$

$$\int_0^l \frac{\cancel{S}\rho\omega^2(l^2 - r^2)}{\cancel{S} \cdot 2E}\,dr = \int_0^{\Delta l} du$$

$$\frac{\rho\omega^2}{2E}\left(l^2 r - \frac{r^3}{3}\right)\Big|_0^l = \Delta l$$

$$\Delta l = \frac{\rho 4\pi^2\nu^2}{2E}\left(l^3 - \frac{l^3}{3}\right) = \frac{4}{3}\frac{\rho\pi^2\nu^2}{E}l^3$$

$$\Delta l = 0{,}3\,mm$$

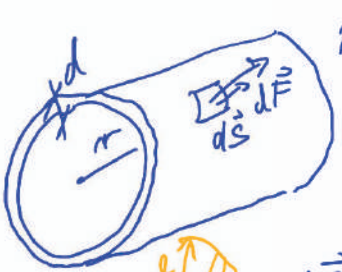
(10) nal 4

$d = 1\,\mathrm{mm}$

$r = 5\,\mathrm{cm}$

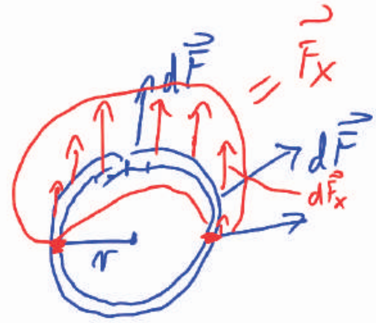
$\sigma = 1{,}4 \cdot 10^{8}\,\frac{N}{m^2}$

$p = ?$



$p = \frac{dF}{dS}$

$d\vec{F} \parallel d\vec{S}$



$dF = p\,dS$

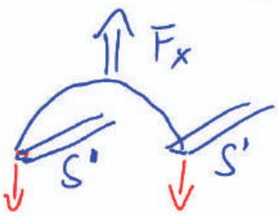
$dS = r\,d\varphi \cdot l$

$\Rightarrow dF_x = dF \cdot \sin\varphi$

$\int_0^{F_x} dF_x = p\,r\,l \int_0^{\pi} \sin\varphi\,d\varphi$

$F_x = p\,r\,l\,(-\cos\varphi)\Big|_0^{\pi}$

$\underline{F_x = p\,r\,l\,2}\,,$



$S' = d \cdot l$

$\sigma = \frac{F_x}{2S'} = \frac{p\,r\,\cancel{l}\cdot\cancel{2}}{\cancel{2}\,d\,\cancel{l}} = \frac{p\,r}{d}$

$\Rightarrow \underline{\underline{p = \frac{\sigma \cdot d}{r}}}$

$= 28\,\mathrm{bar}$

ZA DOMA

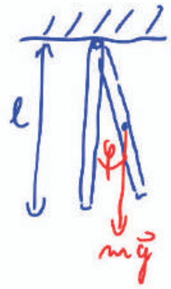
PRISPEVEK STRANIC

(10) nal 6

$\Delta T = 50°C$

$\alpha = 10^{-5} K^{-1}$

$t_0(T+\Delta T) = ?$



NAVOR: $-mg\frac{l}{2}\cdot \sin\varphi = J\ddot{\varphi}$ $\quad \bullet \sin\varphi \to \varphi$

$-\cancel{m}g\frac{\cancel{l}}{2}\varphi = \frac{1}{3}\cancel{m}l^{\cancel{2}}\ddot{\varphi}$ $\quad \bullet J = \frac{1}{3}ml$

$\ddot{\varphi} = -\underbrace{\frac{3}{2}\frac{g}{l}}_{\omega^2}\cdot\varphi$

$\underline{t_0} = \frac{2\pi}{\omega} = \underline{\underline{2\pi\cdot\sqrt{\frac{2}{3}\frac{l}{g}}}}$

$\bullet\ l \to l + \Delta l \overset{\boxed{\Delta l = \alpha\cdot l\Delta T}}{=} l(1+\alpha\Delta T)$

$\underline{t_0(T+\Delta T)} = 2\pi\sqrt{\frac{2}{3}\frac{l}{g}(1+\alpha\Delta T)} \underset{\alpha\Delta T \ll 1}{\doteq} \underline{2\pi\sqrt{\frac{2}{3}\frac{l}{g}}\left(1+\frac{\alpha\Delta T}{2}\right)}$

$\sqrt{1+\alpha\Delta T} \to \left(1+\frac{\alpha\Delta T}{2}\right)$

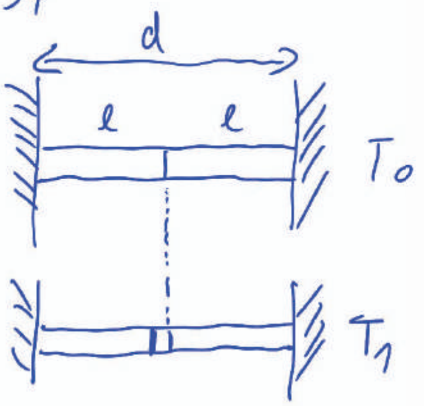
$\to t_0(T+\Delta T) = t_0(T) + \Delta t_0 \quad \boxed{\Delta t_0 = t_0\cdot\frac{\alpha\Delta T}{2}}$

$t_0 = 2\pi\sqrt{\frac{2}{3}\frac{l}{g}} \quad / \ln$

$\ln t_0 = \ln\left(2\pi\sqrt{\frac{2}{3g}}\right) + \frac{1}{2}\ln l \quad \leftarrow \ln\sqrt{x} = \frac{1}{2}\ln x$

$\frac{dt_0}{t_0} = 0 + \frac{1}{2}\frac{dl}{l} \Rightarrow \underline{dt_0} = t_0\cdot\frac{dl}{2l} \overset{dl = \alpha l dT}{=} \underline{\underline{t_0\frac{\alpha dT}{2}}}$

ZBIRKA 9 nal 3/st 34



$d = 1m$

$l = 0.5m$

$T_0 = 20°C$

$T_1 = 30°C$

$\alpha_A = 1{,}1 \cdot 10^{-5} K^{-1}$

$E_A = 2{,}1 \cdot 10^5 \frac{N}{mm^2}$

$\alpha_B = 1{,}7 \cdot 10^{-5} K^{-1}$

$E_B = 9 \cdot 10^4 N/mm^2$

$\sigma = \frac{F}{S} = ?$

$$\frac{F}{S} = E\frac{\Delta l}{l} \Rightarrow \Delta l = l\frac{F}{SE}$$

KER d OSTANE ENAK

• $\Delta l_A + \Delta l_B = 0$

$$\Delta l = \underbrace{\alpha l \Delta T}_{\text{TEMPERATURA}} - \underbrace{l\frac{F}{SE}}_{\text{STISKANJE}} = l\left(\alpha\Delta T - \frac{F}{SE}\right)$$

• $$l\left(\alpha_A\Delta T - \frac{F}{SE_A}\right) + l\left(\alpha_B\Delta T - \frac{F}{SE_B}\right) = 0$$

$$(\alpha_A + \alpha_B)\Delta T = \frac{F}{S}\left(\frac{1}{E_A} + \frac{1}{E_B}\right)$$

$$\frac{F}{S} = \frac{\alpha_A + \alpha_B}{\left(\frac{1}{E_A} + \frac{1}{E_B}\right)} \cdot \Delta T = 17{,}64 \frac{N}{mm^2}$$

$\Delta T = (T_1 - T_0)$

ZBIRKA 9 nal 7/str 35

$\Delta p = 6\,bar$
$\chi = 1{,}15 \cdot 10^{-4}\,bar^{-1}$
$\beta = 7{,}5 \cdot 10^{-4}\,K^{-1}$
$\alpha_j = 1{,}2 \cdot 10^{-5}\,K^{-1}$

$\Delta T = ?$

V: $\boxed{p, T}$ $\longrightarrow$ $V+\Delta V$: $\boxed{p+\Delta p,\ T+\Delta T}$

- $\frac{\Delta V}{V} = \beta \Delta T$

STISKLIVOST

- $\frac{\Delta V}{V} = -\chi \Delta p$

- ALKOHOL:

$$\frac{\Delta V}{V} = \beta \Delta T - \chi \Delta p$$

- JEKLENA POSODA: (SAMO VPLIV T) VPLIV TLAKA → 0

$$\frac{\Delta V}{V} = 3\alpha \Delta T$$

$$V \propto l^3$$

$\Delta l \to 0$

$$V + \Delta V \propto (l + \Delta l)^3$$
$$\propto l^3 + 3l^2 \Delta l + \cancel{3l\Delta l^2} + \cancel{\Delta l^3}$$
$$\cancel{V} + \Delta V \propto \cancel{l^3} + 3l^2 \Delta l$$
$$\Delta V \propto 3l^2 \cdot \alpha l \Delta T$$
$$\propto 3\underbrace{l^3}_{V} \alpha \Delta T$$
$$\underline{\Delta V = \underbrace{3\alpha}_{\beta} V \Delta T}$$

$$\beta \Delta T - \chi \Delta p = 3\alpha \Delta T$$

$$\boxed{\Delta T = \frac{\chi \Delta p}{\beta - 3\alpha}}$$

$$= \underline{1\,K}$$

ZA DOMA (10) nal 8

8 Hidrostatika in hidrodinamika

8.1 Hidrostatika

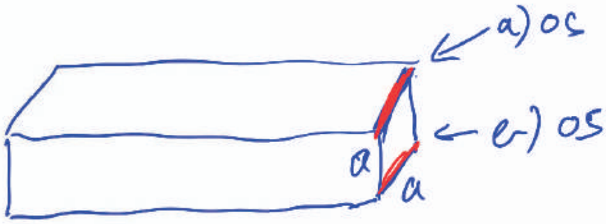
DN (10) nal 10 → ZBIRKA 9 nal 2/st 34

ZBIRKA 9 nal 1/st 26 (SMO RAČUNALI V VAKUUMU)

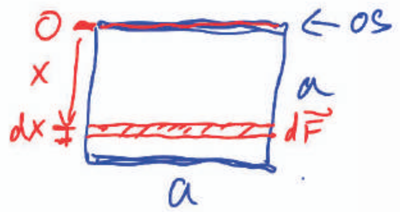
$a = 10\,cm$

a) M(OS NA GLADINI) = ?

b) M(OS PRI DNU) = ?



a)



$$\boxed{p = \rho g h = \frac{F}{S}}$$

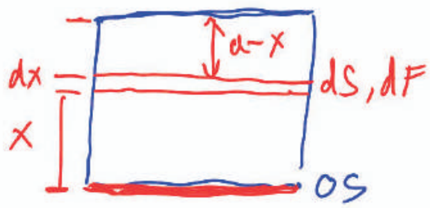
a) $dM = dF \cdot x$

$$dF = dS \cdot p = \overbrace{dx \cdot a}^{dS} \cdot \rho g x$$

$$\int_0^M dM = \rho a g \int_0^a x^2 dx$$

$$M = \rho a g \frac{a^3}{3} = \frac{\rho g a^4}{3} = \underline{\underline{\frac{1}{3}\,Nm}}$$

b)



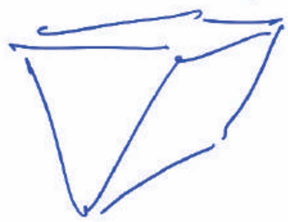
$$dF = dS \cdot p = dx \cdot a\, \rho g (a - x)$$

$$dM = dF\, x$$

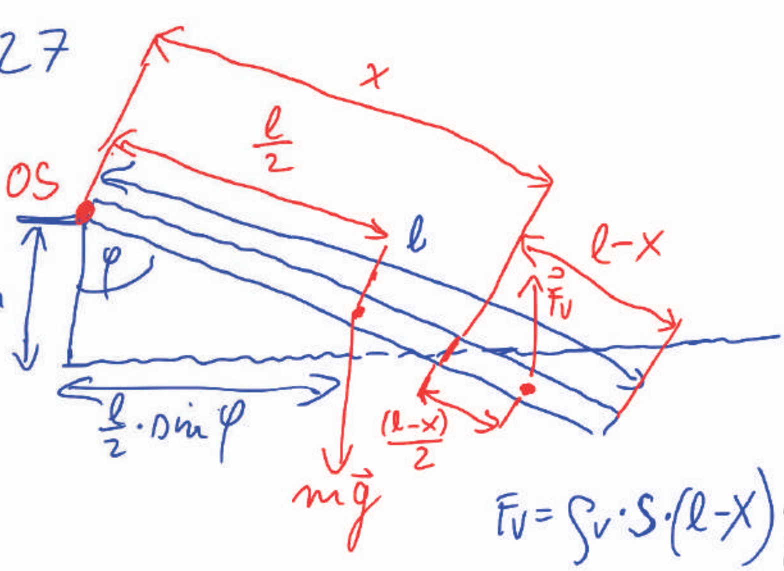
$$\int_0^M dM = \rho a g \int_0^a x(a-x)\,dx$$

$$M = \rho a g \left(\frac{a^2 \cdot a}{2} - \frac{a^3}{3}\right) = \frac{\rho g a^4}{6} = \underline{\underline{\frac{1}{6}\,Nm}}$$

POGLEJ ŠE nal 2/st 26



ZBIRKA 9 nal 10/st 27



$l = 1\,m$
$\rho_D = 0{,}7\,g/cm^3$
$h = 40\,cm$

$\varphi = ?$

$F_V = \rho_V \cdot S \cdot (l-x)\cdot g$

$m = \rho_D \cdot S \cdot l$

- NAVORI:

$$mg\cdot\frac{l}{2}\cdot\cancel{\sin\varphi} = F_V\cdot\left(x+\frac{l-x}{2}\right)\cdot\cancel{\sin\varphi}$$

$$\rho_D \cancel{S}\, l\, \cancel{g}\, \frac{l}{\cancel{2}} = \rho_V \cancel{S}(l-x)\cancel{g}\left(\frac{x+l}{\cancel{2}}\right)$$

$$\rho_D l^2 = (l^2 - x^2)\rho_V$$

$$x = l\sqrt{1-\frac{\rho_D}{\rho_V}} = 0{,}55\,m$$

KOT: $\cos\varphi = \frac{h}{x} \Rightarrow \varphi = 43{,}1^\circ$

ZBIRKA 9 nal 6/str 27

$m = 100\,g$

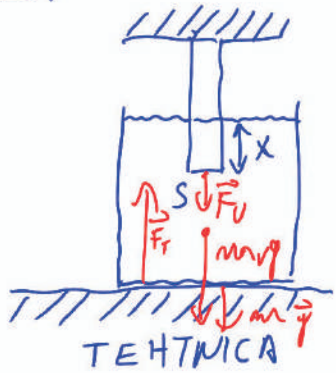
$V = 1\,dm^3$

$x = 5\,cm$

$S = 60\,cm^2$

$\rho_{Al} = 2{,}7\,g/cm^3$

$F_{TEHTNICE} = ?$



$F_V = Sx \cdot \rho_v \cdot g$

$m_{H_2O} = V \cdot \rho_v$

POSODA

$F_T = m_{H_2O}\,g + mg + F_V$

$= [m + \rho_v(V + Sx)] \cdot g = \underline{\underline{14\,N}}$

ZBIRKA 9 nal 14/st 28

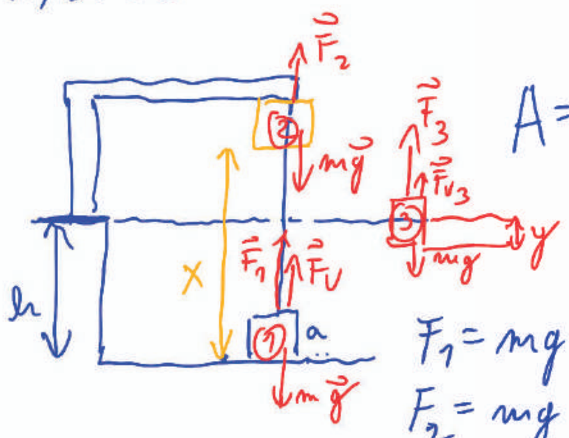
$h = 6\,m$

$a = 1\,m$

$\rho_A = 2{,}5\,g/cm^3$

$X = 10\,m$

$A = ?$



$A = \int F\,ds$

$F_v = a^3 \cdot \rho_v g$

$F_1 = mg - F_v$

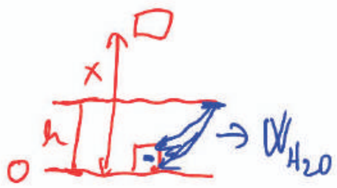
$F_2 = mg$

$F_3 = mg - F_{v_3}$; $F_{v_3} = \underbrace{y \cdot a^2}_{\text{V IZPODRINJENE VODE}} \cdot \rho_v g$

→ ZA DOMA $A = \int F\,ds$ → PO DELIH

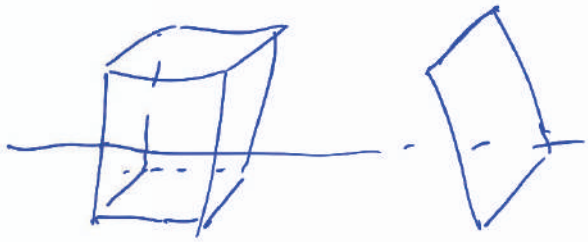
• ALTERNATIVA

$$A = \Delta W = W_p - W_{H_2O} = \underbrace{m}_{a^3\rho_A} gX - a^3\rho_v g\underbrace{\left(h - \frac{a}{2}\right)}_{\text{V POVPREČJU SE VODA SPUSTI DO TEŽIŠČA}} = a^3 g\left[\rho_A X - \rho_v\left(h - \frac{a}{2}\right)\right]$$



$\underline{A = 195 \cdot 10^3\,J}$

• DN (8.1.) nal 4

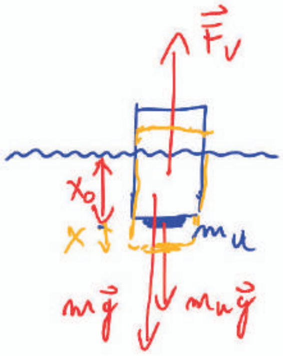


DN ZBIRKA 9 nal 19/st 28

8.1. nal 7

ρ, S, l

m_u



• RAVNOVESJE: (x_0)

$$F_v = mg + m_u g$$

$$\rho_v \cdot S \cdot x_0 \cdot \cancel{g} = \rho \cdot S\, l \cdot \cancel{g} + m_u \cancel{g}$$

$$\boxed{x_0 = \frac{\rho S l + m_u}{\rho_v S}}$$

NIHANJE:

$$(m+m_u)\ddot{x} = \cancel{mg} + \cancel{m_u g} - \rho_v S g(\cancel{x_0} + x)$$

$$(\rho S l + m_u)\ddot{x} = -\rho_v S g x$$

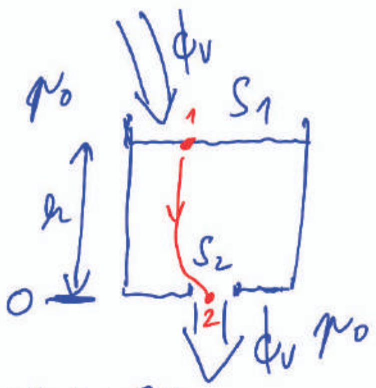
$$\ddot{x} = -\underbrace{\frac{\rho_v S g}{\rho S l + m_u}}_{\omega^2} \cdot x \implies \boxed{\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\rho_v S g}{\rho S l + m_u}}}$$

$$\rightarrow (m+m_u)\ddot{x} = (m+m_u)g - \rho_v S g x$$

$$\ddot{x} + \frac{\rho_v S g}{m+m_u} \cdot x = g \rightarrow x = x_p + x_H$$

8.2 Hidrodinamika, Bernoulli

(8.2) nal 2



$S_1 = 3\,cm^2$

$S_2 = 1\,cm^2$

$\phi_V = 0.1\,l/s$

$h = ?$

• BERNULIJEVA E.:
VZDOLŽ TOKOVNICE:

$$p + \rho g h + \frac{\rho v^2}{2} = konst.$$

• TOKOVNICA: (1) (2)

$$\cancel{p_0} + \cancel{\rho} g h + \frac{\cancel{\rho} v_1^2}{2} = \cancel{p_0} + \frac{\cancel{\rho} v_2^2}{2}$$

$$h = \frac{1}{2g}\left(\frac{\phi_V^2}{S_2^2} - \frac{\phi_V^2}{S_1^2}\right)$$

$$h = \frac{\phi_V^2}{2g}\left(\frac{1}{S_2^2} - \frac{1}{S_1^2}\right)$$

$$\underline{\underline{h = 4{,}4\,cm}}$$

PRETOK:

$$\phi_V = S_1 v_1 = S_2 v_2$$

$\hookrightarrow v_1 = \frac{\phi_V}{S_1}$

$\hookrightarrow v_2 = \frac{\phi_V}{S_2}$

8.2. nal 3 ZBIRKA 9 nal 5/ot 29

→ VENTURIJEVA CEV.

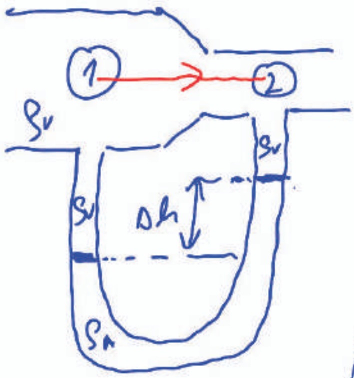
$2r_1 = 2\ cm$

$2r_2 = 1{,}8\ cm$

$\rho_A = 1{,}02\ g/cm^3$

$\Delta h = 25\ cm$

$\phi_V = ?$



• TOKUNICA:

$$p_1 + \frac{\rho_V v_1^2}{2} = p_2 + \frac{\rho_V v_2^2}{2}$$

• $\phi_V = S_1 \cdot v_1 = S_2 v_2$

↳ $v_1 = \frac{\phi_V}{S_1}$; $v_2 = \frac{\phi_V}{S_2}$

$$p_1 - p_2 = \frac{\rho_V}{2}\left(\frac{\phi_V^2}{S_2^2} - \frac{\phi_V^2}{S_1^2}\right)$$

• $p_1 - p_2 = (\rho_A - \rho_V) \cdot g \cdot \Delta h$

$$(\rho_A - \rho_V) g \Delta h = \frac{\rho_V \phi_V^2}{2}\left(\frac{1}{S_2^2} - \frac{1}{S_1^2}\right)$$

$$\phi_V = \sqrt{\frac{2 g \Delta h (\rho_A - \rho_V)}{\rho_V \left(\frac{1}{S_2^2} - \frac{1}{S_1^2}\right)}}$$

$$\phi_V = 0{,}135\ \ell/s$$

8.2. nal 4.

$\Delta h = 20\,\text{cm}$

$v = ?$

3 → $p_0 + \rho g h$ → POTENCIAL

||

2 → $p_0 + \rho g \Delta h$ → DODATNI TLAK ZARADI VODE NAD 2

1

$$p_0 + \cancel{\rho}\frac{v^2}{2} = \cancel{p_0} + \cancel{\rho} g \Delta h$$

$$\underline{\underline{v = \sqrt{2 g \Delta h}}} = \underline{\underline{2\,\text{m/s}}}$$

→ U TOČKI 2 POVEČAN TLAK:

$$\Delta p = \frac{\rho v^2}{2}$$

$$\frac{F}{S} = \frac{\rho v^2}{2} \Rightarrow \boxed{F = c_u \frac{\rho v^2 \cdot S}{2}}$$

SILA VODE NA STOJEČO OVIRO

↳ KVADRATNI ZKON UPORA

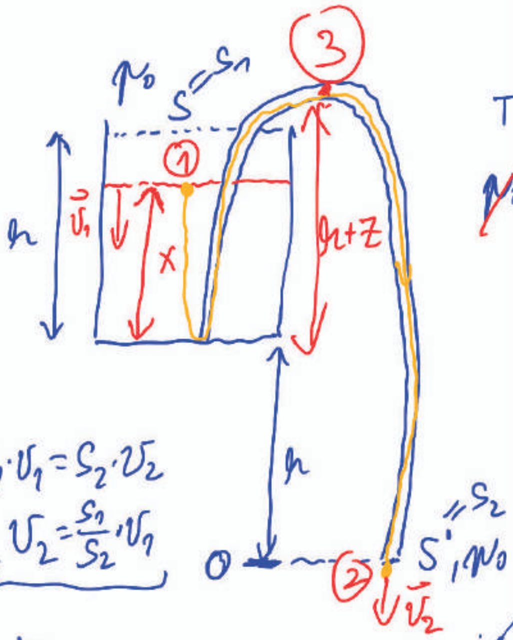
8.2. nal 5

$S = 100\,cm^2$

$h = 10\,cm$

$S' = 1\,cm^2$

$t = ?$



TOKOVNICA:

$$\cancel{p_0} + \rho g(h+x) + \cancel{\frac{\rho v_1^2}{2}} = \cancel{p_0} + \frac{\rho v_2^2}{2}$$

$$2g(h+x) = v_1^2\left(\frac{S_1^2}{S_2^2} - 1\right)$$

$$\underbrace{v_1 = \sqrt{\frac{2g(h+x)}{\left(\frac{S_1^2}{S_2^2} - 1\right)}}}$$

$$\phi_V = S_1 \cdot v_1 = S_2 \cdot v_2$$

$$\underbrace{v_2 = \frac{S_1}{S_2} \cdot v_1}$$

$$\underbrace{v_1 = -\frac{dx}{dt}}$$

$$-\frac{dx}{dt} = \sqrt{\frac{2g}{\left(\frac{S_1^2}{S_2^2} - 1\right)}} \cdot \sqrt{h+x}$$

$$-\sqrt{\frac{\left(\frac{S_1^2}{S_2^2} - 1\right)}{2g}} \int_h^0 \frac{dx}{\sqrt{h+x}} = \int_0^t dt$$

ČE $S_1 >> S_2 \rightarrow v_2 >> v_1 \rightarrow 0$

$$u = h + x$$

$$du = dx$$

$$\rightarrow \int_{2h}^{h} \frac{du}{\sqrt{u}} = 2\sqrt{u}\,\Big|_{2h}^{h}$$

$$\sqrt{\frac{\left(\frac{S_1^2}{S_2^2} - 1\right)}{2g}}\, 2\left(\sqrt{2h} - \sqrt{h}\right) = t$$

$$\underbrace{t = \sqrt{\frac{\left(\frac{S_1^2}{S_2^2} - 1\right)h}{2g}} \cdot 2\left(\sqrt{2} - 1\right)}$$

KGR $S_1 >> S_2$

$$\hookrightarrow t = \sqrt{\frac{h}{2g}} \cdot \frac{S_1}{S} 2\left(\sqrt{2} - 1\right)$$

(3): $p_3 + \rho g(2h+z) + \frac{\rho v_3^2}{2} = p_0 + \frac{\rho v_2^2}{2}$ (2)

$\rightarrow 0$ KGR $v_3 = v_2$

$$p_3 = p_0 + \rho\left[\cancel{\frac{v_2^2 - v_3^2}{2}} - g(2h+z)\right]$$

$$p_3 = p_0 - \rho g(2h+z)$$

$\hookrightarrow$ KO $\quad p_3 = p_{\text{IZPARILNI TLAK VODE}}$

$\hookrightarrow$ TAKRAT V CEVI NASTANEJO MEHURČKI PARE

$\hookrightarrow$ NATEGA NEHA DELOVATI

8.3 Kvadratni zakon upora

(8.3.) nal 1

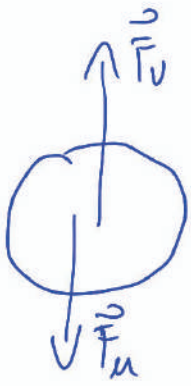
$h = 10\,m$

$\eta = 0.001\,kg/ms$

$\rho_z(h) = 2{,}4\,kg/m^3$

$C_u = 0.4$

$v(r) = ?$



$\boxed{\text{KVADRATNI ZAKON UPORA: } F_u = \frac{1}{2} C_u \rho \cdot S v^2}$

• RAVNOVESNA HITROST:

$$F_v = F_u$$

$$(\rho_v - \rho_z) \cdot \frac{4\pi r^{\cancel{3}}}{3} \cdot g = \frac{1}{2} C_u \rho_v \cancel{\pi} r^{\cancel{2}} \cdot v^2$$

S = PREČNI PRESEK $= \pi r^2$

$$v = \sqrt{\frac{8 g \left(1 - \frac{\rho_z}{\rho_v}\right) r}{3 C_u}}$$

dimenzija

$$R = \frac{\rho \cdot v\, d}{\eta}$$

$d = 2r$

$$R = \frac{\rho_v}{\eta} \sqrt{\frac{8 g \left(1 - \frac{\rho_z}{\rho_v}\right)}{3 C_u}} \cdot 2 r^{3/2}$$

$$R = \begin{cases} > \sim 4000 \rightarrow \text{KVADRATNI ZAKON} \\ < \sim 1 \rightarrow \text{LINEARNI} \end{cases}$$

$r \sim 4\,mm$

(8.3.) nal 2 ZBIRKA 9 nal 17/st 30

$m = 200\,kg$

$S = 2\,m^2$

$v_v = 40\,km/h$

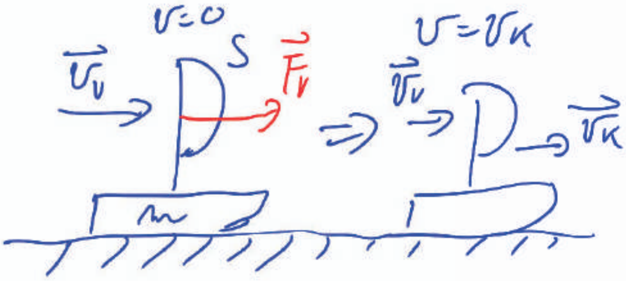
$v_K = 10\,km/h$

$\rho = 1{,}2\,kg/m^3$

$C_u = 0.8$

$t(v_K) = ?$

$F_f = 0$



$$F_u = \frac{1}{2} C_u \rho S v_{REL}^2$$

$$v_{REL} = (v_v - v)$$

JADRNICA ČUTI SILO F_u DOKLER $v_v > v$

$$ma = F_u$$

$$m\frac{dv}{dt} = \frac{1}{2} C_u \rho S (v_v - v)^2$$

$$\frac{2m}{C_u \rho S}\int_0^v \frac{dv}{(v_v-v)^2} = \int_0^t dt$$

$v_v - v = u$

$-dv = du$

$$\frac{2m}{C_u \rho S}\int_{v_v}^{v_v - v} \frac{-du}{u^2} = t$$

$$\frac{2m}{C_u \rho S}\left(\frac{1}{v_v - v} - \frac{1}{v_v}\right) = t$$

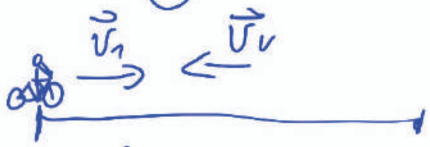
$$\frac{2m}{C_u \rho S}\left(\frac{v}{v_v(v_v - v)}\right) = t$$

$t_K(v = v_K) = 6{,}25\,s$

8.3. nal 3

$v_v = 10\,m/s$
$t_v = 2t_B$
$P_v = P_B$
$v_K = ?$

(1)



$t = t_v$

(2)

$t = t_B = \frac{1}{2} t_v$

• MOČ:

$$P = F_u \cdot v$$

$$P_v = P_B$$

$$\cancel{\tfrac{1}{2} c_u \rho S} (v_v + v_1)^2 \cdot v_1 = \cancel{\tfrac{1}{2} c_u \rho S}\, v_2^2 \cdot v_2$$

$$(v_v + v_1)^2 \cancel{v_1} = 8 v_1^{\cancel{3}\,2}$$

$$v_v^2 + 2 v_v v_1 + v_1^2 = 8 v_1^2$$

$$0 = 7 v_1^2 - 2 v_v v_1 - v_v^2$$

$$v_1 = \frac{2 v_v \pm \sqrt{4 v_v^2 + 4 \cdot 7 \cdot v_v^2}}{14}$$

$$= \frac{1}{7} v_v \left(1 + \sqrt{8}\right) = 5{,}5\,m/s$$

; SILE

$$F_{u1} = \frac{1}{2} c_u \rho S (v_v + v_1)^2$$

$$F_{u2} = \frac{1}{2} c_u \rho S\, v_2^2$$

: HITROST $\left(t_2 = \frac{1}{2} t_1\right)$

$$v_2 = 2 v_1$$

DN 8.3 nal 4 ZBIRKA 9 nal 13/~~21~~30

8.4 Viskoznost in linearni zakon upora

8.4. nal 1   08/09 kol 2, nal 1

$\eta = 0{,}6\ \mathrm{kg/ms}$

$l = 10\ \mathrm{m}$

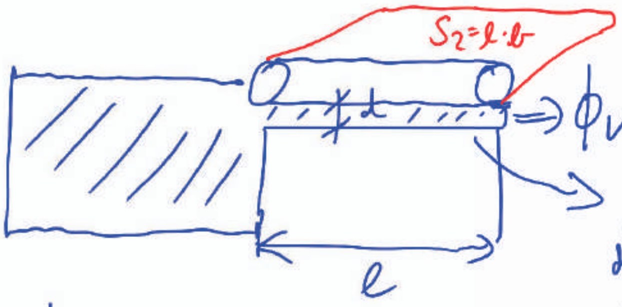
$b = 2\ \mathrm{m}$

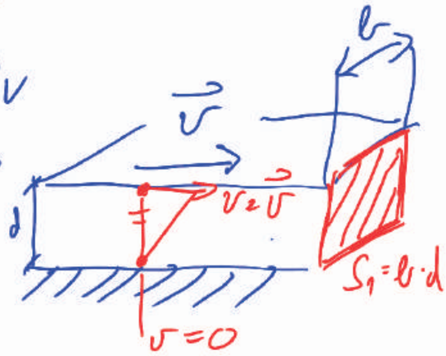
$v = 0.5\ \mathrm{m/s}$

$d = 1\ \mathrm{cm}$

$\Phi_V = ?$

$P = ?$





$$\Phi_V = S_1 \cdot \bar{v} = d \cdot b \cdot \frac{v}{2} = 5 \cdot 10^{-3}\ \mathrm{m^3/s}$$

LINERNI PROFIL HITROSTI

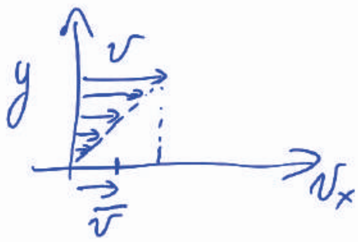
↳ POVPREČNA HITROST

$$\bar{v} = \frac{1}{2} v$$

$$P = F \cdot v$$

$$\boxed{F = \eta \cdot S_2 \left(\frac{d v_x}{d y}\right)}$$

$$= \eta l \cdot b \frac{v}{d}$$



$$P = \eta l b \frac{v^2}{d} = 300\ \mathrm{W}$$

(8.4) nal 2

$R = 5\,cm$

$h = 10\,cm$

$\rho = 13000\,kg/m^3$

$\omega_0 = 10\,s^{-1}$

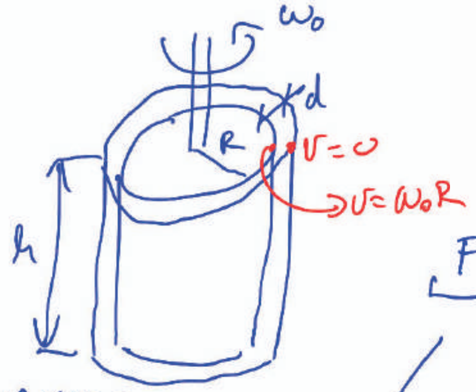
$d = 3\,mm$

$\eta = 0.5\,kg/ms$

$M = ?$ ✓

$P = ?$

$t(\omega = \frac{\omega_0}{2}) = ?$



$S = 2\pi R \cdot h$

$$\underbrace{F = \eta \cdot S \cdot \frac{dv}{dr} = \eta 2\pi R h \frac{\omega_0 R}{d}}$$

$$\frac{dv}{dr} = \frac{\omega_0 \cdot R}{d}$$

• TANKA PLAST

↳ LINEARNI PROFIL HITROSTI

• NAVOR:

$M = F \cdot R$

$$\underbrace{M = \frac{\eta 2\pi h \omega_0 R^3}{d}}$$

$$\underline{M = 0{,}13\,Nm}$$

• MOČ:

$$P = M\omega_0 = \frac{\eta 2\pi h \omega_0^2 R^3}{d} = \underline{\underline{1{,}3\,W}}$$

• USTAVLJANJE

$M = J\alpha$

$$\alpha = -\frac{d\omega}{dt}$$

$$J = \frac{mR^2}{2}$$

$$m = \rho \cdot h\pi R^2$$

$$\frac{\eta 2\cancel{\pi}\cancel{h}\omega \cancel{R^3}}{d} = \frac{\rho \cancel{h}\cancel{\pi} \cancel{R^4}}{2}\left(-\frac{d\omega}{dt}\right)$$

$$\int_0^{t_{1/2}} dt = \frac{\rho R d}{4\eta} \int_{\omega_0}^{\omega_0/2} -\frac{d\omega}{\omega}$$

$$t_{1/2} = \frac{\rho R d}{4\eta} \ln\left(\frac{\cancel{\omega_0}}{\cancel{\omega_0}/2}\right)$$

$$\underbrace{t_{1/2} = \frac{\rho R d}{4\eta} \ln 2} = \underline{\underline{0{,}68\,s}}$$

(8.4.) nal 3

R_2

l

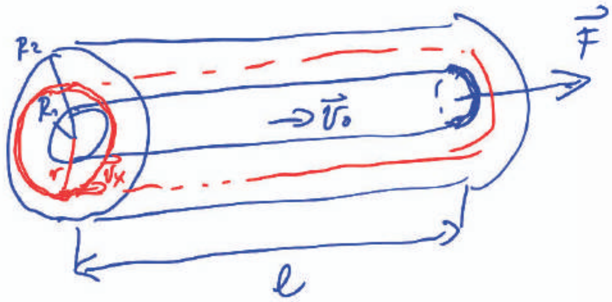
η

R_1

v_0

$F = ?$

$v(r) = ?$



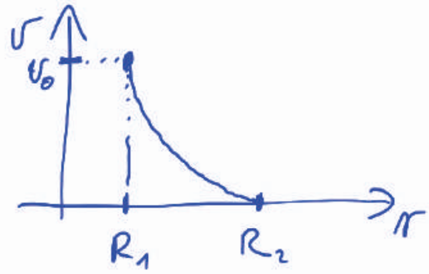
$$F = \eta \cdot S \cdot \frac{dv_x}{dr} = \eta 2\pi r l \left(-\frac{dv}{dr}\right)$$

$$\frac{F}{2\pi\eta l}\int_{R_2}^{r}\frac{dr}{r} = -\int_0^v dv$$

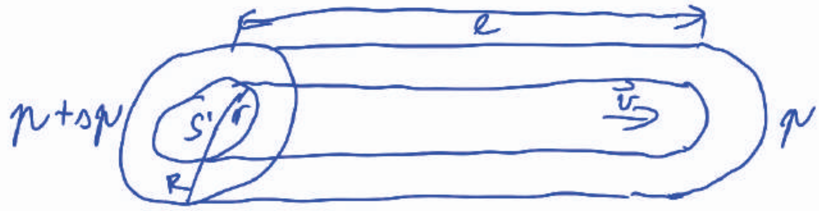
$$\frac{F}{2\pi\eta l}\ln\frac{R_2}{r} = v$$

$$\boxed{F = 2\pi\eta l v_0 \cdot \frac{1}{\ln\frac{R_2}{R_1}}}$$

$$\boxed{v = v_0\,\frac{\ln\frac{R_2}{r}}{\ln\frac{R_2}{R_1}}}$$



POISEVILLE ZAKON $\Rightarrow$ PRETOK VISKOZNE TEKOČINE



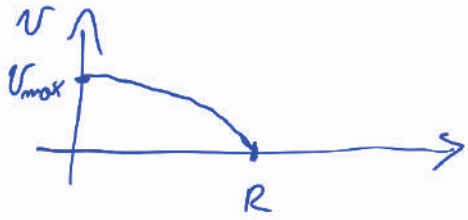
- ZARADI $\Delta p \Rightarrow F = \Delta p \cdot S'$; $S' = \pi r^2$
- VISKOZNOST $\Rightarrow F = \eta \cdot S \dfrac{dv_x}{dr}$; $S = 2\pi r l$; $\dfrac{dv_x}{dr} = -\dfrac{dv}{dr}$

$$\hookrightarrow \Delta p \cancel{\pi} r^{\cancel{2}} = \eta \, 2 \cancel{\pi} \cancel{r} l \left(-\frac{dv}{dr}\right)$$

$\leftarrow$ SILI STA ENAKI, KER GLEDAMO RAVNOVESJE!
$\hookrightarrow \underline{v \neq v(t)}$

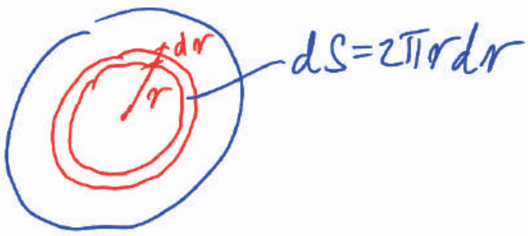
$$\frac{\Delta p}{2\eta l}\int_R^r (-r\,dr) = \int_0^v dv$$

- HITROST: $\boxed{v = \dfrac{\Delta p}{4\eta l}\left(R^2 - r^2\right)}$



$$\boxed{v_{max} = \frac{\Delta p R^2}{4\eta l}}$$

- VOLUMSKI PRETOK



$$d\phi_V = v\,dS$$

$$\phi_V = \frac{\Delta p \cancel{2}\pi}{\cancel{4}_{2}\eta l}\int_0^R (R^2 - r^2)\, r\,dr$$

$$= \frac{\pi \Delta p}{2\eta l}\left(R^2\frac{R^2}{2} - \frac{R^4}{4}\right) = \frac{\pi \Delta p R^4}{8\eta l}$$

$$\boxed{\phi_V = \frac{\pi \Delta p R^4}{8\eta l}}$$

8.4. nal 4

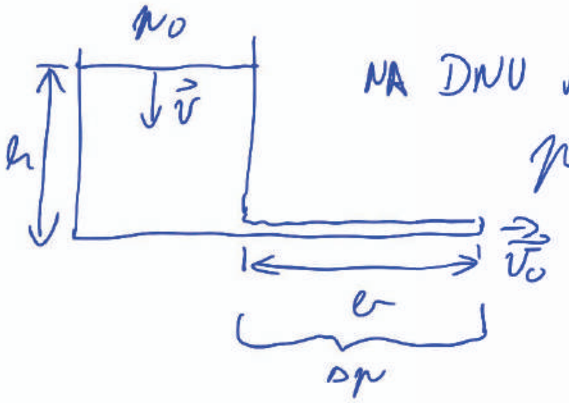
$h_0 = 20\,\mathrm{cm}$

$S = 20\,\mathrm{cm}^2$

$l = 10\,\mathrm{cm}$

$S_0 = 0.2\,\mathrm{mm}^2$

$t_{1/2}\left(h = \frac{h_0}{2}\right) = ?$



NA DNU POSODE:

$$p = p_0 + \rho g h = p_0 + \Delta p$$

$$\boxed{\Delta p = \rho g h}$$

$$\underline{\phi_V = S v} \quad ; \; v = -\frac{dh}{dt}$$

$$\frac{\pi \Delta p R^4}{8 \eta l} = S\left(-\frac{dh}{dt}\right)$$

$$S_0 = \pi R^2$$

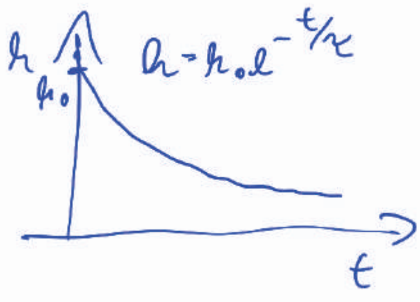
$$\underline{R^2 = \frac{S_0}{\pi}}$$

$$\frac{\cancel{\pi} \rho g h S_0^2}{8 \eta l \pi^{\cancel{2}}} = S\left(-\frac{dh}{dt}\right)$$

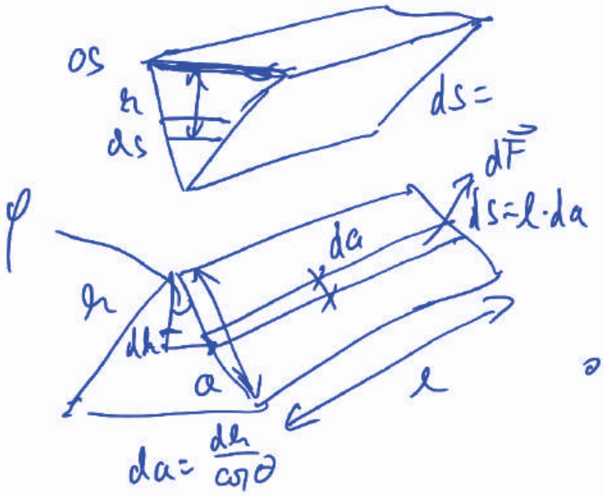
$$\int_0^{t_{1/2}} dt = -\frac{8 \eta l S \pi}{\rho g S_0^2} \int_{h_0}^{h_0/2} \frac{dh}{h}$$

$$\underline{t_{1/2} = \frac{8 \eta l S \pi}{\rho g S_0^2} \ln 2}$$

$$\boxed{t = -\underbrace{\frac{8 \eta l S \pi}{\rho g S_0}}_{\tau = 20{,}1\,\mathrm{s}} \ln \frac{h}{h_0}}$$



ZBIRKA 9 nal 2/2f Z6



$p = \rho g h$

$$\int_0^F dF = p \cdot dS = \rho g h l \frac{dh}{\cos\theta}$$

ZBIRKA 9 nal 11/s130

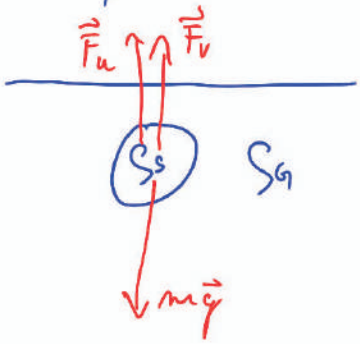
$r = 1\,cm$

$\rho_s = 2{,}5\,g/cm^3$

$\rho_G = 1{,}26\,g/cm^3$

$\eta = 1{,}39\,kg/m\,s$

$X(v = 0{,}99\,v_{max}) = ?$



ZA KROGLO (TOČNO)

$F_u = 6\pi r \eta v$

OCENA:

$$\left(F = \eta S \frac{dv_x}{dy} \sim \eta \frac{4\pi r^{\cancel{2}} v}{\cancel{r}} \quad \left.\begin{array}{l} S \sim 4\pi r^2 \\ dv_x \sim v \\ dy \sim r \end{array}\right] = 4\pi r \eta v \right)$$

$$ma = mg - F_v - F_u$$

$$\frac{4\cancel{\pi} r^{\cancel{3}\,2}}{3}\rho_s a = \frac{4\cancel{\pi} r^{\cancel{3}\,2}}{3} g(\rho_s - \rho_G) - 6\cancel{\pi}\cancel{r}\eta v$$

$$F_v = \frac{4\pi r^3}{3}\cdot\rho_G\, g$$

$$mg = \frac{4\pi r^3}{3}\rho_s\cdot g$$

• MAKSIMALNA $v \Rightarrow a = 0$ (RAVNOVESJE)

$$v_{max} = \frac{4r^2 g(\rho_s - \rho_G)}{18\eta} = \underline{\underline{5\,cm/s}}$$

• POT: $\quad a = \frac{dv}{dt}\cdot\underbrace{\frac{dx}{dx}}_{} = v\frac{dv}{dx}$, $\quad \frac{dx}{dt} = v$

$$\frac{4r^2}{3}\rho_s\cdot v\frac{dv}{dx} = \frac{4r^2}{3}g(\rho_s - \rho_G) - 6\eta v \quad \Big/\cdot\frac{1}{6\eta}$$

$$\frac{4r^2\rho_s}{18\eta}\cdot v\frac{dv}{dx} = v_{max} - v$$

$$\frac{2r^2\rho_s}{9\eta}\int_0^{0{,}99\,v_{max}}\frac{v\,dv}{(v_{max} - v)} = \int_0^X dx$$

$$v_{max} - v = u$$

$$-dv = du$$

$$v = v_{max} - u$$

$$\frac{2r^2\rho_s}{9\eta}\int_{v_{max}}^{0{,}01\,v_{max}}\frac{(v_{max} - u)(-du)}{u} = X$$

$$\frac{2r^2\rho_s}{9\eta}\int_{v_{max}}^{0{,}01\,v_{max}}\left(1 - \frac{v_{max}}{u}\right)du = X$$

$$\frac{2r^2\rho_s}{9\eta}\left(-0{,}99\,v_{max} - v_{max}\ln\frac{0{,}01\,\cancel{v_{max}}}{\cancel{v_{max}}}\right) = X$$

$$\frac{2r^2\rho_s}{9\eta}v_{max}\left(\ln 100 - 0{,}99\right) = X \qquad \underline{\underline{X = 1{,}8\,mm}}$$

SPLOŠNO $\ln\frac{v_{max}}{v_{max} - v}$

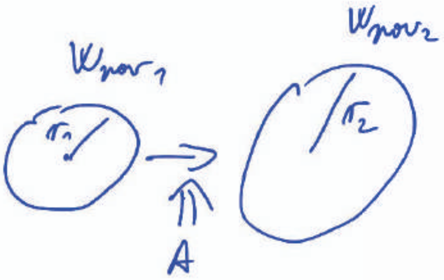
8.5 Površinska napetost

8.5. nal 1

MEHURČEK:

$r_1 \to r_2$

$A = ?$



MEHURČEK (2 POVRŠINI)

$W_{pov} = 2 \cdot 4\pi r^2 \cdot \gamma$

$W_{pov} = \gamma \cdot S$

$$A = W_{pov_2} - W_{pov_1} = \underline{8\pi\gamma\left(r_2^2 - r_1^2\right)}$$

EKVIVALENTNO:

$$dA = -p\,dV$$

$$\int_0^A dA = -\int_{r_1}^{r_2} \frac{4\gamma}{\cancel{r}} \cdot 4\pi r^{\cancel{2}}\,dr$$

MEHURČEK

$V = \frac{4\pi r^3}{3}$

$\underline{dV = 4\pi r^2 dr}$

$\underline{p = \frac{4\gamma}{r}}$

$$A = 16\gamma\pi\left(\frac{r_1^2 - r_2^2}{2}\right)$$

$$A = 8\gamma\pi\left(r_1^2 - r_2^2\right)$$

$$\underline{A = -8\gamma\pi\left(r_2^2 - r_1^2\right)}$$

TLAK: (1 POVRŠINA)

$$\underline{dS = r \cdot d\theta \cdot r\cos\theta\, d\varphi}$$

2π

$$\underline{dF = p\,dS} \qquad \varphi(0 \to 2\pi) \quad \theta\left(0 \to \frac{\pi}{2}\right)$$

$$dF_y = dF \cdot \sin\theta$$

$$= p\, 2\pi r^2 \cos\theta \sin\theta\, d\theta$$

$$\int_0^{F_y} dF_y = p\, 2\pi r^2 \frac{1}{2}\int_0^{\pi/2} \sin 2\theta\, d\theta$$

$$F_y = p\pi r^2\left(-\frac{\cos 2\theta}{2}\right)\Big|_0^{\pi/2}$$

SILA TLAKA V KROGLI TEKOČINE $\rightarrow$ $\boxed{F_y = p\pi r^2}$

$\rightarrow$ SILA POVRŠINSKE NAPETOSTI:

$$F_y = 2\pi r\gamma = p\pi r^2$$

$$\boxed{p = \frac{2\gamma}{r}}$$

TLAK V KAPLJICI $\rightarrow$ (1 POVRŠINA)

TLAK V MEHURČKU $\rightarrow$ (2 POVRŠINI)

$$\underline{p = 2 \cdot \frac{2\gamma}{r} = \frac{4\gamma}{r}}$$

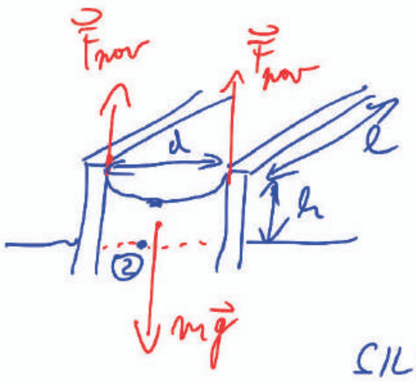
8.5. rel 2

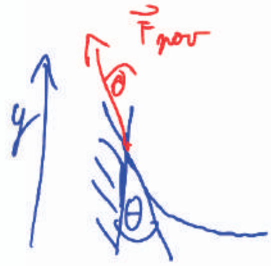
$d = 1\,mm$

$\gamma = 0{,}07\,N/m$

$\theta = 0^\circ$

$h = ?$





$F_{pov} = \gamma \cdot l$

$F_{pov_y} = \gamma \cdot l \cdot \cos\theta$

SILE: $2F_{pov_y} = mg \quad ; \theta = 0$

$\cos\theta\, 2\gamma \cancel{l} = h \cancel{l} d \cdot \rho \cdot g$

$$\boxed{h = \frac{2\gamma \cdot \cos\theta}{\rho g d}}$$

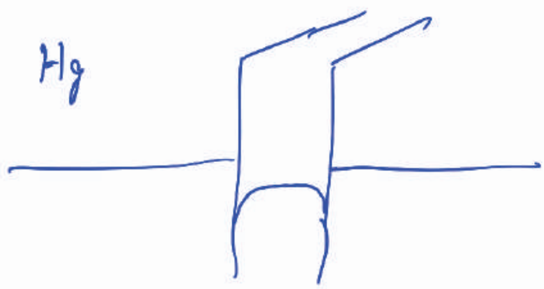
SILE POVRŠIN

TLAK: $\Delta p_{pov} = \frac{2\gamma l \cos\theta}{d l}$

$\underline{\Delta p} = \underline{\frac{2\gamma \cos\theta}{d}}$

$\Delta p = \rho g h \rightarrow \underline{h = \frac{2\gamma \cos\theta}{\rho g d}}$

→ ZA Hg



$\cos\theta > 90^\circ$

$\hookrightarrow \Delta p < 0$

$\underline{h < 0}$

DN → h v KAPILARI (RADIJ r)

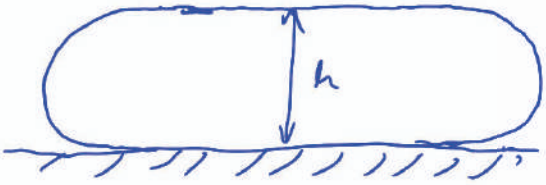
ZBIRKA 9 nal 26/st 29

$\gamma = 0{,}07\ N/m$

$h = ?$

$\theta = 180°$

NE OMOČI $\rightarrow \theta = 180°$



• ENERGIJA:

$$W = W_p + W_{pov}$$

$$= mg\frac{h}{2} + \gamma_{kp} S' + \gamma_{kt} \cdot S'$$

$$W_p = mg\frac{h}{2}$$

$$S' = \frac{V}{h} \longrightarrow W = \rho S' h \cdot g \frac{h}{2} + S'(\gamma_{kp} + \gamma_{kt})$$

$$\underline{W = \rho V \cdot g \frac{h}{2} + \frac{V}{h}(\gamma_{kp} + \gamma_{kt})}$$

$S = 2 \cdot S' + h$ OBSEG

$\underline{S = 2S'}$ ← h → ZA MAJHNE h

$h = ? \Rightarrow$ MINIMUM W:

$\gamma_{kp} \rightarrow$ KAPLJEVINA – PLIN

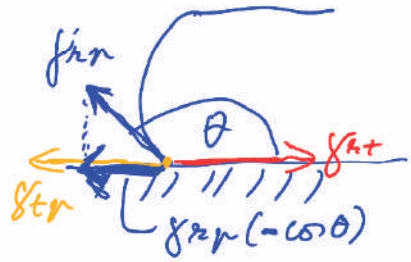
$\gamma_{kt} \rightarrow$ KAPLJEVINA – TRDNINA

$$\frac{dW}{dh} = \rho V g \frac{1}{2} - \frac{V}{h^2}(\gamma_{kp} + \gamma_{kt}) = 0$$

$$h = \sqrt{\frac{2(\gamma_{kp} + \gamma_{kt})}{\rho g}}$$

$$\boxed{h = \sqrt{\frac{4\gamma}{\rho g}}}$$

$$\underline{h = 7{,}6\ mm}$$



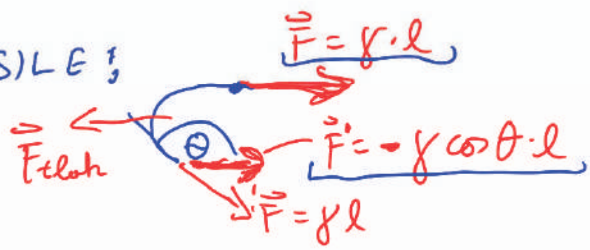
$$\gamma_{tp} - \gamma_{kp}\cos\theta = \gamma_{kt}$$

$$\boxed{\gamma_{kp}\cos\theta = \gamma_{tp} - \gamma_{kt}}$$

$$\theta = 180° \Rightarrow \gamma_{tp} = 0$$

$$\underline{\gamma_{kt} = \gamma_{kp}}$$

• SILE:



$$dF_{tlak} = p \cdot dS = \rho g h \cdot l \cdot dh \Rightarrow F_{tlak} = \rho g l \frac{h^2}{2}$$

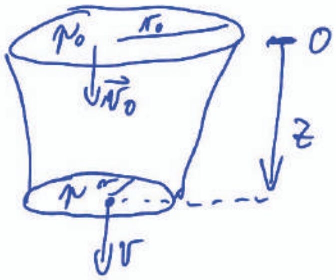
$$F + F' = F_{tlak}$$

$$\gamma l(1 - \cos\theta) = \rho g l \frac{h^2}{2} \Rightarrow \boxed{h = \sqrt{\frac{2\gamma(1-\cos\theta)}{\rho g}}}$$

8.5. nal 4

$r_0 = 1\,cm$

$v_0 = 0.5\,m/s$



$$\phi_V = v_0 \cdot \pi r_0^2 = v \pi r^2$$

$$v = v_0 \frac{r_0^2}{r^2}$$

BERNULIJEVA E.: $p_0 + \frac{\rho v_0^2}{2} = p - \rho g z + \frac{\rho v^2}{2}$

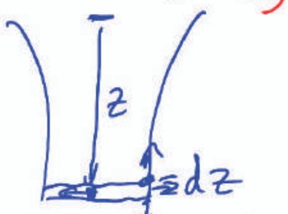
TLAK ZUNAJ MANJŠI

$$\frac{3\gamma}{r_0} + \frac{\rho v_0^2}{2} = \frac{3\gamma}{r} - \rho g z + \frac{\rho v_0^2}{2}\left(\frac{r_0}{r}\right)^4$$

$$\rho g z = 3\gamma\left(\frac{1}{r} - \frac{1}{r_0}\right) + \frac{\rho v_0^2}{2}\left[\left(\frac{r_0}{r}\right)^4 - 1\right]$$

$$\boxed{z = \frac{3\gamma}{\rho g}\left(\frac{1}{r} - \frac{1}{r_0}\right) + \frac{v_0^2}{2g}\left[\left(\frac{r_0}{r}\right)^4 - 1\right]}$$

$$dA = -p\,dV$$
$$dW = -p\,dV$$
$$\hookrightarrow p = -\frac{dW}{dV} = -\gamma \frac{dS}{dV}$$
$$\boxed{p = -\frac{3\gamma}{r}}$$



$$\hookrightarrow dV = \pi r^2 dz$$
$$dW = \gamma \cdot dS = \gamma \cdot 2\pi r\,dz$$
$$\rightarrow V = \pi r^2 z$$
$$S = 2\pi r z$$

$$\frac{dS}{dV} = \frac{\partial S}{\partial z}\left(\frac{\partial z}{\partial V}\right) + \frac{\partial S}{\partial r}\left(\frac{\partial r}{\partial V}\right) = \frac{\frac{\partial S}{\partial z}}{\frac{\partial V}{\partial z}} + \frac{\frac{\partial S}{\partial r}}{\frac{\partial V}{\partial r}}$$

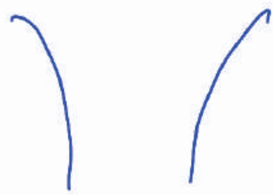
$$= \frac{2\cancel{\pi r}}{\cancel{\pi r} r} + \frac{\cancel{2\pi z}}{\cancel{2\pi} r \cancel{z}} = \frac{3}{r}$$

$$z_{1/2}\left(r = \frac{r_0}{2}\right) =$$

ČE $\gamma \to 0$:

$$z = \frac{v_0^2}{2g}\left[\left(\frac{r_0}{r}\right)^4 - 1\right]$$

$$\hookrightarrow r = r_0 \sqrt{\frac{v_0}{\sqrt{v_0^2 + 2gz}}}$$



9 Mehansko valovanje

ZBIRKA 9 nal 1/st 31

$\nu_p = 33\,\text{min}^{-1}$

$r_1 = 10\,\text{cm}$

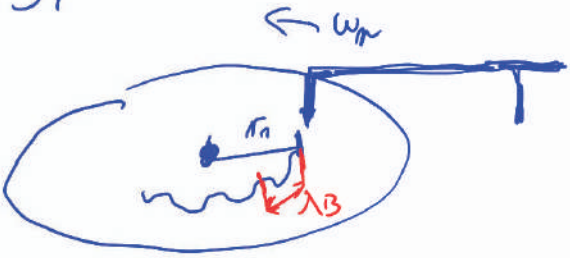
$\nu = 440\,\text{Hz}$

a) $\lambda_B = ?$

b) $r_0 = 3\,\text{cm}$

$a = 20\,\mu\text{m}$

$\nu_{max} = ?$



ČAS NIHAJA:

$t_0 = \frac{1}{\nu}$

POT:

$\lambda_B = t_0 \cdot v_1$

HITROST:

$v_1 = \omega_p r_1 = 2\pi \nu_p \cdot r_1$

a)

$$\underline{\lambda_B = \frac{1}{\nu} \cdot 2\pi \nu_p \cdot r_1}$$

$$\underline{\lambda_B = 0{,}79\,\text{mm}}$$

b)

$\lambda_{B_{min}} = 2a$

$$\lambda_{B_{min}} = \frac{2\pi \nu_p r_0}{\nu_{max}} \Rightarrow \underline{\nu_{max} = \frac{2\pi \nu_p r_0}{2a}}$$

$$\underline{\underline{= 2{,}6\,\text{kHz}}}$$

(9) nal 2

$y(x,t) = a \sin(kx - \omega t - \pi/4)$

$\to$ V SMERI X

$+ \to$ V SMERI $-X$

$a = 0.05\,m$

$k = 5\pi\ m^{-1}$

$\omega = 20\pi\ s^{-1}$

$\lambda = ?$

$t_0 = ?$

$c = ?$

SMER = ?

$\dot{y}(0,0) = ?$

$\lambda = \frac{c}{\nu} = \frac{2\pi}{k} = 0{,}4\,m$

$t_0 = \frac{1}{\nu} = \frac{2\pi}{\omega} = 0{,}1\,s$

$c = \frac{\omega}{k} = 4\,m/s$

GIBLJE SE V (+X)

$$\boxed{c = \lambda \nu \quad ; \quad \omega = 2\pi\nu \qquad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}}$$

$$y(x,t) = y_0 \sin(kx - \omega t + \delta) = y_0 \sin\left(k(x - ct) + \delta\right)$$

VALOVNA ENAČBA:

$$\boxed{\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y(x,t)}{\partial t^2}}$$

- ČASOVNI ODVOD
- 1 KRAJEVNI ODVOD

$\dot{y}(x,t) = a(-\omega)\cos(kx - \omega t - \frac{\pi}{4})$

$\ddot{y}(x,t) = -a\omega^2 \cos(kx - \omega t - \frac{\pi}{4}) = -\omega^2 y(x,t)$

$\dot{y}(0,0) = -\omega a \cos(\frac{\pi}{4})$

ZBIRKA 9 nal 2/st 37

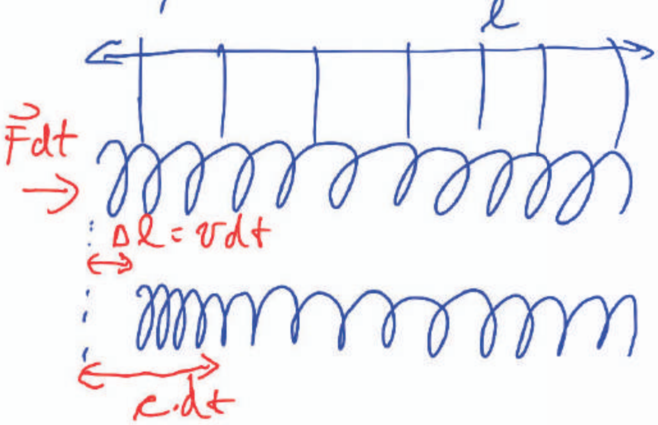
$m = 1\,kg$

$l = 6\,m$

$2r = 10\,cm$

$k = 100\,N/m$

$c = ?$



• GIBALNA KOLIČINA

$$F\cancel{dt} = dm\,v = m\frac{c\cancel{dt}}{l}\cdot v \qquad dm = m\frac{c\,dt}{l}$$

$$F = \frac{mcv}{l}$$

• RAZTEZK:

$$\frac{\Delta l}{l} = \frac{v\,dt}{c\,dt}$$

$$F = k\,\Delta l = k\,l\,\frac{v}{c} \qquad \Delta l = l\,\frac{v\cancel{dt}}{c\cancel{dt}}$$

$$\Rightarrow \frac{mc\cancel{v}}{l} = \frac{kl\cancel{v}}{c}$$

$$c^2 = \frac{kl^2}{m} \Rightarrow \boxed{c = \sqrt{\frac{kl^2}{m}}}$$

• $m = \rho l \cdot S$ $\qquad k = \frac{ES}{l}$

• $\frac{F}{S} = E\frac{\Delta l}{l} \Rightarrow F = E \cdot S\frac{\Delta l}{l}$

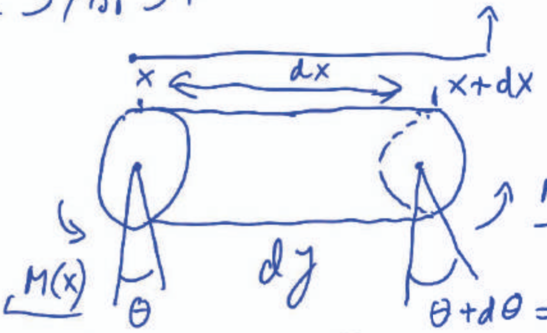
$$c = \sqrt{\frac{E\cancel{S}\,\cancel{l^2}}{\cancel{l}\rho\cancel{l}\cancel{S}}} = \sqrt{\frac{E}{\rho}}$$

ZBIRKA 9 nal 3/st 31

$l = 10\,m$

$\rho = 7{,}8\,g/cm^3$

$G = 80000\,N/mm^2$



$M(x+dx) = M(x) + dM = M(x) + \frac{dM}{dx}\cdot dx$

$\theta + d\theta = \theta + \frac{d\theta}{dx}\cdot dx$

RAZLIKA M VRTI VMESNI DEL ŽICE

$M(x+dx) - M(x) = dJ\,\alpha \quad ; \; \alpha = \ddot{\theta}$

$dJ = \frac{dm\,R^2}{2}$ UZTRAJNOSTNI MOMENT

$= \frac{\rho\pi R^2 \cdot dx \cdot R^2}{2}$

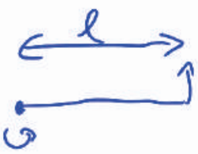
$$dJ = \frac{\rho\pi R^4}{2}\cdot dx$$

$$\frac{dM}{dx}\cdot \cancel{dx} = \frac{\rho\pi R^4}{2}\cdot \cancel{dx}\,\ddot{\theta}$$

$$\frac{G\cancel{\pi R^4}}{\cancel{2}}\frac{d^2\theta}{dx^2} = \frac{\rho\cancel{\pi R^4}}{\cancel{2}}\ddot{\theta}$$

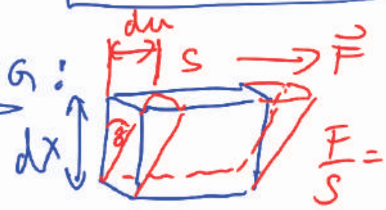
$$\frac{d^2\theta}{dx^2} = \underbrace{\frac{\rho}{G}}_{\frac{1}{c^2}}\cdot\ddot{\theta}$$

$$\Rightarrow \boxed{c = \sqrt{\frac{G}{\rho}}}$$



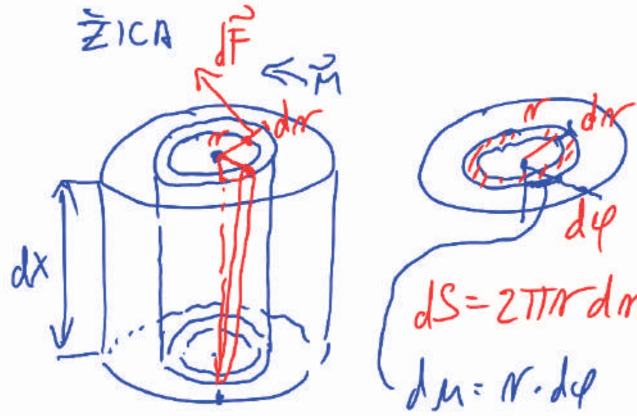
$t = \frac{l}{c} = 3{,}1\,ms$

• STRIG:



$\frac{F}{S} = G\frac{du}{dx}$

ŽICA



$dS = 2\pi r\,dr$

$du = r\cdot d\varphi$

• $dF = G\frac{du}{dx}\cdot dS$

$= G\,\frac{r\,d\varphi\cdot 2\pi r\,dr}{dx}$

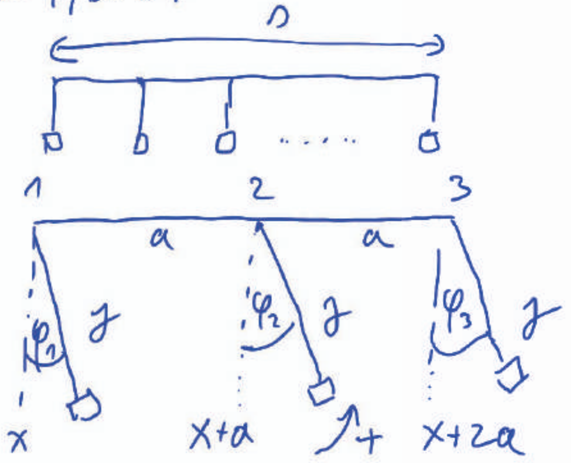
• NAVOR: $dM = dF\,r$

$$\int_0^M dM = G\,2\pi\,\frac{d\varphi}{dx}\int_0^R r^3\,dr$$

$$\boxed{M = \frac{G\pi R^4}{2}\cdot\frac{d\varphi}{dx}}$$

ZBIRKA 9 nal 4/st 31

$D_b = 3000\ Nm$
$b = 1m$
$a = 10m$
$J = 30\ kgm^2$
$s = 100m$
$t = ?$



TORZIJSKI KOEF.

$\underline{M = D \cdot \Delta\varphi}$

$D \propto \frac{1}{l}$

$D_a = D_b \frac{b}{a}$

• NAVOR 1 NA 2

$$M_{12} = D_a \Delta\varphi_{12} = D_a(\varphi_1 - \varphi_2)$$

• NAVOR 3 NA 2:

$$M_{32} = D_a \Delta\varphi_{32} = D_a(\varphi_3 - \varphi_2)$$

$\Rightarrow M_{12} + M_{23} = J\ddot{\varphi}_2$

$D_a[\varphi_1 - \varphi_2 + \varphi_3 - \varphi_2] = J\ddot{\varphi}_2$

$D_a[\varphi_1 - 2\varphi_2 + \varphi_3] = J\ddot{\varphi}_2$

$D_b \frac{b}{\cancel{a}} \cdot a^{\cancel{2}} \frac{\partial^2\varphi(x+a)}{\partial x^2} = J\ddot{\varphi}(x+a)$

$$\boxed{\frac{\partial^2\varphi}{\partial x^2} = \underbrace{\frac{J}{D_b \cdot b \cdot a}}_{= \frac{1}{c_0^2}}\ddot{\varphi}}$$

$$\frac{dy}{dx} = \frac{y_{n+1} - y_n}{\Delta x}$$

$$\frac{d^2y}{dx^2} = \frac{y_{n+1} - 2y_n + y_{n-1}}{\Delta x^2}$$

VZAMEM $\Delta x = a$:

$\varphi = \varphi(x)$

$\varphi_1 = \varphi(x), \varphi_2(x+a), \varphi_3(x+2a)$

$\varphi_1 - 2\varphi_2 + \varphi_3 = \varphi(x) - 2\varphi(x+a) + \varphi(x+2a)$

$= a^2 \frac{d^2\varphi(x+a)}{dx^2}$

$$\boxed{c_0 = \sqrt{\frac{D_b a b}{J}}} = \sqrt{1000}\ m/s$$

$t = \frac{s}{c} = \underline{3{,}16\ s}$

$\rightarrow$ REŠITEV VALOVNE ENAČBE $\varphi = \varphi_0 \cos[k(x - ct) + \delta]$

• VZAMEM KAR REŠITEV • $\varphi = \varphi_0 \cos(k(x-ct))$

• $x = a\cdot n$; $\Delta x = a$

$\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$

• $$\frac{d^2\varphi}{dx^2} = \frac{\varphi_{n+1}-2\varphi_n+\varphi_{n-1}}{\Delta x^2} =$$

$$= \frac{\varphi_0}{a^2}\left[\cos\{k[a(n+1)-ct]\} - 2\cos\{k[an-ct]\} + \cos\{k[a(n-1)-ct]\}\right]$$

$$= \frac{\varphi_0}{a^2}\Big[\cos\{k[an-ct]\}\cos ka - \cancel{\sin\{k[an-ct]\}\sin ka} - 2\cos\{k[an-ct]\} +$$
$$+\cos\{k[an-ct]\}\cos ka + \cancel{\sin\{k[an-ct]\}\sin ka}\Big]$$

$$= \frac{\varphi}{a^2}[2\cos ka - 2]$$

• $$\frac{d^2\varphi}{dt^2} = -k^2c^2\varphi$$

→ VALOVNA ENAČBA: $$\frac{\partial^2\varphi}{\partial x^2} = \frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial t^2}$$

$$\frac{\cancel{\varphi}}{a^2}\cdot 2[\cos ka - 1] = \frac{1}{c_0^2}(-k^2c^2)\cancel{\varphi}$$

$$\frac{2c_0^2}{a^2}[1-\cos ka] = k^2c^2$$

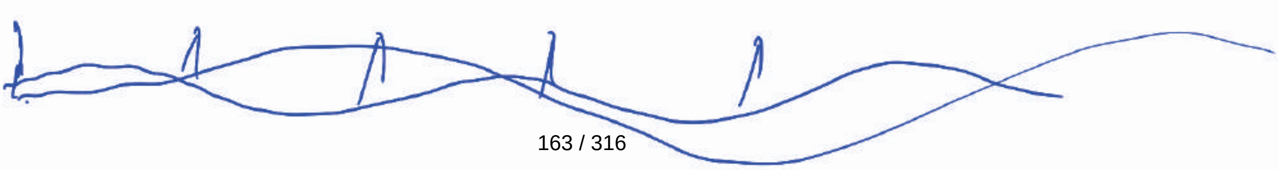
$$\boxed{c^2 = \frac{2c_0^2}{k^2a^2}[1-\cos ka]}$$

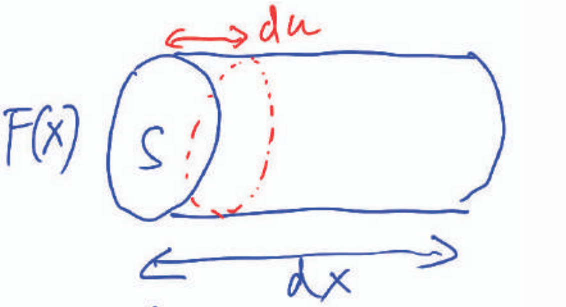
$k = \frac{2\pi}{\lambda}$ HITROST JE ODVISNA OD λ

DISPERZIJA: ODVISNOST $c(k)$

↳ NAZAJ NA HOMOGENO SREDSTVO $a\to 0$: $\cos ka = 1-\frac{k^2a^2}{2}$

↳ $$c^2 = \frac{2c_0^2}{k^2a^2}\left(1-1+\frac{k^2a^2}{2}\right) = c_0^2$$





(9) nal 6

$L = 10\,cm$

$T_1 = 20°C$

$T_2 = 120°C$

$t = ?$

$$F(x+dx) - F(x) = dm\,\ddot{u}$$

$$dF = \rho S dx\,\ddot{u}$$

$$\frac{dF}{dx} = \rho S \ddot{u}$$

$$\frac{\cancel{S}}{\chi}\frac{d^2u}{dx^2} = \rho \cancel{S}\ddot{u}$$

$$\frac{d^2u}{dx^2} = \underbrace{\rho\chi}_{=1/c^2}\ddot{u}$$

$$\Downarrow$$

$$\boxed{c = \frac{1}{\sqrt{\rho\cdot\chi}}}$$

- $dm = \rho\cdot S\cdot dx$
- $F = \Delta p S = \frac{S}{\chi}\frac{du}{dx}$
- STISKLJIVOST:

$$\frac{\Delta V}{V} = -\chi\Delta p$$

$$\Delta p = -\frac{1}{\chi}\frac{\Delta V}{V} = \frac{1}{\chi}\cdot\frac{du}{dx}$$

- $\frac{\Delta V}{V} = \frac{-du\cdot S}{dx S} = -\frac{du}{dx}$

- STISKLJIVOST: ZA ZVOK → ADIABATNA S.

$$pV^{\kappa} = konst \;/\ln \qquad \kappa = 1.4 \text{ (ZA ZRAK)}$$

$$\ln p + \kappa\ln V = konst$$

$$\frac{dp}{p} + \kappa\frac{dV}{V} = 0 \Rightarrow \boxed{\frac{dV}{V} = -\frac{1}{\kappa\cdot p}\cdot dp} \Rightarrow \boxed{\chi = \frac{1}{\kappa p}}$$

ADIABATNA STISKLJIVOST

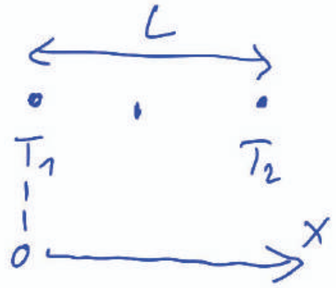
- HITROST: $c = \sqrt{\frac{\kappa p}{\rho}}$

$$\boxed{c = \sqrt{\frac{\kappa RT}{M}}} = c(T)$$

$$pV = \underbrace{\frac{m}{M}}_{=n}\cdot RT \Rightarrow \frac{p}{\rho} = \frac{RT}{M}$$

$M(\text{ZRAK}) = 29$ kg/mol

$$(pV^{\kappa})' = dp\,V^{\kappa} + p\,\kappa V^{\kappa-1}dV$$



• TEMPERATURA (x):

$$T(x) = T_1 + (T_2 - T_1)\frac{x}{L} = T_1\left[1 + \left(\frac{T_2}{T_1} - 1\right)\frac{x}{L}\right]$$

• POT: $c = \dfrac{dx}{dt} = \sqrt{\dfrac{\varkappa R T_1}{M}} \cdot \sqrt{1 + \left(\dfrac{T_2}{T_1} - 1\right)\dfrac{x}{L}}$

$$\sqrt{\frac{M}{\varkappa R T_1}} \int_0^L \frac{dx}{\sqrt{1 + \left(\frac{T_2}{T_1} - 1\right)\frac{x}{L}}} = \int_0^t dt$$

$$1 + \left(\frac{T_2}{T_1} - 1\right)\frac{x}{L} = u$$

$$\left(\frac{T_2}{T_1} - 1\right)\frac{dx}{L} = du$$

$$dx = L \cdot \left(\frac{T_2}{T_1} - 1\right)^{-1} du$$

$$\sqrt{\frac{M}{\varkappa R T_1}} \frac{L}{\left(\frac{T_2}{T_1} - 1\right)} \int_1^{T_2/T_1} \frac{du}{\sqrt{u}} = t$$

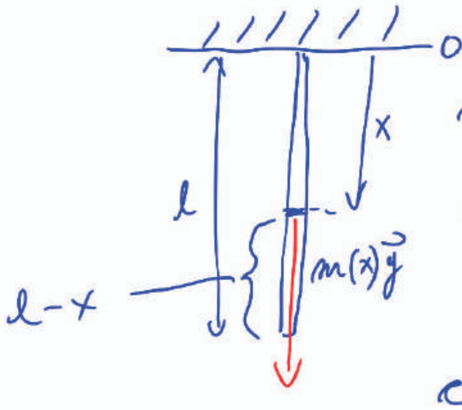
$$\boxed{\sqrt{\frac{M}{\varkappa R T_1}} \frac{L}{\left(\frac{T_2}{T_1} - 1\right)} \cdot 2\left(\sqrt{\frac{T_2}{T_1}} - 1\right) = t}$$

$$t = 2{,}7 \cdot 10^{-4}\ \mathrm{s}$$

(9) nal 7

l, m

$t = ?$



$$c = \sqrt{\frac{F l}{m}}$$

$$F(x) = m(x) g = m \frac{l-x}{l} \cdot g$$

$$\downarrow$$

$$c = \sqrt{\frac{\cancel{m}(l-x)\, g\, \cancel{l}}{\cancel{l}\, \cancel{m}}} = \sqrt{g(l-x)}$$

$$c = \frac{dx}{dt} = \sqrt{g(l-x)}$$

$$\int_0^t dt = \int_0^l \frac{dx}{\sqrt{g(l-x)}}$$

$$g(l-x) = u$$
$$-g\,dx = du$$
$$dx = -\frac{du}{g}$$

$$t = -\frac{1}{g}\int_{gl}^{0} \frac{du}{\sqrt{u}}$$

$$\underline{t} = \frac{1}{g} 2\sqrt{gl} = \underline{2\sqrt{\frac{l}{g}}}$$

ZBIRKA 9 nal 13/st 32

$l = 1m$

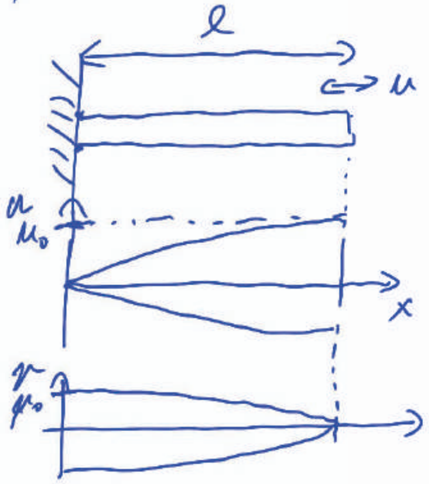
$\rho = 8{,}9\, g/cm^3$

$E = 80000\, N/mm^2$

$\nu_0 = ?$

$u_0 = 3\,\mu m$

$p(u = 2\,\mu m) = ?$



$c = \sqrt{\frac{E}{\rho}}$

$\lambda_0 = 4l \quad \left(l = \frac{\lambda}{4}\right)$

$\boxed{\nu_0 = \frac{c}{\lambda_0} = \frac{c}{4l}}$

$= 750\,Hz$

$\boxed{p = \frac{F}{S} = E\frac{\Delta l}{l} = E\frac{du}{dx}}$

$\frac{\Delta l}{l} = \frac{1}{E}\frac{F}{S}$

$\frac{\Delta l}{l} = \frac{du}{dx}$

STOJEČE VALOVANJE: (KRAJEVNI DEL):

$u = u_0 \sin(kx) \qquad k = \frac{2\pi}{\lambda}$

$= u_0 \sin\left(\frac{2\pi x}{\lambda}\right)$ ← KER TOGO VPETA

$\hookrightarrow \frac{u}{u_0} = \sin\left(\frac{2\pi x}{\lambda}\right)$

$\arcsin\frac{u}{u_0} = \frac{2\pi x}{\lambda} \Rightarrow \boxed{x = \lambda\frac{\arcsin\frac{u}{u_0}}{2\pi}} = 0.465\,m$

TLAK: $\boxed{p = E\frac{du}{dx} = \underbrace{E u_0 \frac{2\pi}{\lambda}}_{p_0} \cos\left(\frac{2\pi x}{\lambda}\right)}$ $\hookrightarrow p(u = 2\mu m) = 281\,kPa$

$p_0 \rightarrow$ AMPLITUDA TLAKA

$p_0 = \frac{2\pi E u_0}{\lambda} = 377\,kPa$

ZBIRKA 9 nal 15/st 32

$l = 80\ cm$

$2r = 0{,}1\ mm$

$\rho = 7{,}8\ g/cm^3$

$F = 100\ N$

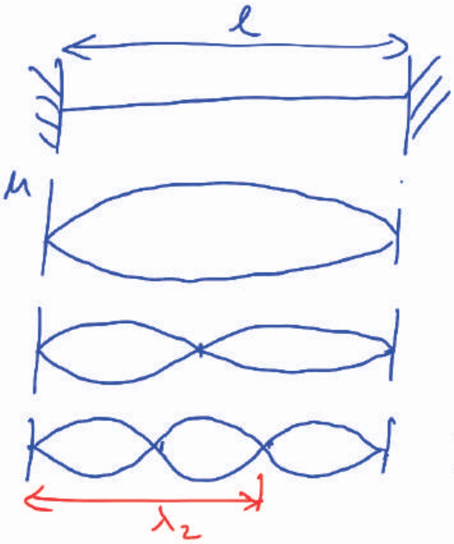
$\nu_0, \nu_1, \nu_2 = ?$

$u_{00} = 1\ mm$

$u_{01} = 0{,}5\ mm$

$u_{02} = 0{,}1\ mm$

W spektar $= ?$



$$c = \sqrt{\frac{F l}{m}} = \sqrt{\frac{F \cdot \cancel{l}}{\cancel{l} \cdot \rho \cdot \pi r^2}} = 638\ m/s$$

$$\boxed{\nu = \frac{c}{\lambda}}$$

$\nu_0, \lambda_0 = 2l \Rightarrow \nu_0 = 400\ Hz$

$\nu_1, \lambda_1 = l \Rightarrow \nu_1 = 800\ Hz$

$\nu_2, \lambda_2 = \frac{2}{3} l \Rightarrow \nu_2 = 1200\ Hz$

ENERGIJA VALOVANJA

↳ ZA STOJEĆE VALOVANJE $W = \frac{1}{4} m \omega^2 u_0^2$

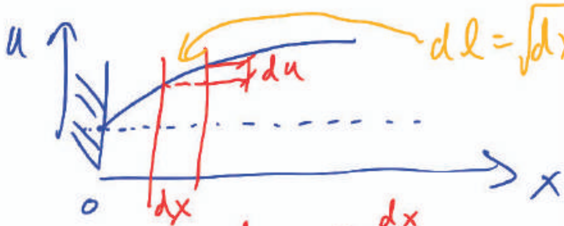
$$W_i = \frac{1}{4} \rho l \pi r^2 4\pi^2 \nu_i^2 u_{0i}^2 \quad (i = 0, 1, 2)$$

$W_0 = 3 \cdot 10^{-4}\ J$

$W_1 = 3 \cdot 10^{-4}\ J$

$W_2 = 0{,}27 \cdot 10^{-4}\ J$

9) nal 11 ENERGIJA STOJEČEGA VALOVANJA



$dl = \sqrt{dx^2+du^2} = dx\sqrt{1+\left(\frac{du}{dx}\right)^2} = dx\left[1+\frac{1}{2}\left(\frac{du}{dx}\right)^2\right]$

$du \ll dx$

$\hookrightarrow dm = m\frac{dx}{l}$

STOJEČE VAL.

$u = u_0 \sin(kx)\cdot\cos(\omega t)$

KRAJEVNI ČASOVNI

$\frac{du}{dt} = -u_0\omega\sin(kx)\sin(\omega t)$

$\frac{du}{dx} = u_0 k\cos(kx)\cos(\omega t)$

F: $c=\sqrt{\frac{Fl}{m}}$; $k=\frac{\omega}{c}$

$\hookrightarrow F = \frac{c^2 m}{l} = \frac{\omega^2 m}{k^2 l}$

ENERGIJE:

- $dW_k = \frac{1}{2}dm\left(\frac{du}{dt}\right)^2$
  $= \frac{1}{2}\frac{m}{l}\cdot dx\; u_0^2\omega^2\sin^2(kx)\sin^2(\omega t)$

- $dW_p = F(dl-dx) = F\left[dx\left[\cancel{1}+\frac{1}{2}\left(\frac{du}{dx}\right)^2\right]-\cancel{dx}\right]$
  $= F\frac{1}{2}\left(\frac{du}{dx}\right)^2 dx = \frac{1}{2}\frac{\omega^2 m}{\cancel{k^2}l}dx\,u_0^2\cancel{k^2}\cos^2(kx)\cos^2(\omega t)$
  $= \frac{1}{2}\frac{m}{l}dx\,u_0^2\omega^2\cos^2(kx)\cos^2(\omega t)$

- $dW = dW_k + dW_p$
  $= \frac{1}{2}\frac{m}{l}dx\,u_0^2\omega^2\left[\sin^2(kx)\sin^2(\omega t)+\cos^2(kx)\cos^2(\omega t)\right]$

POVPREČENJE PO ČASU: $\int^{t_0}\frac{dW}{t_0}\cdot dt$

$\frac{1}{t_0}\int_0^{t_0}\sin^2\left(\frac{2\pi}{t_0}\cdot t\right)dt = \frac{1}{2}$

$\frac{1}{t_0}\int_0^{t_0}\cos^2\left(\frac{2\pi}{t_0}t\right)dt = \frac{1}{2}$

$\hookrightarrow dW_{AVG(t)} = \frac{1}{2}\frac{m}{l}dx\,u_0^2\omega^2\left[\frac{1}{2}\sin^2(kx)+\frac{1}{2}\cos^2(kx)\right]$

$= \frac{1}{4}\frac{m}{l}dx\,u_0^2\omega^2$ ← POVPREČNA W DELČKA dx

CELOTNA ENERGIJA : $\int_0^l dW_{AVG(t)}\,dx = \boxed{W = \frac{1}{4}m u_0^2\omega^2}$

ENERGIJA STRUNE ZA STOJEČE VALOVANJE

POTUJOČE VALOVANJE: $u = u_0 \sin(kx-\omega t)$

$$\frac{du}{dx} = k u_0 \cos(kx-\omega t)$$

$$\frac{du}{dt} = -\omega u_0 \cos(kx-\omega t)$$

ENERGIJE:

• $dW_k = \frac{1}{2} dm \left(\frac{du}{dt}\right)^2 = \frac{1}{2} \frac{m}{l}\left(\frac{du}{dt}\right)^2 \cdot dx$

• $dW_p = F\frac{1}{2} \left(\frac{du}{dx}\right)^2 dx = \frac{1}{2} \frac{m}{l}\frac{\omega^2}{k^2}\left(\frac{du}{dx}\right)^2 dx$

$F = \frac{c^2 m}{l} = \frac{\omega^2 m}{k^2 l}$; $dm = \frac{m}{l} dx$

$$dW = dW_k + dW_p$$

$$dW_k = \frac{1}{2}\frac{m}{l} dx\, \omega^2 u_0^2 \cos^2(kx-\omega t)$$

$$dW_p = \frac{1}{2} \frac{m}{l} dx \frac{\omega^2}{\cancel{k^2}}\cancel{k^2} u_0^2\cos^2(kx-\omega t)$$

$$\Rightarrow dW = \frac{m}{l}\omega^2 u_0^2 \cos^2(kx-\omega t)\,dx$$

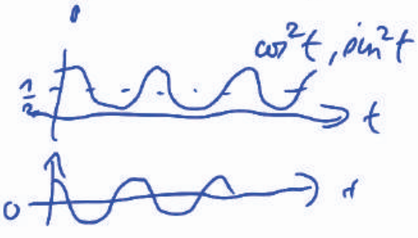
↳ ENERGIJA DELČKA dx V ČASU t

$$\cos^2(kx-\omega t) = (\cos kx \cos\omega t + \sin kx \sin\omega t)^2 =$$

$$= \cos^2 kx \cos^2 \omega t + \frac{1}{2}\sin(2kx)\sin(2\omega t) + \sin^2 kx \sin^2 \omega t$$

ČASOVNO POVPREČJE:

$$\frac{1}{t_0}\int^{t_0}\cos^2(kx-\omega t)\,dt = \frac{1}{2}\cos^2 kx + 0 + \frac{1}{2}\sin^2 kx$$

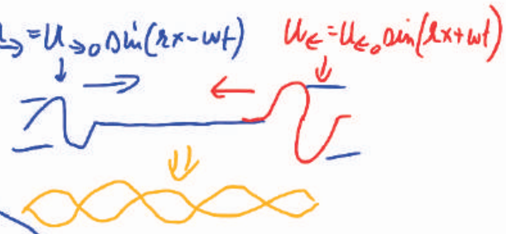


$$dW_{avg(t)} = \frac{m}{l}\omega^2 u_0^2\left(\frac{1}{2}\cos^2 kx + \frac{1}{2}\sin^2 kx\right)dx$$

$$= \frac{1}{2}\frac{m}{l}\omega^2 u_0^2\, dx$$

POTUJOČE VALOVANJE $\Rightarrow$ $\boxed{W = \int_0^l \frac{1}{2} \frac{m}{l}\omega^2 u_0^2\, dx = \frac{1}{2} m\omega^2 u_0^2}$

POVEZAVA S STOJEČIM VALOVANJEM:



W: $W_{stoječe} = W_\rightarrow + W_\leftarrow$

U_0: $U_{0s} = U_{\rightarrow 0} + U_{\leftarrow 0}$; $U_{\rightarrow 0} = U_{\leftarrow 0} = \frac{1}{2} U_{0s}$

$$W_\rightarrow \propto \frac{1}{2} U_{\rightarrow 0}^2 \propto \frac{1}{2}\frac{1}{4} U_{0s}^2 \propto \frac{1}{8} U_{0s}^2 \; ; \; W_\leftarrow \propto \frac{1}{8} U_{0s}^2 \Rightarrow W_s = 2\cdot\frac{1}{8} U_{0s}^2 = \frac{1}{4} U_{0s}^2$$

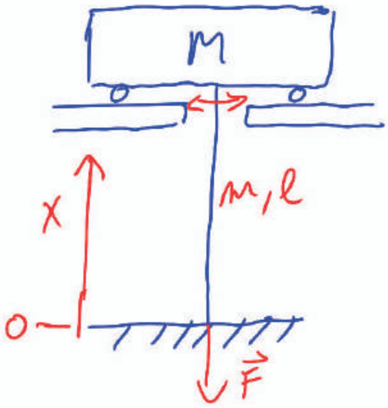
2008/09 2. kol/nal 4

m, l, F

$M \gg m$

$\nu_{0,1,2,\dots} = ?$

$\nu_0 = ?$



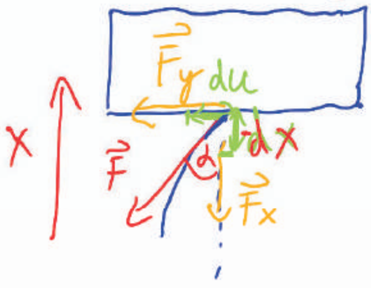
$c = \sqrt{\frac{F \cdot l}{m}}$ KER $u(0,t)=0$!

$$u(x,t) = u_0 \sin(kx)\sin(\omega t)$$

KER $M \neq \infty$ (KER STRUNA NI TOGO UPETA NA OBEH STRANEH): $\lambda_0 \neq 2l$

↳ λ IŠČEMO!

ROBNI POGOJ PRI $x=l$:



$$M\ddot{u}\Big|_{x=l} = -F \cdot \frac{du}{dx}\Big|_{x=l}$$

$F = \frac{c^2 m}{l}$

$\ddot{u} = -\omega^2 u$

$\frac{du}{dx} = k u_0 \cos kx \cdot \sin \omega t \cdot \frac{\sin kx}{\sin kx} = \frac{k}{\mathrm{tg}\, kx} \cdot u$

$k = \frac{\omega}{c}$

$$-M\omega^2 \cancel{u}\Big|_{x=l} = -\frac{c^2 m}{l}\frac{k}{\mathrm{tg}(kl)} \cdot \cancel{u}\Big|_{x=l}$$

$$M\omega^2 = \frac{c^2 m}{l}\frac{\omega}{c}\frac{1}{\mathrm{tg}\left(\frac{\omega l}{c}\right)}$$

$$\boxed{\mathrm{tg}\left(\frac{\omega l}{c}\right) = \frac{m}{M}\left(\frac{c}{\omega l}\right)}$$

$F_y = F\sin\alpha = -F\left(\frac{du}{dx}\right)$

$\mathrm{tg}\,\alpha = \frac{du}{-dx}$

MAJHNI KOTI

$\mathrm{tg}\,\alpha \sim \alpha$

$\sin\alpha \sim \alpha$

ZA $M \to \infty$ → TOGO VPETJE

$$\mathrm{tg}\left(\frac{\omega l}{c}\right) = 0 \Rightarrow \frac{\omega l}{c} = n\pi$$

• NAJNIŽJI ν: RAZVIJEMO $\mathrm{tg}(x)$ OKOLI NIČ: $\mathrm{tg}\, x \approx x + \frac{x^3}{3}$ $\left(x = \frac{\omega l}{c}\right)$

$$\mathrm{tg}(x) = \frac{m}{M}\frac{1}{x} \Rightarrow x + \frac{x^3}{3} = \frac{m}{M}\frac{1}{x}$$

$$\frac{x^4}{3} + x^2 - \frac{m}{M} = 0$$

ZA $\frac{m}{M} \to 0$

$\to \left(1 + \frac{1}{2}\frac{4}{3}\frac{m}{M}\right)$

$$x^2 = \frac{-1 \mp \sqrt{1 + \frac{4}{3}\frac{m}{M}}}{2/3} = \frac{-1 + 1 + \frac{2}{3}\frac{m}{M}}{2/3} = \frac{m}{M} \Rightarrow x = \sqrt{\frac{m}{M}}$$

DOVOLJ: $\mathrm{tg}\, x \approx x$

↳ $x = \frac{m}{M} \cdot \frac{1}{x}$

$x = \sqrt{\frac{m}{M}}$

$$\frac{\omega l}{c} = \sqrt{\frac{m}{M}}$$

$$\Rightarrow \omega_0 = \sqrt{\frac{m}{M}} \cdot \frac{c}{l}$$

ZA OSTALE REŠITVE RAZVOJ $tg\,x$ OKOLI $\underline{\underline{x = n\pi}}$

$$f(x) = \frac{f^{n}(a)}{n!}(x-a)^n$$

$a = n\pi$

↳ NIČLE $tg(x)$

$$tg'(x) = \frac{1}{\cos^2 x}$$

$$\hookrightarrow tg(x) = tg(a) + \frac{tg'(a)}{1}(x-a)$$

$$= tg(a) + \frac{1}{\cos^2 a}(x-a)$$

$$= 0 + 1\cdot(x - n\pi)$$

$$tg(x) = \frac{m}{M}\frac{1}{x}$$

$$x - n\pi = \frac{m}{M}\cdot\frac{1}{x} \Rightarrow x = n\pi + \frac{m}{M}\frac{1}{x} \qquad x = \frac{\omega l}{c}$$

$$\omega = n\frac{\pi c}{l} + \frac{m}{M}\frac{c^2}{\omega l^2}$$

$$\omega_0 = \frac{\pi c}{l} \longrightarrow \omega = n\omega_0 + \frac{m}{M}\frac{\omega_0^2}{\pi^2\omega}$$

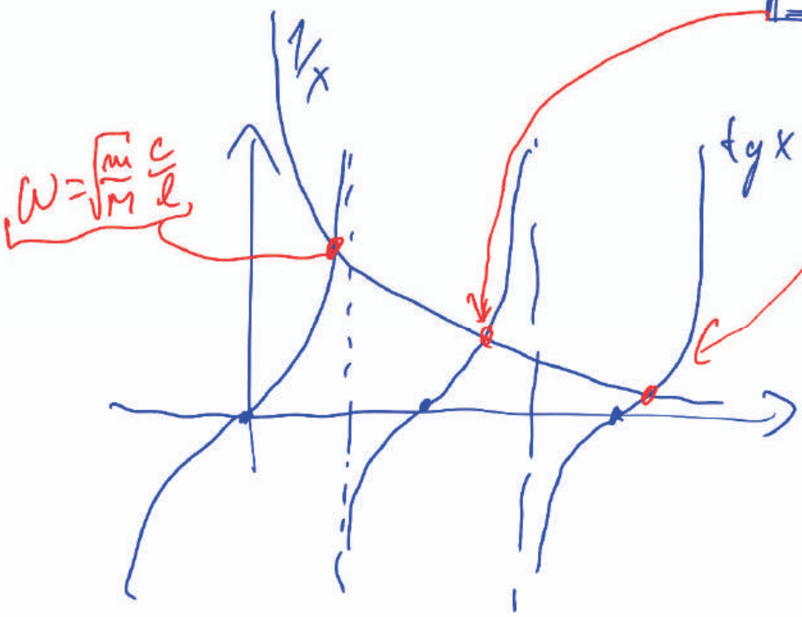
- REKURZIVNO

$\omega \sim n\omega_0$

ZA $\frac{m}{M} \ll 1$

$$\omega = n\omega_0 + \frac{m}{M}\frac{\omega_0^{\cancel{2}}}{\pi^2 n\cancel{\omega_0}}$$

$$\boxed{\omega = \omega_0\left(n + \frac{m}{M}\frac{1}{\pi^2 n}\right)}$$



ZBIRKA 9 nal 19/st 33

$r_0 = 4\,cm$

$\nu = 50\,s^{-1}$

$u_0 = 1\,\mu m$

$r_1 = 25\,cm$

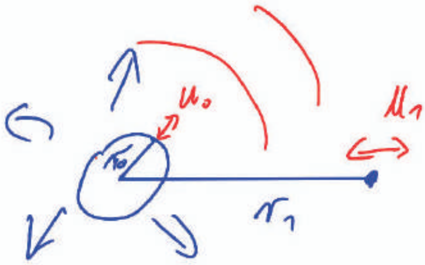
$p = 10\,N/m^2$

$T = 27°C$

$M = 29\,kg/kmol$

$j(r_1) = ?$

$j(r_1, p_2 = 0{,}133\,mbar) = ?$



$P(r_0) \rightarrow$ TEČE SKOZI POVRŠINO KROGLE $4\pi r_0^2$

↓

SE OHRANJA:

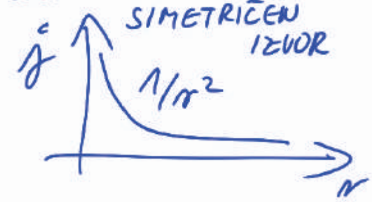
- $P(r_0) = P(r_1)$

$$j(r_0)\cdot 4\pi r_0^2 = j(r_1)\cdot 4\pi r_1^2$$

$$\boxed{j(r_1) = j_0(r_0)\cdot \frac{r_0^2}{r_1^2}}$$

- ENERGIJA ZVOKA: $\langle W\rangle = \frac{1}{2} m u_0^2 \omega^2$
- GOSTOTA W: $\langle w\rangle = \frac{\langle W\rangle}{V}$
- ENERGIJSKI TOK: $P = \langle w\rangle S c$ [W]
- GOSTOTA ENER. TOKA: $j = \frac{P}{S} = \langle w\rangle c$ $[\frac{W}{m^2}]$

ZA SFERNO SIMETRIČEN IZVOR



$$j_0 = \frac{1}{2}\rho u_0^2 4\pi^2 \nu^2 c = 10^{-5}\,\frac{W}{m^2}$$

$$j_1 = 2{,}55\cdot 10^{-7}\,\frac{W}{m^2}$$

- $pV = \frac{m}{M}RT$

$$\rho = \frac{pM}{RT} = 1{,}16\,kg/m^3$$

- $c = \sqrt{\frac{\kappa R T}{M}} = 347{,}4\,\frac{m}{s}$

$$= \sqrt{\frac{\kappa p}{\rho}}$$

ODVISNOST OD TLAKA:

$$j \propto \rho\cdot c = \rho\cdot\sqrt{\frac{\kappa p}{\rho}} = \sqrt{\kappa p \rho} = \sqrt{\frac{\kappa M}{RT}}\cdot p$$

$j \propto p$

↓

$\frac{j}{p} = konst$

$$\frac{j_1}{p_1} = \frac{j_2}{p_2}$$

$$\Rightarrow j(p_2) = j(p_1)\cdot\frac{p_2}{p_1}$$

$$j(p_2) = 3{,}4\cdot 10^{-9}\,\frac{W}{m^2}$$

ZBIRKA 9   nal 21/str 33

$$g = 10 \log\left(\frac{j}{j_0}\right) \quad [dB] \quad [fon] \; ; \; 1 \text{ FON} = 1 \text{dB PRI } 1000 \text{ Hz}$$

$j_0 = 10^{-12}\ \mathrm{W/m^2}$ ← NAJMANJŠA JAKOST, KI JO ČLOVEK ŠE SLIŠI PRI $\nu = 1000$ Hz

$r_1 = 10$ m

$g_1 = 10$ fon

$r_2 = 15$ m

$g_2 = ?$

$\frac{u_0(r_1)}{u_0(r_2)} = ?$

• MOČ IZVORA:

$$P_1 = P_2$$

$$j_1 4\pi r_1^2 = j_2 4\pi r_2^2$$

$$j_2 = j_1\left(\frac{r_1}{r_2}\right)^2$$

$$g_2 = 10 \log\left(\frac{j_2}{j_0}\right) =$$

$$= 10 \log\left[\frac{j_1}{j_0}\cdot\left(\frac{r_1}{r_2}\right)^2\right] =$$

$$= 10\left[\log\left(\frac{j_1}{j_0}\right) + 2 \log\frac{r_1}{r_2}\right] =$$

$$= 10 \text{ fon} + \underbrace{20 \log\frac{r_1}{r_2}}_{\hat{o}} = \underline{6{,}48 \text{ fon}}$$

$$j = \frac{1}{2}\rho \omega^2 u_0^2 \cdot c$$

$$\frac{j_1}{j_2} = \frac{\frac{1}{2}\rho\omega^2 u_{01}^2 c}{\frac{1}{2}\rho\omega^2 u_{02}^2 c} = \frac{u_{01}^2}{u_{02}^2}$$

$$\frac{j_1}{j_2} = \frac{r_2^2}{r_1^2}$$

$$\boxed{\frac{u_{01}}{u_{02}} = \frac{r_2}{r_1} = \frac{15}{10} = \frac{3}{2}}$$

ZBIRKA 9 nal 23/st 33

$P_M = 10^6\ W$

$r = 10\ km$

$j_0 = 10^{-12}\ W/m^2$

a) $\frac{P_Z}{P_M} = ?$

(P_Z = ZA ZVOK)

b) ČE ABSORBCIJA

$\alpha = 0{,}2 \cdot 10^{-4}\ m^{-1}$

$\frac{P_Z}{P_M} : ?$

a) $P_Z = j_0 \cdot 4\pi r^2 = \underline{4\pi \cdot 10^{-4}\ W}$

$\frac{P_Z}{P_M} = \frac{j_0 4\pi r^2}{P_M} = \underline{\underline{4\pi \cdot 10^{-10}}}$

P_Z

$\underline{\underline{P_Z \neq P_Z(r)}}$

b) $\int_{P_0}^{P} \frac{dP}{P} = -\alpha \int_{r_0}^{r} dr$

$\ln \frac{P}{P_0} = -\alpha (r - r_0)$

$\underline{P = P_0 e^{-\alpha (r - r_0)}}$

$\underline{j = j_1 e^{-\alpha (r - r_1)}}$

P_Z'

$\underline{\underline{P_Z' e^{-\alpha r}}}$

JAKOST, KI JO SLIŠIMO JE POSLEDICA ZMANJŠANE MOČI

$\Rightarrow j_0 = \frac{P_Z' \cdot e^{-\alpha r}}{4\pi r^2}$

$\rightarrow P_Z' = j_0 \cdot 4\pi r^2 \cdot e^{\alpha r}$

$\frac{P_Z'}{P_M} = \frac{j_0 4\pi r^2 \cdot e^{\alpha r}}{P_M} = \underline{\underline{1{,}5 \cdot 10^{-9}}}$

ZBIRKA 9 nal 6/str 31

$c = 340\ m/s$

$v_1 = 100\ km/h$

$v_2 = 80\ km/h$

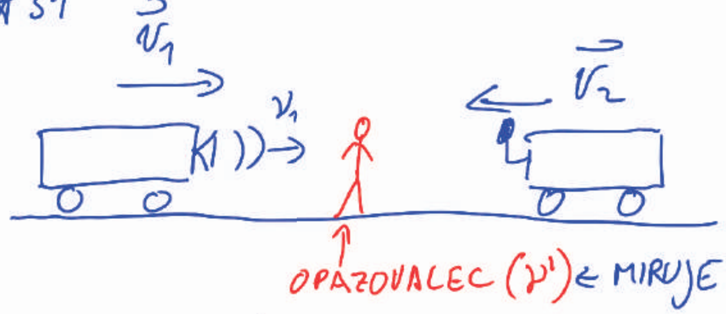
$\nu_1 = 1000\ s^{-1}$

a) $\nu_2 = ?\ (v_z = 0)$

b) $\nu_2 = ?\ (v_z = 30\ km/h)$

$\vec{v}_z \parallel \vec{v}_1$

a)

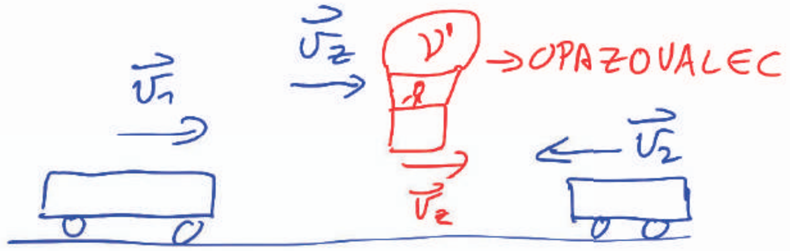


$$\nu' = \nu_1 \frac{1}{1 - \frac{v_1}{c}}$$

$$\nu = \nu_1 \frac{c + v_2}{c - v_1}$$

$$\nu_2 = \nu' \frac{1 + \frac{v_2}{c}}{1} = \nu_1 \frac{1 + \frac{v_2}{c}}{1 - \frac{v_1}{c}} = \underline{\underline{1{,}16\ kHz}}$$

b)



⇒ KER SE OPAZOVALEC PREMIKA Z ZRAKOM JE $\underline{\underline{C = c}}$

$$\nu' = \nu_1 \frac{1}{1 - \frac{(v_1 - v_z)}{c}}$$

$$\nu_2 = \nu' \frac{1 + \frac{v_2 + v_z}{c}}{1} = \nu_1 \frac{1 + \frac{v_2 + v_z}{c}}{1 - \frac{v_1 - v_z}{c}} =$$

$$\nu_2 = \nu_1 \frac{\overbrace{c + v_z}^{c'} + v_2}{\underbrace{c + v_z}_{c'} - v_1} = \underline{\underline{1{,}156\ kHz}}$$

⇒ ZARADI VETRA $\underline{\underline{c' = c + v_z}}$

SPLOŠNO:

IZVOR, ν_{IZ}

SPREJEMNIK, ν_{SP}

$$\boxed{\nu_{SP} = \nu_{IZ} \frac{c \pm v_{SPREJEMNIK}}{c \mp v_{IZVOR}}}$$

ZBIRKA 9 nal 9/st 32

$d = 12\ \text{cm}$

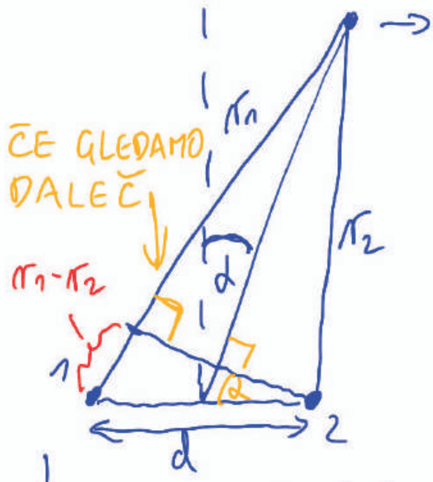
$\nu = 3\ \text{Hz}$

$\delta = -\frac{1}{4}2\pi = -\frac{\pi}{2}$

$c = 25\ \text{cm/s}$

$\alpha_{MAX} = ?$

$\alpha_{MIN} = ?$



$\hookrightarrow r_1 - r_2 = d \cdot \sin\alpha$

$\hookrightarrow$ ZA DALEČ

$\rightarrow u_1 = u_0 \sin(k r_1 - \omega t + \delta)$

$u_2 = u_0 \sin(k r_2 - \omega t)$

SKUPAJ:

$$u = u_1 + u_2$$
$$= u_0\left[\sin(k r_1 - \omega t + \delta) + \sin(k r_2 - \omega t)\right]$$
$$= u_0 \cdot 2\sin\left(\frac{k(r_1+r_2)}{2} - \omega t + \frac{\delta}{2}\right) \cdot \cos\left(\frac{k(r_1-r_2)}{2} + \frac{\delta}{2}\right)$$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$

$$\Rightarrow u = u_0 \cdot 2 \underbrace{\sin\left(\frac{k(r_1+r_2)}{2} - \omega t + \frac{\delta}{2}\right)}_{\text{ČASOVNO ODVISEN}} \cdot \underbrace{\cos\left(\frac{k(r_1-r_2)}{2} + \frac{\delta}{2}\right)}_{\text{NI ODVISEN OD } t}$$

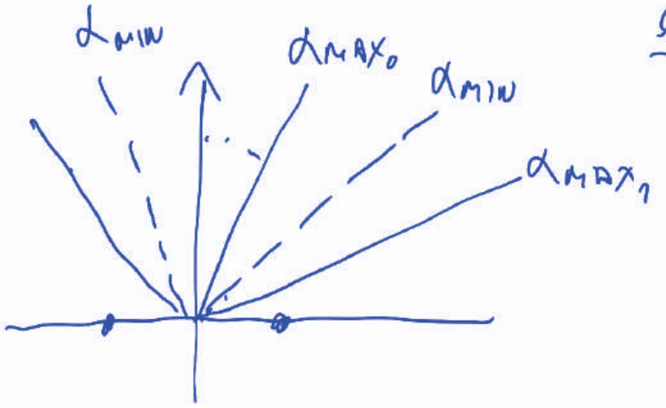
- MAKSIMUMI: $\cos\left(\frac{k(r_1-r_2)+\delta}{2}\right) = \pm 1$

$$\frac{k(r_1-r_2)+\delta}{2} = N\pi$$
$$\frac{\cancel{2}\pi\nu}{c \cdot \cancel{2}} d \cdot \sin\alpha + \frac{\delta}{2} = N\pi$$
$$\frac{\cancel{\pi}\nu}{c} d \sin\alpha = N\cancel{\pi} + \frac{\cancel{\pi}}{4}$$
$$\sin\alpha = \frac{c}{d\nu}\left(N + \frac{1}{4}\right)$$
$$\boxed{\alpha_{MAX} = \arcsin\left[\frac{c}{d\nu}\left(N + \frac{1}{4}\right)\right]}$$

N	α_{MAX}	α_{MIN}
0		
1		
-1		
2		
-2		

- MINIMUMI: $\cos\left[\frac{k(r_1-r_2)+\delta}{2}\right] = 0$

$$\frac{k(r_1-r_2)+\delta}{2} = \frac{\pi}{2} + N\pi$$
$$\boxed{\alpha_{MIN} = \arcsin\left[\frac{c}{d\nu}\left(N + \frac{3}{4}\right)\right]}$$



11 Termodinamika

11.1 Idealni plin

ZBIRKA 9 nal 11/st 35

$V_1 = 10\,m^3$

$p_1 = 1\,bar$

$T_1 = 20°C$

a) $m = ?$

b) $h = 1000\,m$

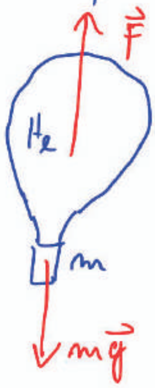
$T_2 = 10°C$

$p_2 = 0.9\,bar$

$M_z = 29\,kg/kmol$

$M_{He} = 4\,kg/kmol$

$R = 8314 \frac{kPal}{mol\,K}$



$pV = \frac{m}{M}RT$

$\underbrace{\rho = \frac{pM}{RT}}$

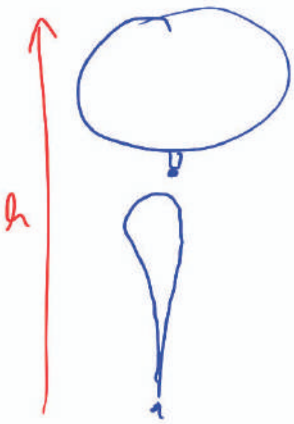
RAVNOVESJE:

a) $F = mg$

$F = V_1(\rho_{ZRAK} - \rho_{He})\cdot g = mg$

$\underbrace{\frac{V_1 p_1}{R T_1}(M_z - M_{He}) = m}$

$\underline{\underline{m_1 = 10,3\,kg}}$



b) $V_2 \Rightarrow$ SE PRILAGODI He

↳ He IMA ISTI p IN T KOT ZRAK

$\underbrace{\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}}$

$m_2 = m_1 = \underline{10,3\,kg}$

BALON

↳ SE RAZTEGNE

$\Rightarrow V_2 = V_1 \cdot \frac{p_1}{p_2} \cdot \frac{T_2}{T_1} = \underline{V_1 \cdot 1,07}$

ZBIRKA 9 nal 13/st 35

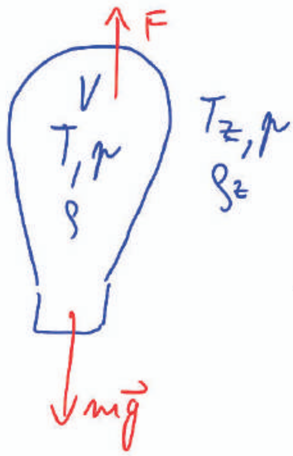
$V = 450\,m^3$

$T_z = 18°C$

$p = 1\,bar$

$m = 160\,kg$

$T = ?$



ZA DVIG:

$$F = mg \qquad pV = \frac{m}{M}RT$$

$$V(\rho_z - \rho)g = mg \qquad \rho = \frac{pM}{RT}$$

$$\frac{VpM}{R}\left(\frac{1}{T_z} - \frac{1}{T}\right) = m$$

$$\frac{1}{T_z} - \frac{1}{T} = \frac{Rm}{VpM}$$

$$T = \left(\frac{1}{T_z} - \frac{Rm}{VpM}\right)^{-1}$$

$$T = \left(\frac{1}{291K} - \frac{8314\frac{J}{kmol\,K}\cdot 160kg}{450m^3\cdot 10^5\frac{N}{m^2}\cdot 29\frac{kg}{kmol}}\right)^{-1}$$

$$\underline{T = 413K = 140°}$$

ZBIRKA 9 nal 15/str 36

$S = 100\,cm^2$

$V_1 = 1\,dm^3$

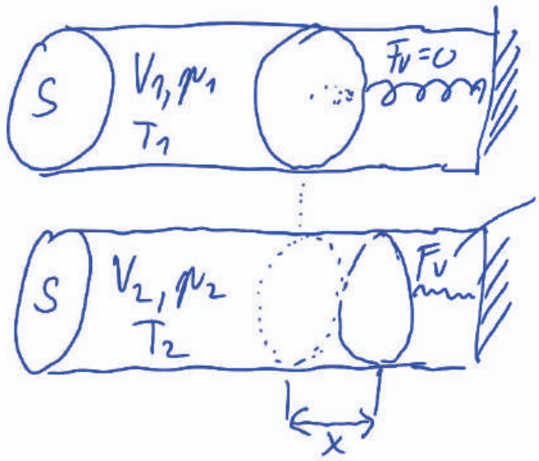
$p_1 = 1\,bar$

$T_1 = 20°C$

$k = 0{,}1\,N/cm$

$T_2 = 80°C$

$V_2 = ?$



RAZLIKA TLAKOV IZENAČI SILO VZMETI

$\Delta p = p_2 - p_1$

$F_V = kx$

$\Delta p \cdot S = F_V$

$V_2 = V_1 + S \cdot x$

$p_2 = p_1 + \frac{F_V}{S} = p_1 + \frac{kx}{S}$

- $\frac{pV}{T} = \frac{m}{M} R = konst.$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{p_1 V_1}{T_1} = \frac{\left(p_1 + \frac{kx}{S}\right)\cdot\left(V_1 + S\cdot x\right)}{T_2}$$

$$\frac{p_1 V_1 \cdot T_2}{T_1} = p_1 V_1 + p_1 S x + \frac{kx}{S} V_1 + k x^2$$

$$0 = \underbrace{k}_{A} x^2 + \underbrace{\left(p_1 S + \frac{k V_1}{S}\right)}_{B} x + \underbrace{p_1 V_1\left(1 - \frac{T_2}{T_1}\right)}_{C}$$

$B = 10^3\,N$

$C = -20\,Nm$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \underline{\underline{2{,}05\,cm}}$$

$$V_2 = V_1 + x \cdot S = \underline{\underline{1{,}2\,dm^3}}$$

ZBIRKA 9 nal 17/str 36

$V = 10\,dm^3$

$p_0 = 1\,bar$

$t = 1\,min$

$\Phi_V = 0.2\,dm^3/s$

$T = konst.$

$p(t) = ?$

GLEDAMO PREOSTANEK ZRAKA:

$$p(V-dV) = (p+dp)V$$

$$\cancel{pV} - p\,dV = \cancel{pV} + V\,dp$$

$$\underline{\frac{dp}{p} = -\frac{dV}{V}}$$

ALTERNATIVA:

$pV = konst. \quad / \ln$

$\ln p + \ln V = konst. \quad / d$

$$\underline{\frac{dp}{p} + \frac{dV}{V} = 0}$$

$$dV = \Phi_V \cdot dt \rightarrow \int_{p_0}^{p} \frac{dp}{p} = -\int_0^t \frac{\Phi_V\,dt}{V}$$

$$\ln\frac{p}{p_0} = -\frac{\Phi_V t}{V}$$

$$\underline{p = p_0 e^{-\frac{\Phi_V t}{V}} = p_0 e^{-\frac{t}{\tau}}} \;;\; \underline{\tau = \frac{V}{\Phi_V}}$$

$$\underline{\underline{p = 0.3\,bar}}$$

ČE GLEDAMO DELCE:

$$n - \frac{dn}{dt} \cdot dt$$

$$n = n_0 - \int_0^t \frac{dn}{dt}\,dt$$

$$pV = nRT \Rightarrow n = pV\frac{1}{RT}$$

$$dn = \frac{1}{RT}\,p\,dV$$

$$\frac{dn}{dt} = \frac{p}{RT} \cdot \Phi_V$$

KER GLEDAMO dn PRI TRENUTNEM TLAKU V POSODI!

$$\frac{d}{dt} \Big\backslash \; pV_0 \cancel{\frac{1}{RT}} = p_0 V_0 \cancel{\frac{1}{RT}} - \cancel{\frac{1}{RT}} \int \Phi_V p\,dt$$

$$dp\,V_0 = 0 - \Phi_V p\,dt$$

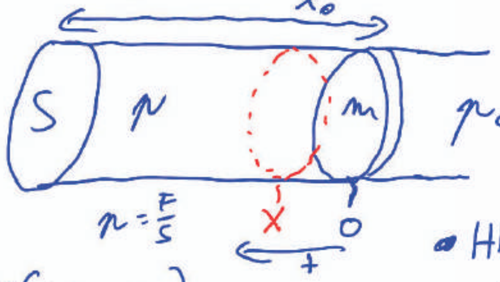
$$\underline{\frac{dp}{p} = -\frac{\Phi_V}{V_0} \cdot dt} \Longrightarrow \text{ISTO KO ZGORAJ}$$

86/87 2. pop. kol./nal 2

$S = 1\,cm^2$

$m = 400\,g$

$\frac{t_{0H}}{t_{0V}} = ?$

a) 

$p = \frac{F}{S}$

VOLUMNI:

$V_0 = S \cdot x_0$

$V = S(x_0 - x)$

• HITRA SPREMEMBA

↳ ADIABATNA S.:

$$pV^{\varkappa} = p_0 V_0^{\varkappa}$$

ZRAK

$$\left(\varkappa = \frac{c_p}{c_v};\ \varkappa \sim 1{,}4 \quad R = c_p - c_v\right)$$

$$p = p_0\left(\frac{V_0}{V}\right)^{\varkappa} = p_0\left(\frac{x_0}{x_0 - x}\right)^{\varkappa} = p_0\left(\frac{1}{1-\frac{x}{x_0}}\right)^{\varkappa} \simeq p_0\left(1 + \varkappa\frac{x}{x_0}\right)$$

$x \ll x_0$

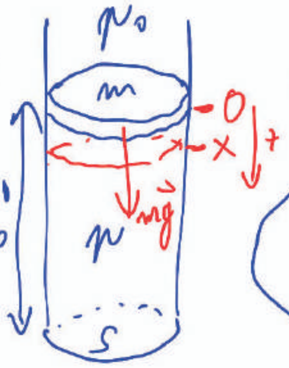
$$\left(1 - \frac{x}{x_0}\right)^{-\varkappa} \simeq \left(1 - (-\varkappa)\frac{x}{x_0}\right)$$

$$-m\ddot{x} = S(p - p_0)$$

$$-m\ddot{x} = S p_0\left[\left(\cancel{1} + \varkappa\frac{x}{x_0}\right) - \cancel{1}\right]$$

$$\ddot{x} = -\underbrace{\frac{S p_0 \varkappa}{m x_0}}_{\omega_H^2} \cdot x$$

$$\hookrightarrow t_{0H} = \frac{2\pi}{\omega_H} = 2\pi\sqrt{\frac{m x_0}{S p_0 \varkappa}}$$

b) 

• RAVNOVESNI TLAK:

$$p_0' = p_0 + \frac{mg}{S} = p_0\left(1 + \frac{mg}{S p_0}\right)$$

• SPREMEMBA $x_0 \Rightarrow x_0'$ IZOTERMNA S.:

$$p_0' V_0' = p_0 V_0 \quad \begin{vmatrix} V_0 = S x_0 \\ V_0' = S x_0' \end{vmatrix}$$

$$x_0' = x_0 \cdot \frac{p_0}{p_0'}$$

• $-m\ddot{x} = S(p - p_0')$

$$-m\ddot{x} = S p_0' \varkappa \frac{x}{x_0'}$$

$$\ddot{x} = -\frac{S p_0}{m}\left(1 + \frac{mg}{S p_0}\right)\varkappa\frac{x}{x_0}\cdot\frac{\cancel{p_0}\left(1 + \frac{mg}{S p_0}\right)}{\cancel{p_0}}$$

$$\ddot{x} = -\underbrace{\frac{S p_0 \varkappa}{m x_0}\left(1 + \frac{mg}{S p_0}\right)^2}_{\omega_V^2} \cdot x$$

$$p = p_0'\left(1 + \varkappa\frac{x}{x_0'}\right)$$

↳ ENAK IZRAZ KOT PREJ LE DRUGE RAVNOVESNE VREDNOSTI; $p_0 \Rightarrow p_0'$, $x_0 \Rightarrow x_0'$

$$\Rightarrow t_{0V} = \frac{2\pi}{\omega_V} = 2\pi \cdot \sqrt{\frac{m x_0}{S p_0 \varkappa}}\left(1 + \frac{mg}{S p_0}\right)^{-1}$$

$$\frac{t_{0H}}{t_{0V}} = \left(1 + \frac{mg}{S p_0}\right) = \underline{\underline{1{,}39}}$$

(11.1.) nal 6.

$p, T, \rho\ (h)$ ZA IZOTERMNO IN IZENTROPNO ATMOSFERO

a) IZOTERMNA: $T = \text{konst.}$; $\underline{p \cdot V} = p_0 V_0 = \underline{\frac{m}{M} RT}$

$\hookrightarrow \frac{p}{\rho} = \frac{RT}{M} = \frac{p_0}{\rho_0}$

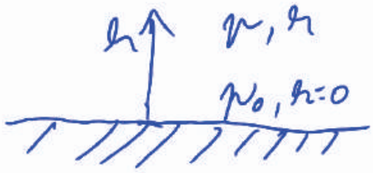
$$\rho = \frac{\rho_0}{p_0} \cdot p$$

PRITISK STOLPCA ZRAKA:

$dF = g\rho \cdot S\, dh$

$\underline{dp} = \underline{-\rho g\, dh}$

*KER p Z VIŠINO PADA



$dp = -p \frac{\rho_0}{p_0} g\, dh$

$\int_{p_0}^{p} \frac{dp}{p} = -\frac{\rho_0 g}{p_0} \int_0^h dh$

$\ln \frac{p}{p_0} = -\frac{h}{h_0}$

$$\boxed{p = p_0 \cdot e^{-\frac{h}{h_0}}}$$

$$\boxed{\rho = \rho_0 e^{-\frac{h}{h_0}}}$$

$\frac{\rho_0 g}{p_0} = \frac{Mg}{RT} = h_0^{-1}$

$\underline{h_0(T = 293\,K) = 8{,}5\,km}$

b) IZENTROPNA: $pV^{\varkappa} = p_0 V_0^{\varkappa} \Rightarrow \left(V = \frac{m}{\rho}\right) \Rightarrow \underline{p\rho^{-\varkappa} = p_0 \rho_0^{-\varkappa}}$,

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\varkappa} \Big/ \frac{1}{\varkappa}$$

$$\left(\frac{p}{p_0}\right)^{\frac{1}{\varkappa}} = \frac{\rho}{\rho_0}$$

$$\underline{\underline{\rho = \rho_0 \left(\frac{p}{p_0}\right)^{1/\varkappa}}}$$

$$dp = -\rho g\, dh$$

$$dp = -\rho_0 g p_0^{-\frac{1}{\varkappa}} p^{\frac{1}{\varkappa}} dh$$

$$\int_{p_0}^{p} p^{-\frac{1}{\varkappa}} dp = -\frac{\rho_0 g}{p_0^{1/\varkappa}} \int_0^h dh$$

$$\left.\left(\frac{p^{-\frac{1}{\varkappa}+1}}{-\frac{1}{\varkappa}+1}\right)\right|_{p_0}^{p} = -\frac{\rho_0 g}{p_0^{1/\varkappa}} h \Big/ \cdot \frac{\varkappa - 1}{\varkappa}$$

$$-\frac{1}{\varkappa}+1 = \frac{\varkappa-1}{\varkappa}$$

$$p^{\frac{\varkappa-1}{\varkappa}} - p_0^{\frac{\varkappa-1}{\varkappa}} = -\frac{\varkappa-1}{\varkappa} \cdot \frac{\rho_0 g \cdot p_0}{p_0^{1/\varkappa} \cdot p_0^{-1}} \cdot h \qquad p_0^{1/\varkappa} \cdot p_0^{-1} = p_0^{\frac{1-\varkappa}{\varkappa}}$$

$$p^{\frac{\varkappa-1}{\varkappa}} = p_0^{\frac{\varkappa-1}{\varkappa}}\left(1 - \frac{\varkappa-1}{\varkappa} \cdot \frac{h}{h_0}\right)$$

$$\underbrace{h_0 = \frac{p_0}{\rho_0 g} = \frac{RT}{Mg}}$$

TLAK $\rightarrow$ $\boxed{p = p_0 \left(1 - \frac{\varkappa-1}{\varkappa} \frac{h}{h_0}\right)^{\frac{\varkappa}{\varkappa-1}}}$

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\varkappa}$$

GOSTOTA $\rightarrow$ $\boxed{\rho = \rho_0 \left(1 - \frac{\varkappa-1}{\varkappa} \frac{h}{h_0}\right)^{\frac{1}{\varkappa-1}}}$

$$\underbrace{p = \frac{RT}{M}\rho}$$

$$\frac{T\rho}{T_0\rho_0} = \left(\frac{\rho}{\rho_0}\right)^{\varkappa}$$

TEMPERATURA $\rightarrow$ $\boxed{T = T_0 \left(1 - \frac{\varkappa-1}{\varkappa} \frac{h}{h_0}\right)}$

$$\underbrace{\frac{T}{T_0} = \left(\frac{\rho}{\rho_0}\right)^{\varkappa-1}}$$

$$\frac{T_0}{h_0} \frac{\varkappa-1}{\varkappa} = \frac{300\,K}{8{,}5\,km} \cdot \frac{0{,}4}{1{,}4} \sim \underline{\underline{10\,K/km}}$$

ZBIRKA 9 nal 8 / st 35

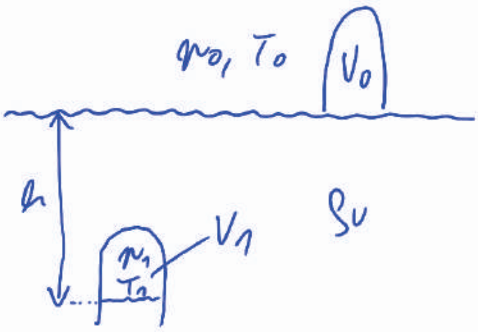
$h = 8\,m$

$T_1 = 12\,°C$

$T_0 = 25\,°C$

$p_0 = 1\,bar$

$\frac{V_1}{V_0} = ?$



$p_1 = p_0 + \rho_v g h$

$\frac{p_1 V_1}{T_1} = \frac{p_0 V_0}{T_0}$

$\frac{V_1}{V_0} = \frac{p_0}{p_1}\frac{T_1}{T_0} \sim \underline{\underline{0{,}53}}$

11.5 Kalorimetrija

ZBIRKA 9 nal 2/st 37

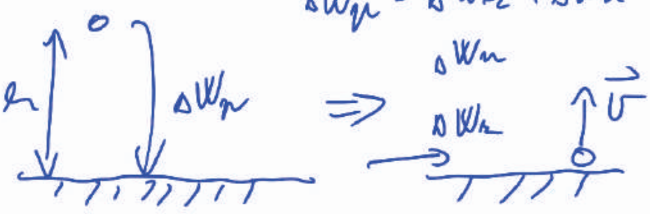
$h = 20\,m$

$v = 3\,m/s$

$c_{pb} = 130\,J/kgK$

$\Delta T =$

$\Delta W_p = \Delta W_k + \Delta W_n$



$$\Delta W_n = \Delta W_p - \Delta W_k = mgh - \frac{mv^2}{2}$$

$$m c_{pb} \cdot \Delta T = m\left(gh - \frac{v^2}{2}\right)$$

$$\Delta T = \frac{gh - \frac{v^2}{2}}{c_{pb}} = 1{,}47\,K$$

ZBIRKA 9 nal 6/st 37

ZBIRKA 9 nal 6/str 37

$m = 8\,t$

$v_0 = 10\,m/s$

$m_1 = 10\,kg$

$c = 450\,J/kgK$

$\Delta T = ?$

G: $m v_0 = 2 m v_1$

$$\underline{v_1 = \frac{v_0}{2}}$$

W: $\Delta W_n = W_{k0} - W_{k1} = \frac{m v_0^2}{2} - \frac{2 m v_1^2}{2}$

$$= \frac{1}{2} m v_0^2 \left(1 - \frac{2}{4}\right) = \underline{\frac{1}{4} m v_0^2}$$

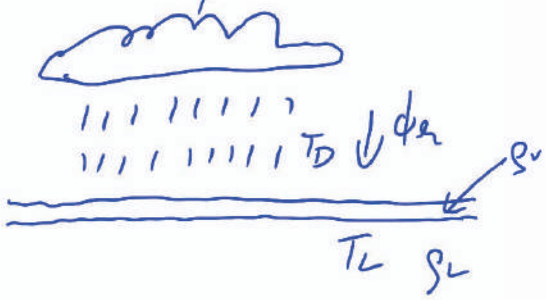
$$\Delta W_n = m_1 \cdot c\, \Delta T = \frac{1}{4} m v_0^2$$

$$\Delta T = \frac{m v_0^2}{4 m_1 c} = \frac{8 \cdot 10^3\, \cancel{kg} \cdot 10^2\, \cancel{m^2/s^2}}{4 \cdot 10\, \cancel{kg}\; 450\, \cancel{J}/\cancel{kg}K}$$

$Nm = \cancel{kg}\, \cancel{\frac{m^2}{s^2}}$

$$\underline{\underline{\Delta T = 44{,}4\,K}}$$

ZBIRKA 9 nal 13/st 38



$T_L = 0°C$

$\phi_R = 1\,mm/h$

$T_D = 6°C$

$t = 12\,h$

$\frac{\rho_V}{\rho_L} = 1{,}1$

$X = ?$

$q_t = 3{,}34 \cdot 10^5\,J/kg$

$c_V = 4200\,J/kgK$

$\Delta T = (T_D - T_L)$

• TOPLOTA, KI JO ODDA DEŽ:

$Q_D = m_D \cdot c_V \cdot \Delta T$

$m_D = V_D \cdot \rho_V = S \cdot \underbrace{\phi_R \cdot t}_{\text{VIŠINA}} \cdot \rho_V$

$Q_D = S\,\phi_R\,t \cdot \rho_V \cdot c_V \cdot \Delta T$

$W_k : Q_D$

$\frac{v^2}{2} : c_V \Delta T$

$50 : 24000$

$W_k << Q_D$

• TALJENJE:

$Q_T = m_L q_t = S X \rho_L q_t$

$m_L = S \cdot X \rho_L$

$\cancel{S} X \rho_L q_t = \cancel{S} \phi_R t \cdot \rho_V c_V \cdot \Delta T$

$X = \frac{\rho_V c_V \Delta T \phi_R \cdot t}{\rho_L q_t}$

$X = 1\,mm$

ZBIRKA 9 nal 17/st 39

$V_0 = 40\,m^3$

$\eta = 65\%$

$\phi_m = 3\,kg/h$

$g_s = 10{,}47\,MJ/kg$

$T_0 = 15°C$

$p_0 = 1\,bar$

$\varkappa = 1{,}4$

$t = 1\,min$

$\Delta T = ?$

$M_z = 29\,kg/kmol$

• KURJENJE:

$$\underbrace{Q_K = m_p \cdot g_s \cdot \eta = \phi_m t \cdot g_s \eta}$$

• SEGREVANJE ZRAKA:

$$\underbrace{Q_s = m_z c_v \Delta T}$$

↳ KER JE V = konst.

• $p_0 V_0 = \frac{m_z}{M_z} R T_0$

$$\underbrace{m_z = \frac{p_0 V_0 M_z}{R T_0}}$$

• $\boxed{\varkappa = \frac{c_p}{c_v};\ c_p - c_v = \frac{R}{M}}$

↳ $\varkappa c_v - c_v = \frac{R}{M}$

$$\underbrace{c_v = \frac{1}{\varkappa - 1} \cdot \frac{R}{M}}$$

$$Q_s = \frac{p_0 V_0 \cancel{M_z}}{\cancel{R} T_0} \cdot \frac{1}{(\varkappa-1)} \cdot \frac{\cancel{R}}{\cancel{M_z}} \cdot \Delta T$$

$$\underbrace{Q_s = \frac{p_0 V_0}{T_0(\varkappa-1)} \Delta T}$$

↳ $Q_s = Q_K$

$$\frac{p_0 V_0 \Delta T}{T_0(\varkappa-1)} = \phi_m t \cdot g_s \eta$$

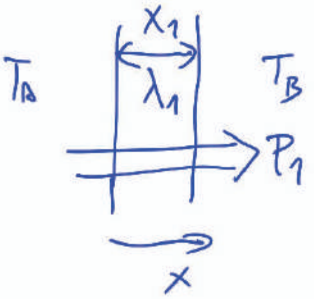
$$\underbrace{\Delta T = \frac{(\varkappa-1) T_0 \phi_m t\, g_s \eta}{p_0 V_0}} = \underline{\underline{9{,}8\,K}}$$

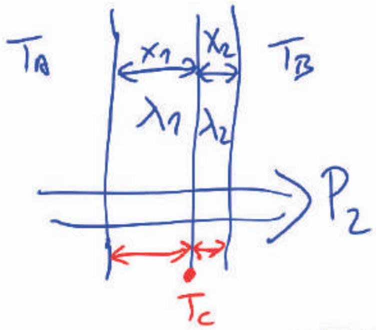
11.6 Prevajanje toplote

ZBIRKA 9 nal 55/st 43

$X_1 = 12\ \mathrm{cm}$
$\lambda_1 = 1{,}1\ \mathrm{W/mK}$
$X_2 = 2\ \mathrm{cm}$
$\lambda_2 = 0{,}05\ \mathrm{W/mK}$

$\frac{P_2}{P_1} = ?$





TOPLOTNI TOK (MOČ)

$$\boxed{P = \frac{dQ}{dt} = \lambda \cdot S\left(-\frac{dT}{dx}\right)}$$

P TEČE OD VIŠJE T. K NIŽJI T.

$$P_1 = \lambda_1 S\left(-\frac{T_B - T_A}{x_1}\right)$$

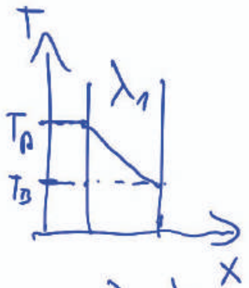
$$P_1 = \lambda_1 S\,\frac{T_A - T_B}{x_1}$$

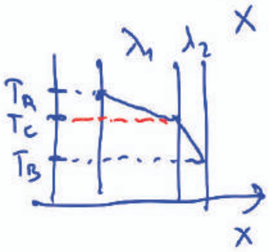
• ISTI TOPLOTNI TOK TEČE SKOZI x_1 IN x_2

$$P_2 = \lambda_1 \cancel{S}\left(-\frac{T_C - T_A}{x_1}\right) = \lambda_2 \cancel{S}\left(-\frac{T_B - T_C}{x_2}\right)$$

$$\frac{\lambda_1 \cdot T_A}{x_1} + \frac{\lambda_2 T_B}{x_2} = T_C\left(\frac{\lambda_2}{x_2} + \frac{\lambda_1}{x_1}\right) \quad / \cdot x_1 x_2$$

$$\frac{\lambda_1 x_2 T_A + \lambda_2 x_1 T_B}{\lambda_2 x_1 + \lambda_1 x_2} = T_C$$





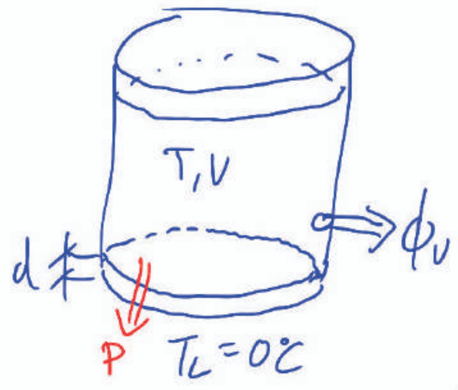
$$P_2 = \frac{\lambda_1 S}{x_1} \cdot \frac{T_A(\lambda_2 x_1 + \cancel{\lambda_1 x_2}) - \cancel{\lambda_1 x_2 T_A} - \lambda_2 x_1 T_B}{\lambda_2 x_1 + \lambda_1 x_2}$$

$$P_2 = \underbrace{\frac{\lambda_1}{x_1} S (T_A - T_B)}_{P_1} \frac{\lambda_2 x_1}{\lambda_2 x_1 + \lambda_1 x_2}$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{\lambda_2 x_1}{\lambda_2 x_1 + \lambda_1 x_2} = 0{,}21$$

ZBIRKA 9 nal 54/st 43

$2r = 30\,cm$
$d = 1\,cm$
$\lambda = 1\,W/mK$
$V_0 = 20\,l$
$T_0 = 30\,°C$
$t = 3\,min$
$\phi_V = 5\,l/min$
$T = ?$



- $P = \frac{dQ}{dt} = \lambda S\left(-\frac{T_L - T}{d}\right)$
- $dQ = m\,c\,(-dT)$
- $m = m_0 - \phi_V \rho_V t = \rho_V (V_0 - \phi_V t)$

$$\frac{\rho_V (V_0 - \phi_V t)\, c\, (\cancel{-}dT)}{dt} = \lambda S\left(\cancel{-}\,\frac{T_L - T}{d}\right)$$

$$\int_{T_0}^{T} \frac{dT}{T_L - T} = \frac{\lambda S}{d c \rho_V V_0} \cdot \int_0^t \frac{dt}{1 - \frac{\phi_V}{V_0} \cdot t}$$

- $T_L - T = u$
  $-dT = du$
- $1 - \frac{\phi_V}{V_0} t = v$
  $-\frac{\phi_V}{V_0} dt = dv$
  $dt = -\frac{V_0}{\phi_V} dv$

$$\int_{T_L - T_0}^{T_L - T} \frac{-du}{u} = \frac{\lambda S}{d c \rho_V \cancel{V_0}}\left(-\frac{\cancel{V_0}}{\phi_V}\right)\int_1^{1 - \frac{\phi_V}{V_0} t} \frac{dv}{v}$$

$$\ln \frac{T_L - T_0}{T_L - T} = -\frac{\lambda S}{d c \rho_V \phi_V} \cdot \ln\left(1 - \frac{\phi_V}{V_0} t\right)$$

$$\frac{T_L - T_0}{T_L - T} = \left(1 - \frac{\phi_V}{V_0} t\right)^{-\frac{\lambda S}{d c \rho_V \phi_V}}$$

$$(T_L - T_0)\left(1 - \frac{\phi_V}{V_0} t\right)^{\frac{\lambda S}{d c \rho_V \phi_V}} = T_L - T$$

$$\longrightarrow \underline{T = T_L + (T_0 - T_L)\left(1 - \frac{\phi_V}{V_0} t\right)^{\frac{\lambda \pi r^2}{d c \rho_V \phi_V}}}$$

- VELJA DOKLER NE ZMANJKA VODE
  $\underline{t < 4\,min}$

$$\underline{T = 29{,}17\,°C}$$

ZBIRKA 9 nal. 56/st 43

$l = 1 m$

$2r = 1 cm$

$x = 3 mm$

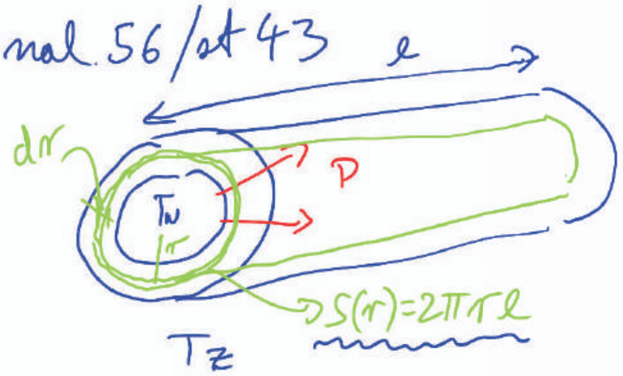
$\lambda = 70 W/mK$

$\phi_m = 0,1 kg/min$

$p = 1 bar \Rightarrow T_N = 100°C$

$T_z = ?$

$q_i = 2,26 \cdot 10^6 J/kg$



• TOPLOTNI TOK:

→ ZA UTEKOČINJEVANJE:

$$\underbrace{\frac{dQ}{dt} = \phi_m \cdot q_i}_{}$$

$\phi_m = \frac{dm}{dt}$

→ PREVAJANJE:

$$P = \frac{dQ}{dt} = \lambda S \left(-\frac{dT}{dx}\right)$$

$$= \lambda 2\pi r l \left(-\frac{dT}{dr}\right)$$

$$\phi_m q_i = -\lambda 2\pi r l \frac{dT}{dr}$$

$$-\frac{\phi_m q_i}{2\pi\lambda l}\int_{r_1}^{r_2}\frac{dr}{r} = \int_{T_N}^{T_z} dT$$

$$\frac{\phi_m q_i}{2\pi\lambda l}\ln\frac{r_2}{r_1} = T_N - T_z$$

$r_2 = r$

$r_1 = r - x$

$$T_z = T_N - \frac{\phi_m q_i}{2\pi l \lambda}\ln\frac{r_2}{r_1}$$

$$\underline{\underline{T_z = 368,6 K}}$$

ZA $r_1 \to r_2$

$$\ln\frac{r_2}{r_1} = \ln\left(1 + \frac{r_2 - r_1}{r_1}\right) \underset{\uparrow}{\simeq} \frac{r_2 - r_1}{r_1}$$

$r_2 - r_1 \ll r_1$

11.6. nal 3

$r_1 = 1\,cm$

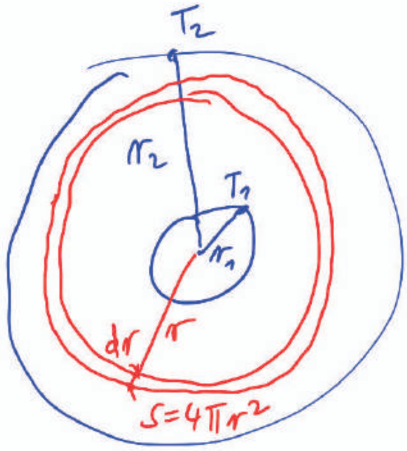
$r_2 = 10\,cm$

$P = 1000\,W$

$T_2 = 0°C$

$\lambda = 390\,W/mK$

$T_1 = ?$



$$P = \lambda S\left(-\frac{dT}{dr}\right)$$

$$P = -\lambda 4\pi r^2 \frac{dT}{dr}$$

$$\frac{P}{4\pi\lambda}\int_{r_1}^{r_2}\frac{dr}{r^2} = -\int_{T_1}^{T_2} dT$$

$$\frac{P}{4\pi\lambda}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = T_1 - T_2$$

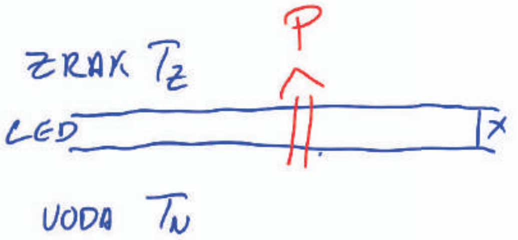
$$T_1 = T_2 + \frac{P}{4\pi\lambda}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\underline{\underline{T_1 = 291{,}4\,K}}$$

ZBIRKA 9 nal 58/st 43

$T_N = 0°C$
$T_Z = -10°C$
$t = 12\,h$
$\lambda_L = 2{,}2\,W/mK$
$\rho_L = 0{,}9\,g/cm^3$

$x(t) = ?$



$$P = \frac{dQ}{dt} = \lambda_L S\left(-\frac{dT}{dx}\right) = \lambda_L S\left(-\frac{T_Z - T_N}{x}\right)$$

$$dQ = q_t \cdot dm = q_t \rho_L S\, dx$$

$$dm = \rho_L S\, dx$$

$$P = \frac{q_t \rho_L \cancel{S}\, dx}{dt} = \lambda_L \cancel{S}\, \frac{T_N - T_Z}{x}$$

$$\int_0^x x\, dx = \frac{\lambda_L (T_N - T_Z)}{q_t \rho_L} \int_0^t dt$$

$$\frac{x^2}{2} = \frac{\lambda_L (T_N - T_Z)}{q_t \rho_L}\, t$$

$$x = \sqrt{\frac{2 \lambda_L (T_N - T_Z)}{q_t \rho_L} \cdot t} = \underline{\underline{8\,cm}}$$

ZAČNE NAVPIČNO

ZBIRKA 9 nal 20/st 39

$m = 1\,kg$
$p_1 = 1\,bar$
$p_2 = 50\,bar$
$T = konst.$
$\chi = 5 \cdot 10^{-5}\,bar^{-1}$

$A = ?$

$\underline{V_1 = 1\,l}$

DELO:

$$\boxed{A = -\int p\,dV}$$

↳ KO SE SREDSTVO RAZTEGUJE ODDA A

$$\int_{V_1}^{V_2} \frac{dV}{V} = -\chi \int_{p_1}^{p_2} dp$$

$$\ln \frac{V_2}{V_1} = -\chi (p_2 - p_1)$$

$$V_2 = V_1 e^{-\underbrace{\chi(p_2-p_1)}_{\sim 10^{-4}}}$$

$$dV = -\chi V dp \qquad \sim 1$$

$$\Downarrow$$

$$dV = -\chi V_1 dp$$

$$A = +\chi V_1 \int_{p_1}^{p_2} p\,dp$$

$$A = +\chi V_1 \frac{(p_2^2 - p_1^2)}{2}$$

$= +6{,}25\,J$

↳ VODA PREJME DELO

ZBIRKA 9 nal 21/st 39

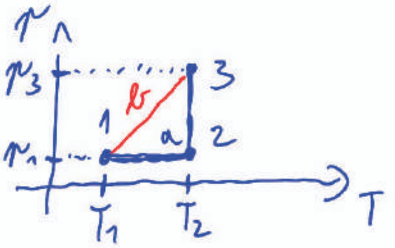
$V_1 = 1\,l$

$m = 1\,kg$

$p_1 = 1\,bar$

$T_1 = 19°C$

$T_2 = 25°C$



$A = -\int p\,dV$

$dV = V(\beta\,dT - \chi\,dp)$

$\frac{dV}{V} = \beta\,dT - \chi\,dp$

$p_2 = p_1$

$p_3 = 35\,bar$

$T_3 = T_2$

$\chi = 5\cdot 10^{-5}\,bar^{-1}$

$\beta = 2\cdot 10^{-4}\,K^{-1}$

a) $A_{1\to 3} = ?$

b) $A_{1\to 3}(p \propto T) = ?$

a)

• $1 \to 2: dp = 0:$ $dV = V\beta\,dT \Rightarrow V = V_0 e^{\underbrace{\underbrace{\beta(T-T_0)}_{\sim 10^{-3}}}_{\sim 1}}$, $V \sim V_1$

$A_{12} = -\int_{T_1}^{T_2} p_1 V_1 \beta\,dT$

$\boxed{A_{12} = -p_1 V_1 \beta (T_2 - T_1)} = \underline{\underline{-0{,}12\,J}}$ $\quad V_2 \sim V_1$

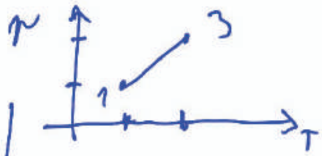
• $2 \to 3: dT = 0:$ $dV = -V\chi\,dp \Rightarrow V_3 = V_2 e^{\underbrace{\underbrace{-\chi(p_3-p_2)}_{\sim 10^{-4}}}_{\sim 1}}$

$A_{23} = +\int_{p_2}^{p_3} p V_2 \chi\,dp$

$\underbrace{A_{23} = V_2 \chi \frac{(p_3^2 - p_2^2)}{2}}_{V_2 = V_1} = \underline{\underline{3{,}1\,J}}$

$\boxed{A_{13} = A_{12} + A_{23} = 2{,}98\,J}$

b)



$p = kT + n$

$k = \frac{p_3 - p_1}{T_3 - T_1} = 5{,}67\,\frac{bar}{K}$

$n = p_1 - k T_1 = -1654\,bar$

$dp = k\,dT$

$A = -\int p\,dV$

$dV = V(\beta\,dT - \chi\,dp) = V(\beta - \chi k)\,dT$

$\Rightarrow V = V_0 e^{\underbrace{(\beta-\chi k)(T-T_0)}_{\sim 1}} \longrightarrow \boxed{dV = V_1(\beta - \chi k)\,dT}$

$A_{13} = -\int p\,dV =$

$= -\int_{T_1}^{T_3} (kT + n) V_1 (\beta - \chi k)\,dT =$

$\boxed{= -V_1(\beta - \chi k)\left[k\frac{T_3^2 - T_1^2}{2} + n(T_3 - T_1)\right]}$

$= \underline{\underline{0{,}89\,J}}$?

ZBIRKA 9 nal 22/st 39

$m = 1\,kg$
$p_1 = 1\,bar$
$T_1 = 20°C$
$T_2 = 21°C$
$dV = 0$
$c_v = 4160\,J/kgK$
$\beta = 2\cdot 10^{-4}\,K^{-1}$
$\chi = 4{,}6\cdot 10^{-5}\,bar^{-1}$

$\Delta H = ?$

- $\underline{dW_n = dQ + dA}$ ← SPREMBA W_n

$dA = -p\,dV$

→ PRI $V = konst.$ ⇒ $\boxed{dW_n = dQ|_{V=konst} = m c_v dT}$

↳ $dA = 0$

- H → ENTALPIJA

$H = W_n + pV$

$dH = dW_n + V dp + p dV = dQ \underbrace{- \cancel{p dV}}_{dA} + V dp + \cancel{p dV}$

$\underline{dH = dQ + V dp}$

→ PRI $p = konst.$ ⇒ $\boxed{dH = dQ|_{p=konst.} = m c_p dT}$

RESEVANJE NALOGE:

$$\frac{dV}{V} = \beta dT - \chi dp = 0 \Rightarrow \underline{dp = \frac{\beta}{\chi} dT}$$

$$dH = dW_n + V dp + \underbrace{\cancel{p dV}}_{0,\ dV=0}$$

$$= m c_v dT + V \frac{\beta}{\chi} dT$$

$$\int_0^{\Delta H} dH = \left(m c_v + V \frac{\beta}{\chi}\right) \int_{T_1}^{T_2} dT$$

$$\underline{\Delta H = \left(m c_v + V \frac{\beta}{\chi}\right)(T_2 - T_1)}$$

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$p = \text{konst}$

$T_1 = 200\,°C$, $T = \text{konst.}$

$m = 1\,kg$

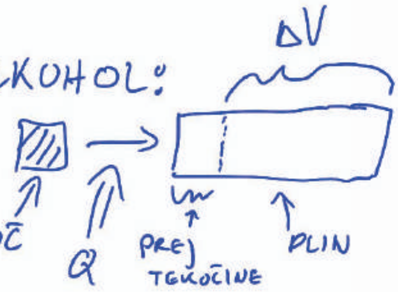
$\Delta W_n = 4{,}02 \cdot 10^5\,J$

$\underline{M = 46\,kg/kmol}$

$q_i(T_1) = ?$

$\rho_A = 800\,kg/m^3$

ALKOHOL:



$p = \text{konst.}!$

$T = \text{konst.}!$

$\Delta V = V_{PLINA} - V_{TEKOČINE}$

• V_{PLIN}:

$$pV_p = \frac{m}{M}RT$$

$$\underline{V_p = \frac{mRT}{Mp}}$$

• $\underline{V_T = \frac{m}{\rho_A}}$

$$\Delta W_n = Q + A = m q_i - p\Delta V$$

$$q_i = \frac{\Delta W_n + p\Delta V}{m}$$

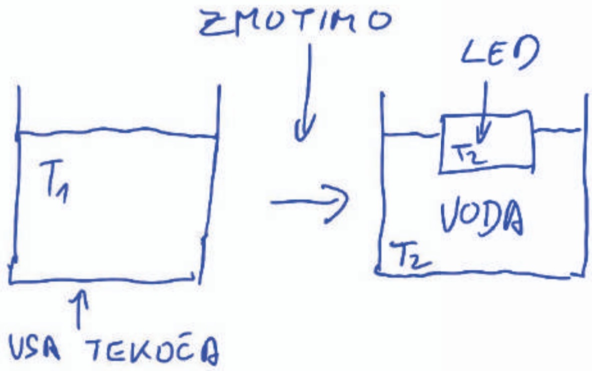
$$q_i = \frac{\Delta W_n + p\left(\frac{mRT}{Mp} - \frac{m}{\rho_A}\right)}{m}$$

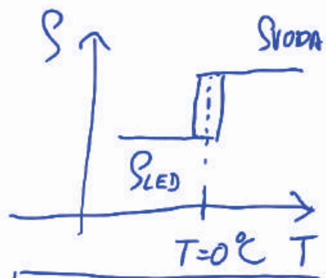
$$\boxed{q_i = \underbrace{\frac{\Delta W_n}{m}}_{10^5} + \underbrace{\frac{RT}{M}}_{\sim 10^5}} - \underbrace{\frac{p}{\rho_A}}_{\frac{10^5\,N\,m^3}{m^2\,10^3\,kg} \sim 10^2} \to 0$$

$$\underline{\underline{q_i(200\,°C) = 4{,}88 \cdot 10^5\,J/kg}}$$

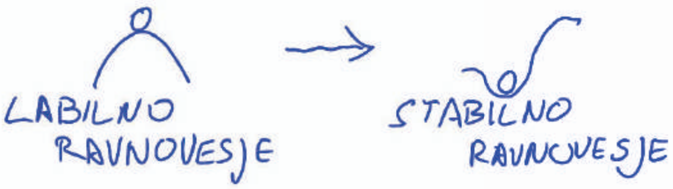
11.5. nal g

$T_1 = -6°C$
$T_2 = 0°C$
$m = 2\,kg$





FAZNI PREHOD 1. REDA
↓
HISTEREZA



- LED
- VODA

POTREBUJE NUKLEACIJSKO JEDRO
↓
SMET ALI MOTNJO

- VODA SE SEGREJE

$$\Delta Q = m c_v \Delta T = m c_v (T_2 - T_1)$$

- TOPLOTO DOBI OD VODE, KI ZMRZNE

$$\Delta Q = m_L q_t$$

$$\Rightarrow m_L q_t = m c_v (T_2 - T_1)$$

$$\underline{m_L = m \frac{c_v (T_2 - T_1)}{q_t}} = \underline{\underline{0{,}15\,kg}}$$

$\Rightarrow$ DA ZMRZNE VSA VODA:

$$\frac{c_v \Delta T}{q_t} = 1 \Rightarrow \Delta T = \frac{q_t}{c_v} = \frac{3{,}3 \cdot 10^5\,J/kg}{4{,}2 \cdot 10^3\,J/kgK}$$

$$\underline{\underline{\Delta T = -79{,}5\,K}}$$

$\Rightarrow$ NAJNIŽJA T ZA PODHLAJENO VODO $\sim -46°C$

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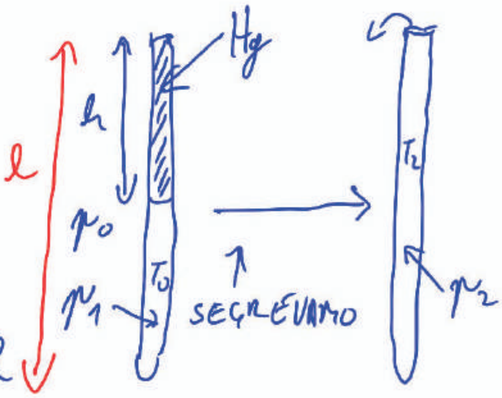
$l = 75\,\text{cm}$
$p_0 = 1\,\text{bar}$
$2r = 1\,\text{mm}$
$T_0 = 27\,°C$
$h = 20\,\text{cm}$
$M = 29\,\text{kg/kmol}$
$\varkappa = 1{,}4$

$\rho_{Hg} = 13\,\text{g/cm}^3$



a) $c_V = ?$
b) $p_1 = ?$
c) $T_2 = ?$
d) $Q = ?$

a) $\varkappa = \frac{c_p}{c_V}$; $c_p - c_V = \frac{R}{M}$

$\hookrightarrow c_V(\varkappa - 1) = \frac{R}{M}$

$$c_V = \frac{1}{(\varkappa-1)}\frac{R}{M}$$

b) $p_1 = p_0 + \rho g h = 1{,}255\,\text{bar}$

c) $T_2 > T_1$, $p_2 = p_0$, $m_z = \text{konst}$, $T_1 = T_0$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad ; \quad V_1 = \pi r^2 (l-h), \quad V_2 = \pi r^2 l$$

$$T_2 = T_1 \frac{p_2 \pi r^2 l}{p_1 \pi r^2 (l-h)} = T_0 \frac{p_0 l}{p_1 (l-h)} = 326\,K$$

d) $\Delta W_u = Q + A$

$Q = \Delta W_u - A$

$$\Delta W_u = m_z c_V \cdot \Delta T = \frac{p_2 V_2 \cancel{M}}{T_2 \cancel{R}} \frac{1}{(\varkappa-1)} \frac{\cancel{R}}{\cancel{M}} (T_2 - T_1) = p_2 V_2 \frac{1}{(\varkappa-1)}\left(1 - \frac{T_1}{T_2}\right),$$

• $p_2 V_2 = \frac{m}{M} R T_2$

$m_z = \frac{p_2 V_2 M}{T_2 R}$

$$A = -\int p\,dV = +\int_h^0 (p_0 + \rho g h)\pi r^2\,dh = -\pi r^2 \left(p_0 + \frac{\rho g h}{2}\right) h,$$

; $p = p_0 + \rho g h$, $V = \pi r^2 (l - h)$, $dV = -\pi r^2\,dh$,

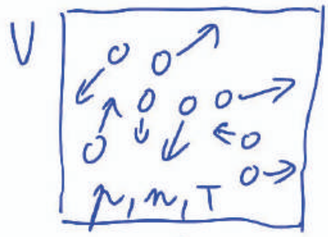
$p_2 = p_0$, $V_2 = \pi r^2 l$
$\downarrow$

$$Q = p_0 \pi r^2 l \frac{1}{\varkappa - 1}\left(1 - \frac{T_1}{T_2}\right) + \pi r^2 h \left(p_0 + \frac{\rho g h}{2}\right)$$

$$Q = p_0 \pi r^2 \left[\frac{l}{\varkappa-1}\left(1 - \frac{T_1}{T_2}\right) + \left(h + \frac{\rho g}{p_0}\frac{h^2}{2}\right)\right] = 0{,}0294\,J$$

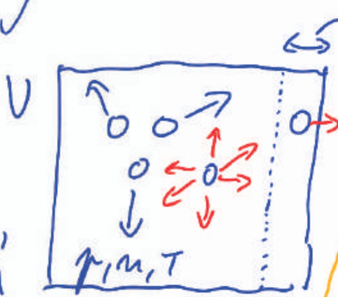
12 Kinetična teorija plinov

KINETIČNA TEORIJA PLINOV:



← RAZLIČNE HITROSTI

POENOSTAVITVE:
→ ENAKE HITROSTI
→ ELASTIČNI TRKI



V ČASU Δt SE ZGODI ΔN TRKOV:

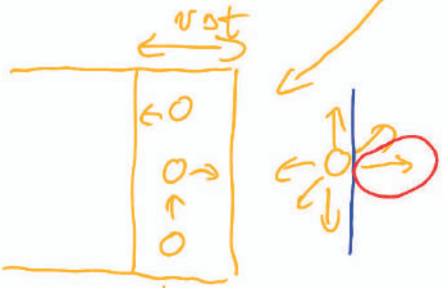
$\Delta N = \frac{1}{6} n \Delta V$

$\Delta V = v \cdot \Delta t \cdot S$

$\underline{\Delta N = \frac{1}{6} n S v \Delta t}$

$n = \frac{N}{V}$

N = št delcev



$\frac{1}{6}$ DELCEV V VOLUMNU ΔV SE ZALETI V STENO IN DELA TLAK

- TRKI: SPREMEMBA G

$\Delta G = [mv - (-mv)]\Delta N$ ← KER SE v OBRNE

$= 2mv\Delta N$

ZATO KER SE TA $\frac{1}{6}$ DELCEV ZALETI PRAVOKOTNO

$\underline{\Delta G = \frac{2}{6} m v^2 n S \Delta t}$

- TLAK: $p = \frac{F}{S}$; $\Delta G = F \Delta t \Rightarrow F = \frac{\Delta G}{\Delta t}$

$p = \frac{2}{3} \frac{mv^2}{2} n$

$\underline{p = \frac{2}{3} \overline{W_k} n}$ $\Rightarrow \overline{W_k}$ = POVPREČNA KINETIČNA ENERGIJA DELCA

- TEMPERATURA: $pV = \frac{m}{M} RT$ $\quad$ $n = \frac{N}{V} = \frac{\frac{m}{M} \cdot N_A}{V}$

$\underline{p = \frac{m}{M} \frac{RT}{V}}$

$p = \frac{2}{3} \overline{W_k} n$

$\frac{\cancel{m}\, RT}{\cancel{M}\, \cancel{V}} = \frac{2}{3} \overline{W_k} \frac{\cancel{m}\, N_A}{\cancel{M}\, \cancel{V}}$

$\frac{R}{N_A} = k_B = 1{,}38 \cdot 10^{-23}\, J/K$

<?> $\underline{\overline{W_k} = \frac{3}{2} \frac{R}{N_A} \cdot T = \frac{3}{2} k_B T}$

- POVPREČNA PROSTA POT: $\langle l \rangle$

$\langle l \rangle = v\Delta t$

⇒ VOLUMEN, KI PRIPADA ENI MOLEKULI:

$\underline{V' = \pi (2r)^2 \langle l \rangle}$

$\frac{kT}{p} = \pi (2r)^2 \langle l \rangle \Rightarrow \boxed{\langle l \rangle = \frac{kT}{\pi (2r)^2 p}}$

- $n = \frac{N}{V} = \frac{1}{V'} \Rightarrow \underline{V' = \frac{1}{n} = \frac{kT}{p}}$
- $p = \frac{2}{3} \overline{W_k} n = kTn \Rightarrow \underline{n = \frac{p}{kT}}$

$\overline{W_k} = \frac{3}{2} kT$

POPRAVEK, KER SE GIBLJEJO TUDI OSTALI DELCI:

$$\boxed{\langle l \rangle = \frac{kT}{\sqrt{2}\, \pi (2r)^2 p}}$$

MOLEKULA O_2 V ZRAKU: $(M = 32\,kg/kmol)$

$T = 300\,K$

$p = 1\,bar$

$\underline{r \sim 10^{-10}\,m = 1\,Å}$

$\langle v \rangle = ?$

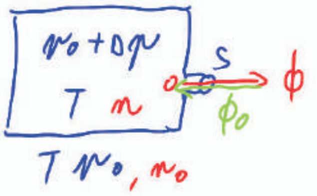
$\langle l \rangle = ?$

$$m = \frac{M}{N_A}$$
$$k = \frac{R}{N_A}$$

$$\overline{W}_k = \frac{3}{2} kT$$
$$\frac{mv^2}{2} = \frac{3}{2} kT$$
$$v = \sqrt{\frac{3kT}{m}}$$
$$v = \sqrt{\frac{3 R \cancel{N_A} T}{\cancel{N_A} M}}$$
$$\underline{v \sim 500\,m/s}$$

$$\langle l \rangle \doteq \frac{kT}{\pi (2r)^2 p}$$
$$\underline{\langle l \rangle \sim \frac{1}{3}\,\mu m}$$

UHAJANJE PLINA SKOZI LUKNJICO:



TOK DELCEV IZ POSODE:

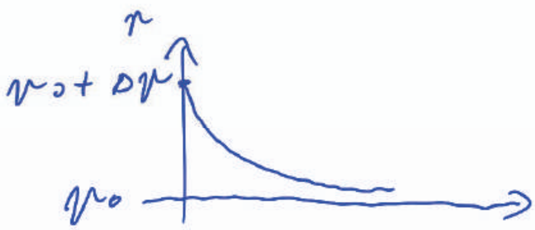
$$\phi = \phi_V \cdot n \cdot \frac{1}{6} = S \langle v \rangle \cdot n \frac{1}{6}$$

TOK DELCEV V POSODO:

$$\phi_0 = S \langle v \rangle n_0 \frac{1}{6}$$

GOSTOTA DELCEV:

$$n = \frac{p}{kT} = \frac{N}{V} \Rightarrow p = \frac{N}{V} \cdot kT$$
$$n = \frac{N}{V} \qquad p = nkT$$



• SPREMEMBA ŠT. DELCEV V POSODI:

$$dN = (\phi_0 - \phi)\,dt$$
$$dN = S \langle v \rangle \frac{1}{6} (n_0 - n)\,dt \;/\cdot \frac{kT}{V}$$
$$dp = \frac{S \langle v \rangle}{6\,V} (p_0 - p)\,dt$$
$$\int_{p_0+\Delta p}^{p} \frac{dp}{(p_0 - p)} = \frac{S \langle v \rangle}{6\,V} \int_0^t dt \qquad p_0 - p = u, \quad -dp = du$$
$$-\int_{-\Delta p}^{p_0 - p} \frac{du}{u} = \frac{S \langle v \rangle}{6\,V} t$$
$$\ln \frac{p_0 - p}{-\Delta p} = -\frac{S \langle v \rangle t}{6\,V}$$
$$p_0 - p = -\Delta p\, e^{-\frac{S\langle v \rangle}{6V} t}$$
$$\underline{p = p_0 + \Delta p\, e^{-\frac{S\langle v \rangle}{6V} t}}$$

11.2 Energijski in entropijski zakon

11.2. nal 1

1. ATOMNI 2. ATOMNI

$$c_V = \frac{3}{2}\frac{R}{M} + \frac{R}{M}$$

He

$p(V) = C/\sqrt{V}$

$C = 2\,\text{bar}\sqrt{l}$

$p_1 = 1\,\text{bar}$

$V_1 = 4\,l$

$p_2 = 2\,\text{bar}$

$V_2 = 1\,l$

$c_V = \frac{3}{2}\frac{R}{M}$

$A = ?$

$\Delta W_n = ?$

$Q = ?$

$\Delta S = ?$

$p_1V_1 \rightarrow p_2V_2$

• $A = -\int p\,dV = -C\int_{V_1}^{V_2}\frac{1}{\sqrt{V}}\,dV = -C\,2\left(\sqrt{V_2}-\sqrt{V_1}\right) = 400\,J$

• $\Delta W_n = m c_V \Delta T \quad \Delta T = (T_2 - T_1)$

$= \frac{\cancel{m}\,3\,\cancel{R}\,MC}{2\,\cancel{M}\,\cancel{m}\,\cancel{R}}\left(\sqrt{V_2}-\sqrt{V_1}\right)$

$pV = \frac{m}{M}RT$

$C\sqrt{V} = \frac{m}{M}RT$

$\frac{\sqrt{V}}{T} = \frac{mR}{MC} = \text{konst.}$

$T = \sqrt{V}\,\frac{MC}{mR}$

$\Delta W_n = \frac{3}{2}C\left(\sqrt{V_2}-\sqrt{V_1}\right)$

$= -300\,J$

Q: $\Delta W_n = Q + A$

$Q = \Delta W_n - A = C\left(\sqrt{V_2}-\sqrt{V_1}\right)\left(\frac{3}{2}+2\right) \quad \frac{7}{2}$

$= -700\,J$

• ENTROPIJA $dS = \frac{dQ}{T}$ $\quad [S] = \frac{J}{K}$ $\quad$ 2. ZAKON TD: $dS \geq 0$ ZA IZOLIRAN SISTEM

• ADIABATNA = IZENTROPNA SPREMEMBA:

$dS = 0 \Rightarrow dQ = 0 \Rightarrow dW_n = d\cancel{Q}^{\,0} - p\,dV \Rightarrow m c_V dT = -p\,dV$

• $c_V = \frac{1}{\varkappa-1}\frac{R}{M} \rightarrow \frac{mR}{M}\cdot\frac{dT}{(\varkappa-1)} = -p\,dV$

• $p = \frac{mRT}{MV} \longrightarrow \frac{\cancel{mR}}{\cancel{M}}\frac{dT}{(\varkappa-1)} = -\frac{\cancel{mR}T}{\cancel{M}V}\,dV$

$\int_{T_1}^{T_2}\frac{dT}{T} = -(\varkappa-1)\int_{V_1}^{V_2}\frac{dV}{V}$

$\ln\frac{T_2}{T_1} = (1-\varkappa)\ln\frac{V_2}{V_1}$

$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{(1-\varkappa)}$

$TV^{\varkappa-1} = \text{konst.}$

$\Delta S = \int\frac{dQ}{T}$; $\quad dW_n = dQ + dA$

$\quad \hookrightarrow dQ = dW_n - dA$

$= \int\frac{m c_V dT + p\,dV}{T}$

$= m c_V \frac{1}{2}\int_{V_1}^{V_2}\frac{dV}{V} + \int_{V_1}^{V_2}\frac{\cancel{C}}{\sqrt{V}}\frac{mR}{M\cancel{C}\sqrt{V}}\,dV$

$= m\frac{R}{M}\left(\frac{3}{4}+1\right)\ln\frac{V_2}{V_1}$

$\Delta S = m\frac{R}{M}\frac{7}{4}\ln\frac{V_2}{V_1}$

• OD ZGORAJ:

• $\frac{\sqrt{V}}{T} = \text{konst.}$

$\frac{1}{2}\ln V - \ln T = \text{konst.}$

$\frac{1}{2}\frac{dV}{V} = \frac{dT}{T}$

• $T = \sqrt{V}\,\frac{MC}{mR}$

11.2. nal 2

$O_2 \rightarrow c_V = \frac{5}{2}\frac{R}{M}$

$T_1 = 0\,^\circ C$

$V_1 = 3\,l$

$h = 10\,m \Rightarrow p = 2\,bar$

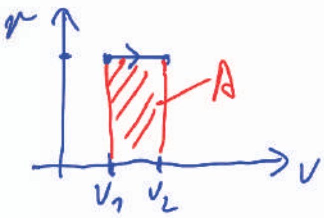
$V_2 = 4\,l$

$A = ?$

$Q = ?$

$\Delta S = ?$

$p = konst \Rightarrow \frac{V}{T} = konst$



• $A = -\int_{V_1}^{V_2} p\,dV = -p(V_2 - V_1) = \underline{-200\,J}$

• Q: $\Delta W_n = Q + A \Rightarrow \underline{Q = \Delta W_n - A}$

$\Delta W_n = m c_V (T_2 - T_1)$

$= \frac{p\cancel{M}V_1}{\cancel{R}\cancel{T_1}} \cdot \frac{5\cancel{R}}{2\cancel{M}}\cancel{T_1}\left(\frac{V_2}{V_1} - 1\right)$

$= \frac{5}{2} p (V_2 - V_1) = \underline{500\,J}$

• $\frac{V_2}{T_2} = \frac{V_1}{T_1} \Rightarrow \underline{T_2 = T_1 \frac{V_2}{V_1}}$

• $pV_1 = \frac{m}{M} R T_1$

$\underline{m = \frac{pMV_1}{RT_1}}$

$\underline{Q = \Delta W_n - A = \frac{7}{2} p (V_2 - V_1)} = \underline{700\,J}$

$\Delta S = \int \frac{dQ}{T} = \int \frac{dW_n}{T} - \int \frac{dA}{T} = m c_V \int_{T_1}^{T_2} \frac{dT}{T} + \int_{V_1}^{V_2} \frac{p\,dV}{T}$

$= m \frac{5}{2}\frac{R}{M} \ln \frac{V_2}{V_1} + \frac{mR\cancel{p}}{M\cancel{p}} \int_{V_1}^{V_2} \frac{dV}{V}$

$= \frac{7}{2} m \frac{R}{M} \ln \frac{V_2}{V_1} = \frac{7}{2} \frac{pV_1}{T_1} \ln \frac{V_2}{V_1} = 0{,}022\,J/K$?

• $\frac{V}{T} = konst$

$\underline{\frac{dV}{V} = \frac{dT}{T}}$

$pV = \frac{m}{M} R T$

$\underline{\frac{1}{T} = \frac{mR}{MpV}}$

$m = \frac{pV_1M}{RT_1}$

$\underline{dQ = \frac{7}{2} p\,dV}$

$\rightarrow dS = \int \frac{dQ}{T} = \;....$ ISTI REZULTAT

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$m = 0.5\,kg$

$T_1 = 10°C$

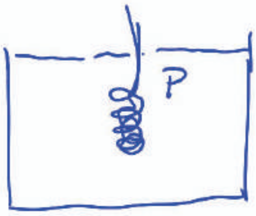
$P = 500\,W$

$t = 5\,min$

REVERZ/IREVERZ = ?

$\Delta S = ?$

$c_v = 4200\,J/kgK$



IREVERZIBILNA SPREMEMBA

↳ KER SE VODA NE MORE OHLADITI IN PREDATI ELEKTRIKE

KER A=0! → KER VODA NI OPRAVILA DELA SAJ SE JI VOLUMEN NI SPREMENIL

$$Q = P \cdot t = \Delta W_n = m c_v (T_2 - T_1)$$

$$\hookrightarrow T_2 = T_1 + \frac{Pt}{m c_v}$$

$$\Delta S = \int \frac{dQ}{T} = \int \frac{dW_n}{T} = m c_v \int_{T_1}^{T_2} \frac{dT}{T} = m c_v \ln \frac{T_2}{T_1} = m c_v \ln\left(1 + \frac{Pt}{m c_v T_1}\right)$$

↳ S VODE SE POVEČA

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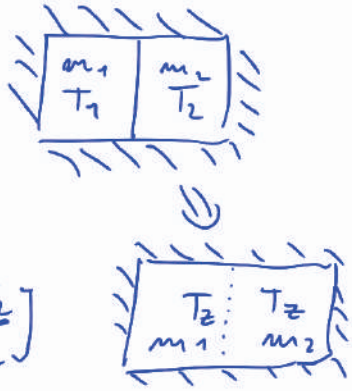
a) $m_1 = 0{,}1\,kg$

$T_1 = 100°C$

$m_2 = 0{,}9\,kg$

$T_2 = 0°C$

$$dS = \frac{dQ}{T} \underset{\substack{\uparrow \\ dA=0\,(\text{VODA})}}{=} \frac{dW_u}{T} = m\,c_v \frac{dT}{T}$$



$$\Delta S = \Delta S_1 + \Delta S_2 = c_v\left[m_1 \int_{T_1}^{T_z} \frac{dT_1}{T_1} + m_2 \int_{T_2}^{T_z} \frac{dT_2}{T_2}\right]$$

b) $m_1 = 0{,}4\,kg$

$T_1 = 80°C$

$m_2 = 0{,}6\,kg$

$T_2 = 40°C$

$\Delta S_a = ?$

$\Delta S_b = ?$

$$\Delta W_u = 0$$

$$\Delta W_{u1} + \Delta W_{u2} = 0$$

$$m_1 c_v (T_z - T_1) = -m_2 c_v (T_z - T_2)$$

$$T_z (m_1 + m_2) = m_2 T_2 + m_1 T_1$$

$$\underline{T_z = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2}} = \begin{cases} a)\ 283\,K \\ b)\ 329\,K \end{cases}$$

$$\underline{\Delta S = c_v\left[m_1 \ln\frac{T_z}{T_1} + m_2 \ln\frac{T_z}{T_2}\right]} = \begin{cases} a)\ 20\,J/K \\ b)\ 7{,}3\,J/K \end{cases}$$

$m = m_1 + m_2$

$m_1 = x\,m;\ x < 1$

$m_2 = (1-x)\,m$

$T_z = x \cdot T_1 + (1-x)\,T_2$

$$\Delta S = c_v m\left[x \ln\frac{T_z}{T_1} + (1-x)\ln\frac{T_z}{T_2}\right] =$$

$$= c_v m\left[x\left(\ln\frac{T_z}{T_1} - \ln\frac{T_z}{T_2}\right) + \ln\frac{T_z}{T_2}\right] =$$

$$= c_v m\left[x \ln\left(\frac{\cancel{T_z}}{T_1}\cdot\frac{T_2}{\cancel{T_z}}\right) + \ln\left(x\frac{T_1}{T_2} + (1-x)\right)\right] =$$

$$= c_v m \ln\underbrace{\left[\left(\frac{T_2}{T_1}\right)^x \cdot \left(1 + x\left(\frac{T_1}{T_2} - 1\right)\right)\right]}_{\geq 1}$$

$$\Delta S \geq 0 \Rightarrow$$

$$\frac{1 + x\left(\frac{T_1}{T_2} - 1\right)}{\left(\frac{T_1}{T_2}\right)^x} \overset{?}{\geq} 1$$

• MATEMATIČNA NEENAKOST:

$\underline{az + by \geq z^a y^b;\ \check{c}e\ a + b = 1}$

$$1 - x + xt \overset{?}{\geq} t^x \quad ;\ a = x,\ b = (1-x)$$

$$b + at \overset{?}{\geq} t^a \cdot 1 \quad ;\ z = t,\ y = 1$$

$$\rightarrow \underline{by + az \geq z^a y^b}$$

11.2. nal 5

$V_1 = 3\,l$

$m_1 = 10\,g$

$T_1 = 0°C$

$V_2 = 5\,l$

$m_2 = 20\,g$

$T_2 = 50°C$

ZRAK: (1)

$M_1 = 29\,kg/kmol$

$\varkappa_1 = 1{,}4$

$c_{v1} = 715\,J/kgK$

ARGON: (2)

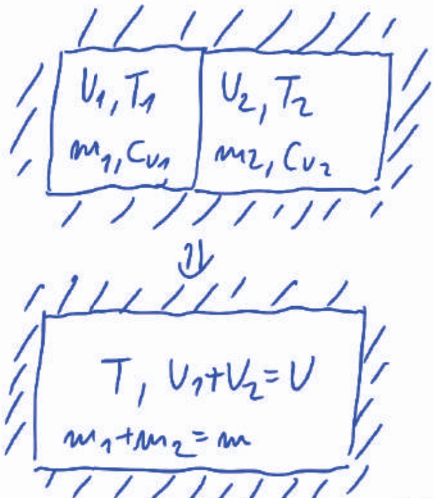
$M_2 = 40\,kg/kmol$

$\varkappa_2 = 1{,}67$

$c_{v_1} = 310\,J/kg\,K$

$T = ?$

$\Delta S = ?$



• KONČNA T: $\Delta W_n = \Delta W_{n_1} + \Delta W_{n_2} = 0$ ← KER JE SISTEM IZOLIRAN

$$m_1 c_{v_1}(T-T_1) = -m_2 c_{v_2}(T-T_2)$$

$$\underline{\underline{T = \frac{m_1 c_{v1} T_1 + m_2 c_{v2} T_2}{m_1 c_{v1} + m_2 c_{v2}}}}$$

$$\underline{\underline{T = 296{,}2\,K}}$$

• TLAKI NA KONCU:

$$p_1 = \frac{m_1}{M_1}\frac{RT}{V} \leftarrow \text{PARCIALNI TLAK}$$

$$p_2 = \frac{m_2}{M_2}\frac{RT}{V}$$

$$p = p_1 + p_2 = \left(\frac{m_1}{M_1} + \frac{m_2}{M_2}\right)\frac{RT}{V}$$

• MOLSKI DELEŽ: $(X_1 + X_2 = 1)$

$$X_1 = \frac{p_1}{p} = \frac{\frac{m_1}{M_1}}{\underbrace{\frac{m_1}{M_1} + \frac{m_2}{M_2}}_{\frac{m}{M} \leftarrow \text{SKUPEN}}}$$

$$\underline{X_2 = 1 - X_1}$$

SAMO ZA REVERZIBILNE DRUGAČE >!

ΔS: $\Delta S = \Delta S_1 + \Delta S_2$; $dW_n = dQ + dA \Rightarrow dQ = dW_n - dA$

$$\Delta S_1:\ dS_1 = \frac{dQ_1}{T_1} = \frac{dW_{n1}}{T_1} - \frac{dA_1}{T_1} = m_1 c_{v1}\frac{dT_1}{T_1} + p_1\frac{dV_1}{T_1}$$

$$\underline{\Delta S_1} = m_1 c_{v_1}\int_{T_1}^{T}\frac{dT_1}{T_1} + m_1\frac{R}{M_1}\int_{V_1}^{V}\frac{dV_1}{V_1} = \underline{\underline{m_1\frac{R}{M_1}\left(\frac{1}{\varkappa_1-1}\ln\frac{T}{T_1} + \ln\frac{V}{V_1}\right)}} = 3{,}4\,J/K$$

$pV = \frac{m}{M}RT$

$\underline{\frac{p}{T} = \frac{m}{M}\frac{R}{V}}$

$\varkappa = \frac{c_p}{c_v}$

$c_p - c_v = \frac{R}{M}$

$\underline{c_v = \frac{1}{\varkappa-1}\frac{R}{M}}$

$$\underline{\Delta S_2 = m_2\frac{R}{M_2}\left[\frac{1}{\varkappa_2-1}\ln\frac{T}{T_2} + \ln\frac{V}{V_2}\right]} = 1{,}4\,J/K$$

$$\underline{\Delta S = 4{,}8\,J/K}$$

ZBIRKA 9 zad 28/str 40

He, IZENTROPNA S. $\Rightarrow dS=0 \Rightarrow dQ=0$

$m = 1\,kg$

$T_1 = 90°C$

$p_2 = 5\,bar$

$T_2 = 120°C$

$M = 4\,kg/kmol$

$\varkappa = 1{,}67$

$A = ?$

$\Delta H = ?$

a) $dW_u = d\cancel{Q}^{\,0} + dA \Rightarrow dA = dW_u = m c_v dT$

$$\underline{A = m c_v (T_2 - T_1) = \frac{m}{(\varkappa-1)}\frac{R}{M}(T_2 - T_1)} = \underline{93\,kJ}$$

$H = W_u + pV \qquad pV = \frac{m}{M} RT$

$$\underline{dH} = dW_u + d(pV) = m c_v dT + m\frac{R}{M} dT =$$

$$= m\left(c_v + \frac{R}{M}\right)dT = m(\cancel{c_v} + c_p - \cancel{c_v})dT = \underline{m c_p dT}$$

$$\underline{\Delta H} = m\left(c_v + \frac{R}{M}\right)(T_2 - T_1) = m\frac{R}{M}\left(\frac{1}{\varkappa-1} + 1\right)(T_2 - T_1)$$

$$= \underline{m\frac{R}{M}\frac{\varkappa}{\varkappa-1}(T_2 - T_1)} = \underline{155{,}3\,kJ}$$

b) $dA = -p\,dV$

$A = -\int p\,dV$

$\underline{pV^{\varkappa} = konst}$; $pV = \frac{m}{M}RT$

$\frac{T}{V}V^{\varkappa} = konst$

$\underline{T V^{\varkappa-1} = konst}$

$p\left(\frac{T}{p}\right)^{\varkappa} = konst$

$\underline{p^{1-\varkappa}T^{\varkappa} = konst}$

$\underline{p = \frac{m}{M}R\frac{T}{V}}$

$\underline{V = \frac{m}{M}R\frac{T}{p}}$

$\ln T + (\varkappa-1)\ln V = konst \,/\, d$

$\frac{dT}{T} = -(\varkappa-1)\frac{dV}{V}$

$dV = -\frac{V}{\varkappa-1}\frac{dT}{T}$

$\underline{dV = -\frac{mR\cancel{T}}{M(\varkappa-1)p}\frac{dT}{\cancel{T}}}$

$$\underline{A = -\int_{T_1}^{T_2} \cancel{p}\left(-\frac{m R}{M(\varkappa-1)\cancel{p}}\right)dT = m\frac{R}{M}\frac{1}{\varkappa-1}(T_2 - T_1)}$$

$(1-\varkappa)\ln p + \varkappa\ln T = konst \,/\, d$

$(1-\varkappa)\frac{dp}{p} = -\varkappa\frac{dT}{T}$

$\hookrightarrow dp = \frac{\varkappa}{\varkappa-1}p\frac{dT}{T}$

$\underline{dp = \frac{\varkappa}{\varkappa-1}\frac{mR\cancel{T}}{MV}\frac{dT}{\cancel{T}}}$

$$dH = \underline{dW_u} + p\,dV + V\,dp$$

$$= d\underbrace{Q}_{0} + \cancel{dA} + \cancel{p\,dV} + V\,dp$$

$dH = V\,dp$

$$H = \int V\,dp = \frac{\varkappa}{\varkappa-1}\frac{mR}{M}\int_{T_1}^{T_2}\frac{\cancel{V}}{\cancel{V}}dT$$

$$\underline{H = \frac{\varkappa}{\varkappa-1}m\frac{R}{M}(T_2 - T_1)}$$

ZBIRKA 9 nal 31/st 40

$V_1 = 1\ dm^3$

$T_1 = 20°C = T_0$

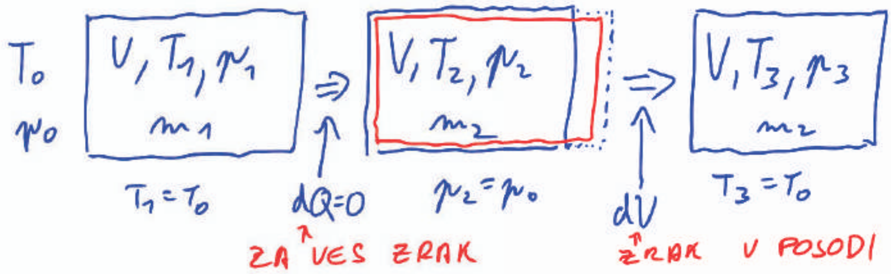
$p_1 = 1{,}2\ bar$

$p_0 = 1\ bar$

$\varkappa = 1{,}4$

$p_3 = ?$

$\Delta W_n = ?$



$1 \to 2:\ dQ=0:\quad p_1^{1-\varkappa} T_1^{\varkappa} = p_2^{1-\varkappa} T_2^{\varkappa}$

$$\underline{T_2 = T_1 \left(\frac{p_1}{p_2}\right)^{\frac{1-\varkappa}{\varkappa}}} < T_1$$

$2 \to 3:\ dV=0:\quad \frac{p_2}{T_2} = \frac{p_3}{T_3}$

$p_2 = p_0,\ T_3 = T_0,\ T_1 = T_0$

$$p_3 = p_2 \frac{T_3}{T_2} = p_0 \frac{T_0}{T_0}\left(\frac{p_0}{p_1}\right)^{\frac{1-\varkappa}{\varkappa}} = \underline{p_0\left(\frac{p_1}{p_0}\right)^{\frac{\varkappa-1}{\varkappa}}}$$

$$\underline{\underline{p_3 = 1{,}05\ bar}}$$

$$\Delta W_n = W_{n3} - W_{n1} = m_3 c_v T_3 - m_1 c_v T_1 =$$

$$= (m_3 - m_1) c_v T_0 \leftarrow \bullet\ \underline{T_3 = T_1 = T_0}$$

$$= \frac{MV}{R T_0}(p_3 - p_1) c_v T_0 \leftarrow \bullet\ pV = \frac{m}{M} RT \Rightarrow m = \frac{MV}{RT} p$$

$$\underline{\Delta W_n} = \frac{M}{R} V (p_3 - p_1) c_v = \underline{\frac{1}{\varkappa - 1} V (p_3 - p_1)} = \underline{37{,}5}\ J$$

$$c_v = \frac{1}{\varkappa-1}\frac{R}{M}$$

11.4 Toplotni stroji

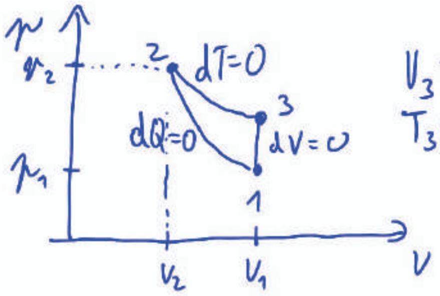
ZBIRKA 9 nal 49/st 42

$T_1 = 20°C$

$T_2 = 300°C$

$\underline{V_3 = V_1}$

$\eta = ?$



$V_3 = V_1$

$T_3 = T_2$

IZKORISTEK:

$$\eta = \frac{-A_{SKUPNO}}{Q_{DOVEDENA}}$$

$1 \to 2$: $dQ_{12} = 0$: $dW_{m_{12}} = dA_{12} \Rightarrow A_{12} = m c_v (T_2 - T_1) > 0$

PLIN DELO PREJME

$2 \to 3$: $dT = 0$: $dW_{m_{23}} = 0$: $dA_{23} = -dQ_{23}$, $pV = p_2 V_2$

$$A_{23} = -\int_2^3 p\,dV \qquad p = \frac{p_2 V_2}{V}$$

$$= -p_2 V_2 \int_{V_2}^{V_3} \frac{dV}{V}$$

$$= -p_2 V_2 \ln \frac{V_3}{V_2} \qquad T_1 V_1^{\varkappa - 1} = T_2 V_2^{\varkappa - 1}$$

$$= -p_2 V_2 \ln \frac{V_1}{V_2} \qquad \frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\varkappa - 1}}$$

$$= -p_2 V_2 \frac{1}{\varkappa - 1} \ln \frac{T_2}{T_1}$$

$$A_{23} = -\frac{m R T_2}{M(\varkappa - 1)} \ln \frac{T_2}{T_1} = -m c_v T_2 \ln \frac{T_2}{T_1} < 0$$

PLIN DELO ODDA

$$Q_{23} = -A_{23} = m c_v T_2 \ln \frac{T_2}{T_1} > 0$$

DOVEDENA TOPLOTA

$3 \to 1$: $dV = 0$: $dA_{31} = 0$: $dW_{m_{31}} = dQ_{31}$

$$Q_{31} = m c_v (T_1 - T_3) = m c_v (T_1 - T_2) < 0$$

ODDANA TOPLOTA

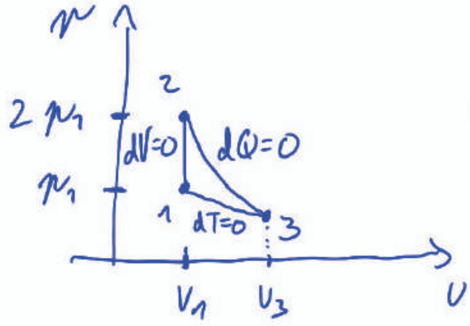
$$\eta = -\frac{A_{12} + A_{23} + \overset{0}{A_{31}}}{Q_{23}} = -\frac{m c_v (T_2 - T_1) - m c_v T_2 \ln \frac{T_2}{T_1}}{m c_v T_2 \ln \frac{T_2}{T_1}} =$$

$$= -\frac{1 - \frac{T_1}{T_2} - \ln \frac{T_2}{T_1}}{\ln \frac{T_2}{T_1}} = 1 - \frac{1 - \frac{T_1}{T_2}}{\ln \frac{T_2}{T_1}} = \underline{\underline{0{,}27}}$$

(11,4) nal 4

$m = 1\,kg$
$T_1 = 0°C$
$p_2 = 2p_1$
$M = 58\,kg/kmol$
$c_V = 360\,J/kgK$
$\varkappa = 1{,}4$

$\eta = ?$



$V_1 = V_2$
$p_2 = 2p_1$
$T_3 = T_1$

$1 \to 2: \; dV = 0 : dA_{12} = 0 : dQ_{12} = dW_{m_{12}}$

$$\underline{Q_{12}} = m\,c_V(T_2 - T_1) = \underline{m\,c_V T_1}$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$
$$T_2 = T_1\frac{p_2}{p_1} = 2T_1$$

$2 \to 3: \; dQ_{23} = 0 : dA_{23} = dW_{m_{23}}$

$$\underline{A_{23}} = m\,c_V(T_3 - T_2) = \underline{-m\,c_V T_1}$$

$3 \to 1: \; dT = 0 : dW_{m_{31}} = 0 : \; A_{31} = -Q_{31}$

$$A_{31} = -\int_3^1 p\,dV = -p_1V_1\int_{V_3}^{V_1}\frac{dV}{V}$$
$$= -p_1V_1 \ln\frac{V_1}{V_3}$$
$$= -\frac{mRT_1}{M}\cdot\frac{1}{\varkappa - 1}\cdot\ln\frac{T_3}{T_2}$$
$$= -m\,c_V T_1 \ln\frac{T_1}{T_2}$$
$$= \underline{m\,c_V T_1 \ln 2}$$

$$Q_3 = -A_{31} < 0$$

- $pV = p_1V_1$

$$\underline{p = p_1V_1\frac{1}{V}}$$

- $T_2V_2^{\varkappa-1} = T_3V_3^{\varkappa-1}$

$$\frac{V_1}{V_3} = \underbrace{\frac{V_2}{V_3} = \left(\frac{T_3}{T_2}\right)^{\frac{1}{\varkappa-1}}}$$

- $p_1V_1 = \frac{m}{M}RT_1$

$$\eta = -\frac{A_{23} + A_{31}}{Q_{12}} = -\frac{-T_1 + T_1\ln 2}{T_1} = \underline{1 - \ln 2} = \underline{0{,}31}$$

11.4. nal 5

V_{min}

V_{max}

$\varkappa = 1{,}4$

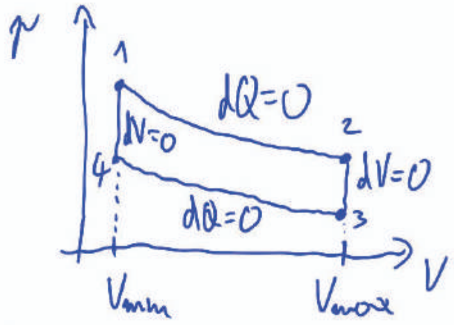
$V_{max}/V_{min} = 8$

$\eta = ?$

$\Delta S = ?$

↳ ∮ POVRATEK V ISTO STANJE

↳ $\Delta S = 0$



$1 \to 2: dQ_{12} = 0 \qquad T_1 V_1^{\varkappa-1} = T_2 V_2^{\varkappa-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\varkappa-1} < T_1$

$A_{12} = W_{m12} = m c_v (T_2 - T_1) < 0$

↳ DELO ODDA

$3 \to 4: dQ_{34} = 0 \qquad T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\varkappa-1}$

$A_{34} = W_{m34} = m c_v (T_4 - T_3) > 0$

↳ DELO PREJME

$2 \to 3: dV = 0: dA_{23} = 0 \to \frac{p_3}{T_3} = \frac{p_2}{T_2} \Rightarrow T_3 = T_2 \frac{p_3}{p_2} < T_2$

$Q_{23} = W_{m23} = m c_v (T_3 - T_2) < 0$

↳ TOPLOTO ODDA

$4 \to 1: dV = 0: dA_{41} = 0 \qquad T_1 = T_4 \frac{p_1}{p_4} > T_4$

$Q_{41} = W_{m41} = m c_v (T_1 - T_4) > 0$

↳ TOPLOTO PREJME ↳ Q_{DOV}

$$\eta = \frac{-A}{Q_{DOV}} = \frac{-\left[\cancel{m c_v}(T_2 - T_1) + \cancel{m c_v}(T_4 - T_3)\right]}{\cancel{m c_v}(T_1 - T_4)}$$

$$= \frac{T_1 - T_2 + T_3 - T_4}{T_1 - T_4} = 1 - \frac{T_2 - T_3}{T_1 - T_4} = 1 - \frac{T_2}{T_1}\frac{\left(1 - \frac{T_3}{T_2}\right)}{\left(1 - \frac{T_4}{T_1}\right)} = 1 - \frac{T_2}{T_1} = \eta_C$$

$$\eta = 1 - \left(\frac{V_1}{V_2}\right)^{\varkappa-1} = 1 - \left(\frac{V_{min}}{V_{max}}\right)^{\varkappa-1} = 0{,}57$$

ADIABATNO

$$\Delta S = 0 = \Delta S_{23} + \Delta S_{41} = \int_2^3 \frac{m c_v dT}{T} + \int_4^1 \frac{m c_v dT}{T} = m c_v \left[\ln\frac{T_3}{T_2} + \ln\frac{T_1}{T_4}\right] = 0$$

$dA_{23} = dA_{41} = 0$

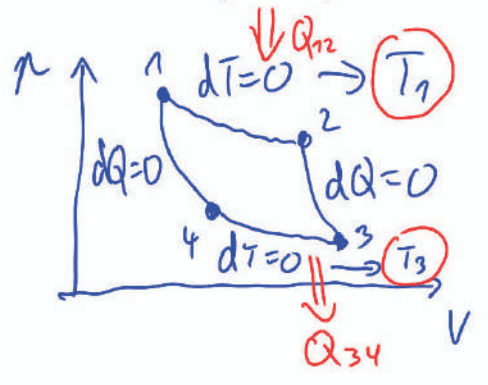
$dQ_{23,41} = m c_v dT$

$$\ln\frac{T_3}{T_2} = -\ln\frac{T_1}{T_4}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1}$$

• CARNOT TOPLOTNI STROJ:

↳ NAJVEČJI MOŽEN IZKORISTEK!



$A + Q_{12} + Q_{34} = 0 \Rightarrow A = -(Q_{12} + Q_{34})$

$$\eta_C = \frac{-A}{Q_{DOV}} = \frac{Q_{12} + Q_{34}}{Q_{12}} = 1 + \frac{Q_{34}}{Q_{12}} = 1 - \frac{T_3}{T_1}$$

$$\Delta S = 0 = \frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3} \Rightarrow \frac{Q_{34}}{Q_{12}} = -\frac{T_3}{T_1}$$

$T_1, T_3 = $ konst.

$T_1 > T_3$

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$$\Delta S = 0 = \frac{Q_H}{T_H} + \frac{Q_Z}{T_Z}$$

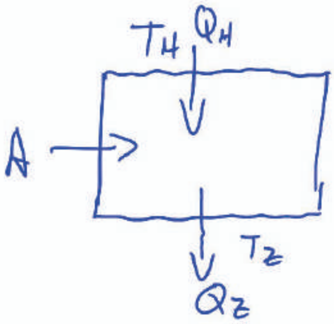
$\frac{dm}{dt} = 1\,kg/10\,min$

$T_H = 0°C$

$T_Z = 30°C$

$\eta = \frac{\eta_c}{2.5}$

$P = ?$



IZKORISTEK HLADILNIKA:

$$\eta_c = \frac{|Q_H|}{|A|} = \frac{|Q_H|}{|Q_Z|-|Q_H|} = \frac{T_H}{T_Z - T_H}$$

$A = |Q_Z| - |Q_H|$, CARNOT

$q_t = 3{,}34 \cdot 10^5\,J/kg$

$$P = \frac{dA}{dt}$$

$$\frac{dQ}{dt} = \frac{dm}{dt} \cdot q_t$$

$$\eta = \frac{dQ}{dA} \Rightarrow dA = dQ\frac{1}{\eta}$$

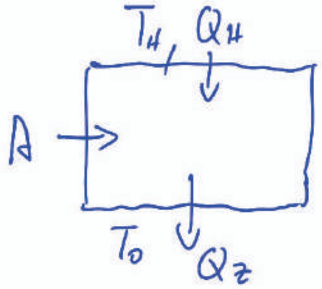
$$P = \frac{dm \cdot q_t \cdot 2.5}{dt\ \eta_c}$$

$$\eta_c = \frac{T_H}{T_Z - T_H} = 9{,}1$$

$$P = 153\,W$$

11.4. nal 6

$m = 10\,kg$
$T_0 = 20°C$
$T_1 = -193°C$
$c_p = 1\,kJ/kgK$
$A = ?$



$$\eta = \frac{dQ_H}{dA} = \frac{T_H}{T_z - T_H} = \frac{1}{\frac{T_z}{T_H} - 1}$$

↳ $\eta(T_H)$: ODVISEN OD T_H

⇒ UČINKOVITEJE, ČE VODO OHLAJAMO SKUPAJ S HLADILNIKOM

⇒ SLABŠE, ČE VODO DAMO V HLADEN HLADILNIK

$$dA = \frac{1}{\eta}\, dQ_H$$

• $dQ_H = dH = m c_p (-dT_H)$

GLEDAMO PRI $p =$ konst.

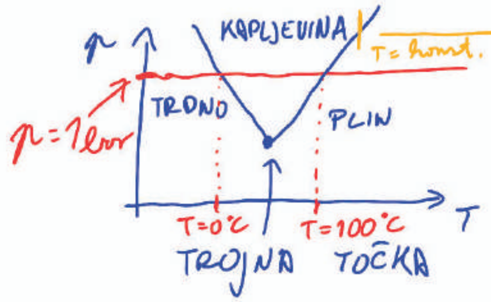
• $\eta = \dfrac{1}{\frac{T_0}{T_H} - 1}$

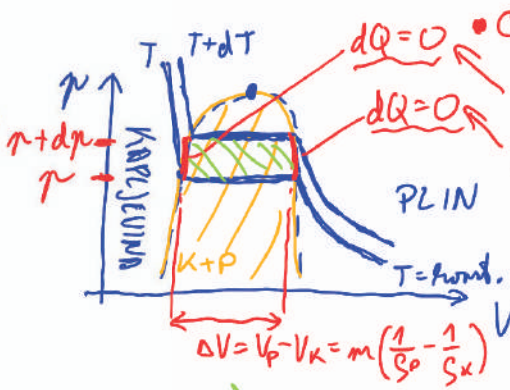
$$\int_0^A dA = -\int_{T_0}^{T_1} \left(\frac{T_0}{T_H} - 1\right) m c_p \, dT_H$$

$$A = -m c_p \left[T_0 \ln\frac{T_1}{T_0} - (T_1 - T_0)\right] = -m c_p T_0 \left[\ln\frac{T_1}{T_0} + 1 - \frac{T_1}{T_0}\right] = +1{,}67 \cdot 10^6\,J$$

11.3 Fazne spremembe

FAZNI DIAGRAM VODE:

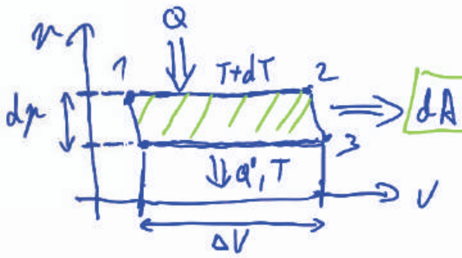




• CLAUSIUS CLAPEYRON:
PRIBLIŽEK ZA $dT \to 0$

• DELO V KROŽNI SPREMEMBO:
$$dA = \Delta V \cdot dp$$

• TOPLOTA, K JO DODAMO:
$$Q = m\,q_i$$



$\boxed{dA = \Delta V \cdot dp}$

• TOPLOTA, KI JO ODDA:
$$Q' = Q - dA$$

• ENTROPIJA KROŽNE SPREMEMBE: $\Delta S = 0$

↳ KER SO VODORAVNE LINIJE IZOTERME: $\frac{|Q|}{T+dT} = \frac{|Q'|}{T} = \frac{|Q| - |dA|}{T}$

$$Q \cdot \cancel{T} = Q\cancel{T} - dA \cdot T + Q\,dT - \cancel{dA}\,dT$$

$$\frac{dT}{T} = \frac{dA}{Q}$$

$$\frac{dT}{T} = \frac{\Delta V \cdot dp}{Q}$$

CLAUSIUS-CLAPEYRON → ZA FAZNI PREHOD

$$\boxed{\frac{dp}{dT} = \frac{L}{T\Delta V}}$$

LATENTNA TOPLOTA

↳ ZA PLIN → KAPLJEVINA: $L = m\,q_i$
IN KAPLJEVINA → TRDNINA: $L = m\,q_t$

ZBIRKA 9 nal 41/str 41

$p_0 = 1\,bar$
$\Delta T = 1\,K$
$T_0 = 273\,K$
$\rho_L = 0{,}92\,g/cm^3$

$\Delta p = ?$

$$\frac{dp}{dT} = \frac{L}{T\Delta V} \quad ; \quad L = m\,q_t \quad ; \quad q_t = q_t(p_0, T_0)$$

$$\Delta V = m\left(\frac{1}{\rho_V} - \frac{1}{\rho_L}\right)$$

TEKOČA VODA

$$\frac{dp}{dT} = \frac{\cancel{m}\,q_t}{T\cancel{m}\left(\frac{1}{\rho_V} - \frac{1}{\rho_L}\right)}$$

$$\int_{p_0}^{p_0+\Delta p} dp = \frac{q_t}{\left(\frac{1}{\rho_V} - \frac{1}{\rho_L}\right)} \int_{T_0}^{T_0-\Delta T} \frac{dT}{T}$$

$$\Delta p = -\frac{q_t}{\left(\frac{1}{\rho_L} - \frac{1}{\rho_V}\right)} \ln\frac{T_0-\Delta T}{T_0} = \frac{q_t}{\left(\frac{1}{\rho_L} - \frac{1}{\rho_V}\right)} \ln\frac{T_0}{T_0-\Delta T}$$

$$\underline{\Delta p = 140{,}95\,bar}$$

$$\ln\left(\frac{T_0}{T_0-\Delta T}\right) = -\ln\left(\frac{T_0-\Delta T}{T_0}\right) = -\ln\left(1-\frac{\Delta T}{T_0}\right) \approx -\left(-\frac{\Delta T}{T_0}\right) \approx \frac{\Delta T}{T_0},$$

$\frac{\Delta T}{T} \ll 1$

$T \sim$ konst,
$dT \to \Delta T,\ dp \to \Delta p$

$$\Delta p = \frac{q_t}{\left(\frac{1}{\rho_L} - \frac{1}{\rho_V}\right)} \cdot \frac{\Delta T}{T_0} = \underline{140{,}7\,bar}$$

ZBIRKA 9 nal 23/of 36

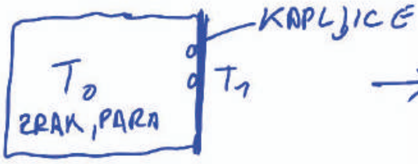
$V = 1\,m^3$

$T_0 = 22\,°C$

$p_0 = 1\,bar$

$T_1 = 13\,°C$

$m_p = ?$



$\rightarrow$ VRELIŠČE SE SPUSTILI SPUSTI NA T_1 ZARADI MAJHNEGA PARCIALNEGA TLAKA PARE

$$\frac{dp}{dT} = \frac{m\,q_i}{T \Delta V}$$

$$\underline{\Delta V = V_p - V_K \sim V_p = \frac{mRT}{Mp}}$$

$$V_p >> V_K$$

$$pV = \frac{m}{M} RT \Rightarrow V = \frac{mRT}{Mp}$$

$$\frac{dp}{dT} = \frac{\cancel{m}\, q_i M p}{T \cancel{m} R T}$$

$$\int_{p_0}^{p} \frac{dp}{p} = \frac{q_i M}{R} \int_{T_0}^{T} \frac{dT}{T^2}$$

$$\ln \frac{p}{p_0} = \frac{q_i M}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right)$$

$$\underline{p = p_0\, e^{\frac{q_i M}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)}}$$

$\hookrightarrow$ ODVISNOST TLAKA VRELIŠČA/ROSIŠČA OD TEMPERATURE

· PODATKI:

NASIČEN PARNI TLAK PARE:

$p_N(15°C) = 17{,}1\,mbar$

$p_N(20°C) = 23{,}3\,mbar$

$p_N(25°C) = 31{,}2\,mbar$

$$p = p_N(15°C)\, e^{\frac{q_i M}{R}\left(\frac{1}{288\,K} - \frac{1}{T}\right)}$$

$$\underline{p_N(13°C) = 15{,}2\,mbar}$$

$$p_N(13°C) = p_p(T_0)$$

$$p_p V = \frac{m_p}{M_p} R T_0$$

$$\underline{m_p = \frac{p_p V M_p}{R T_0}}$$

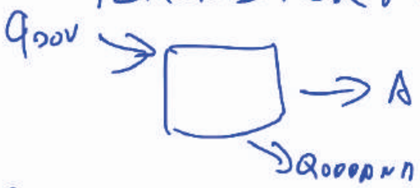
$$\underline{\underline{11{,}2\,g}}$$

VPRAŠANJA:

$p = V \cdot C$

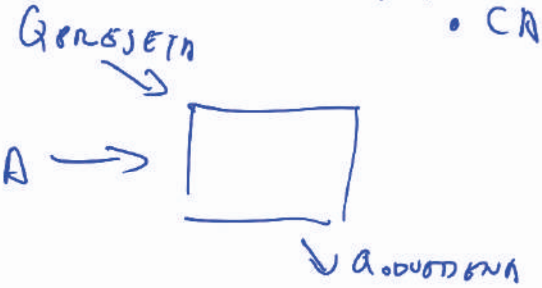
$$A = -\int p\,dV = -C\int_{V_1}^{V_2} V\,dV =$$

IZKORISTEK: • CARNOTOV TOPLOTNI STROJ



$$\eta_C = \frac{|A|}{|Q_{dov}|} = 1 - \frac{T_{nižja}}{T_{višja}}$$

• CARNOTOV HLADILNIK



$$\eta_C = \frac{|Q_{prejeta}|}{|A|} = \frac{T_{nižja}}{T_{višja} - T_{nižja}} = \frac{1}{\eta_C(\text{TOPLOTNI STROJ})}$$

$$= \frac{1}{1 - \frac{T_{nižja}}{T_{višja}}}$$

ZBIRKA 9 nal 42/st 41

$T = 0°C$

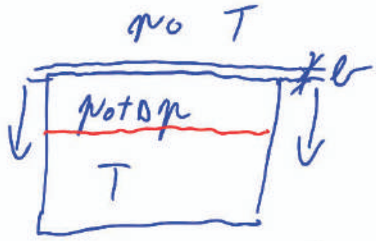
$b = 0{,}4\ mm$

$\lambda = 400\ W/mK$

$\Delta p = 6\ bar$

$\rho_L = \frac{\rho_V}{1.1}$

$v = ?$



$$\Delta V = m\underbrace{\left(\frac{1}{\rho_V} - \frac{1}{\rho_L}\right)}_{\hat{0}}$$

$$\frac{dp}{dT} = \frac{\overbrace{m q_t}^{L}}{T \Delta V}$$

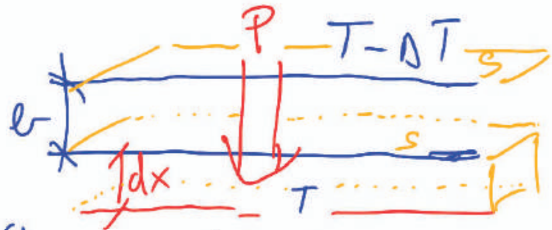
$$dT = \frac{T \Delta V}{m q_t} \cdot dp$$

$\Delta T << T \Rightarrow T \approx konst$

$$\Delta T = \frac{T \cancel{m}\left(\frac{1}{\rho_V} - \frac{1}{\rho_L}\right)}{\cancel{m}\, q_t} \Delta p$$

$$\Delta T = -0{,}05\ K$$

Z INTEGRALOM: $\Delta T = -0{,}055K$



$$P = \frac{dQ}{dt}$$

$$dQ = dm\, q_t = \rho_L \cdot S dx \cdot q_t$$

$$P = \rho_L S \cdot q_t \cdot \frac{dx}{dt} \qquad P = \lambda S\left(-\frac{\Delta T}{b}\right)$$

$$\rho_L \cancel{S} q_t \underbrace{\frac{dx}{dt}}_{v} = \lambda \cancel{S}\left(-\frac{\Delta T}{b}\right)$$

$$v = \frac{\lambda(-\Delta T)}{b\, q_t\, \rho_L} = 0{,}16\ mm/s$$

(11.3) nal 3

↳ DN

113. {3}

$p = 1\,\mathrm{bar}$

$T = 20^\circ C$

$V = 1\,m^3$

$m = 0{,}9\,g$

$M_z = 29\,\frac{kg}{kmol}$

$M_p = 18\,\frac{kg}{kmol}$

$q_i = 2{,}3\,MJ/kg$

$p_p, p_z, m_z = ?$

$\eta = \frac{p_p}{p_{pNAS}} = ?$

$V_2\,(p_{p2} = p_{pNAS}) = ?$

$$p_i \cdot V = m_i \cdot R \cdot T$$

$$p_i = m_i \frac{R \cdot T}{V}$$

$$p_i = \frac{m_i}{M_i}\,\frac{R \cdot T}{V}$$

$$p_p = \frac{9 \cdot 10^{-4}\,kg\,kmol \cdot 8314\,J \cdot 293\,K}{18\,kg \cdot kmol K \cdot 1\,m^3}$$

$$p_p = 121.8\,N/m^2$$

$$\underline{p_p = 1{,}22\,mbar}$$

$$p = p_z + p_p$$

$$\hookrightarrow p_z = p - p_p \sim 1\,\mathrm{bar}$$

$$p \cdot V = n \cdot R \cdot T$$

$$p = p_z + p_p = (n_z + n_p)\frac{R \cdot T}{V}$$

$$p = \left(\frac{m_z}{M_z} + \frac{m_p}{M_p}\right)\frac{R \cdot T}{V}$$

$$m_z = \left(\frac{pV}{RT} - \frac{m_p}{M_p}\right) \cdot M_z$$

$$m_z = \left(\frac{10^5\,N \cdot 1\,m^3\,K\,kmol}{m^2\,8314\,J \cdot 293\,K} - \frac{9 \cdot 10^{-4}\,kg\,kmol}{18\,kg}\right) 29\,\frac{kg}{kmol}$$

$$\underline{m_z = 1{,}189\,kg}$$

CLAUSIUS - CLAPEYRON

$$\frac{dp}{dT} = \frac{L}{T\Delta V}$$

$$\frac{dT}{T} = \frac{\Delta V}{L} \cdot dp$$

KAPLJEVINA → PLIN

$\Delta V \sim V_{PLINA}$

$$\frac{dT}{T} = \frac{1}{L} V dp$$

$V = m \frac{RT}{M p}$

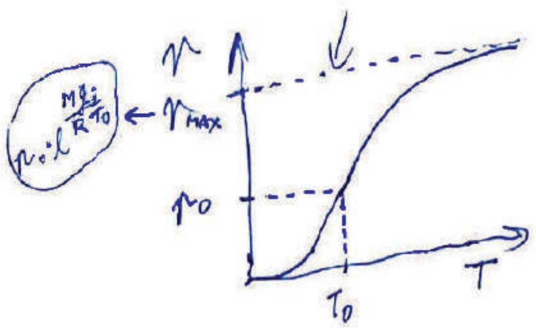
$L = m \cdot q_i$

$$\frac{dT}{T} = \frac{\cancel{m} R T}{\cancel{m} q_i M p} \cdot dp$$

$$\int_{T}^{T_0} \frac{dT}{T^2} = \frac{R}{M q_i} \int_{p}^{p_0} \frac{dp}{p}$$

$$\frac{1}{T} - \frac{1}{T_0} = \frac{R}{M q_i} \ln \frac{p_0}{p}$$

$$p = p_0 \cdot e^{\frac{M q_i}{R \cdot T_0}\left(1 - \frac{T_0}{T}\right)}$$



p_0, T_0 → DOLOČIMO IZ VRELIŠČA

$\hookrightarrow p_0 = 1\,\mathrm{bar}, T_0 = 100^\circ C$

$$p_{pNAS} = 1\,\mathrm{bar} \cdot e^{\frac{18 \cdot 2{,}3 \cdot 10^6\,J\,kg\,kmol\,K}{kg\,kmol \cdot 8314\,J\,373\,K}\left(1 - \frac{373}{293}\right)}$$

$$= 1\,\mathrm{bar}\, e^{\underbrace{\frac{18 \cdot 2{,}3 \cdot 10^6}{8314 \cdot 373}\left(1 - \frac{373}{293}\right)}_{= -3{,}64}}$$

$$\underline{p_{pNAS}(T) = 26{,}1\,mbar}$$

$$\eta = \frac{p_p}{p_{pNAS}} = \frac{1{,}22\,mbar}{26{,}1\,mbar} = \underline{4{,}67\%}$$

IZOTERMNO STISNEMO DA SE ZAČNEJO IZLOČATI KAPLJICE

$$V_2\,(p_{p2} = p_{pNAS}) = ?$$

$$p_{p2} V_2 = p_{p1} V_1 \qquad V_1 = 1\,m^3$$

$$V_2 = V_1 \cdot \frac{p_{p1}}{p_{pNAS}} = V_1 \cdot 0{,}0467 = 46{,}7\,l$$

13 Elektromagnetizem

13.1 Elektricno polje

ZBIRKA 9 nal 1/st 47

$$\vec{F}_e = \frac{e_1 e_2}{4\pi\varepsilon_0 r_{12}^2} \cdot \underbrace{\frac{\vec{r}_{12}}{r_{12}}}_{\text{SMER}}$$

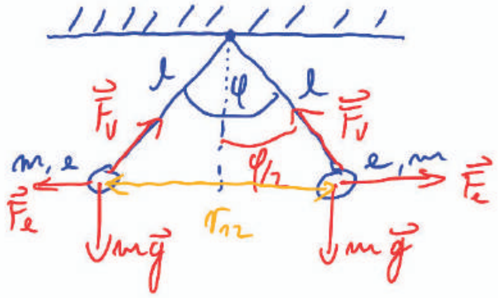
$$\varepsilon_0 = 8{,}85 \cdot 10^{-12} \frac{As^2}{Nm^2}$$

$l = 12\,cm$

$m = 1\,g$

$e = 10^{-6}\,As$

$\varphi = ?$



$\Rightarrow$

$$\Rightarrow \ tg\frac{\varphi}{2} = \frac{F_e}{mg}$$

$$\bullet\ F_e = \frac{e^2}{4\pi\varepsilon_0 r_{12}^2} = \frac{e^2}{4\pi\varepsilon_0 \cdot 4l^2 \sin^2\frac{\varphi}{2}}$$

$$r_{12} = 2 \cdot l \sin\frac{\varphi}{2}$$

$$tg\frac{\varphi}{2} = \underbrace{\frac{e^2}{16\pi\varepsilon_0 l^2 mg}}_{\alpha = 15{,}9} \frac{1}{\sin^2\frac{\varphi}{2}} = \alpha \cdot \frac{1}{\underbrace{\sin^2\frac{\varphi}{2}}_{\leq 1}} \geq \alpha$$

V PRVEM PRIBLIŽKU: $\frac{\varphi}{2} \Rightarrow$ BO VELIK

$\bullet\ tg\frac{\varphi}{2} \geq \alpha$

$\hookrightarrow \frac{\varphi}{2} \geq 86{,}4°$

$$tg\frac{\varphi}{2} = \alpha \frac{1}{\sin^2\frac{\varphi}{2}}$$

$$\frac{\sin\frac{\varphi}{2}}{\cos\frac{\varphi}{2}} = \alpha \frac{1}{\sin^2\frac{\varphi}{2}}$$

$$\cos\frac{\varphi}{2} = \frac{1}{\alpha}\sin^3\frac{\varphi}{2} \doteq \frac{1}{\alpha}$$

$\hookrightarrow \frac{\varphi}{2} = 86{,}3° \Rightarrow \underline{\varphi = 172{,}65°}$

$F_e \gg mg$

ZBIRKA 9 nal 2/st 48

$e = +3 \cdot 10^{-7} As$

$r_1 = 10\,cm$

$r_2 = 3\,cm$

$A = ?$

$F = \frac{e^2}{4\pi\varepsilon_0 r^2}$

$\Rightarrow dr = -ds$

$$A = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{s} = - \int_{r_1}^{r_2} \frac{e^2\, dr}{4\pi\varepsilon_0 r^2} = \frac{-e^2}{4\pi\varepsilon_0} \left(-\frac{1}{r}\right)\Big|_{r_1}^{r_2} =$$

$$\underline{A = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$\Rightarrow$ DELO JE ODVISNO OD ZAČETNEGA IN KONČNEGA POLOŽAJA

$$A = \Delta W_e = W_e(r_2) - W_e(r_1)$$

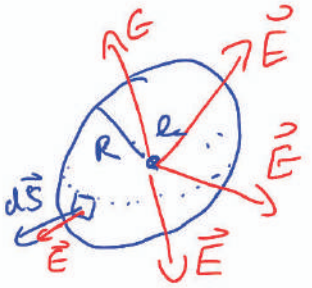
SPREMEMBA EL. POTENCIALNE ENERGIJE

$$\boxed{W_e = \frac{e_1 e_2}{4\pi\varepsilon_0 r}}$$

$\rightarrow$ EL. POTENCIALNA ENERGIJA

$$\underline{\vec{F}_e = -\vec{\nabla} W_e = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) W_e}$$

GAUSSOV IZREK:



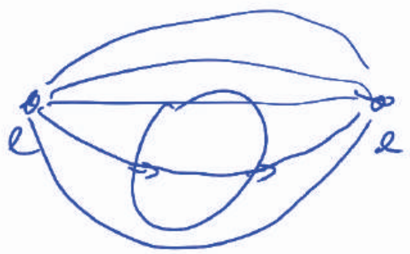
$$\boxed{\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = e}$$

GAUSSOV IZREK

$\hookrightarrow$ ZA KROGLO OKOLI TOČKASTEGA NABOJA

$d\vec{S} \parallel \vec{E}$

$\rightarrow$ KAJ ČE GLEDAMO POVRŠINO MED DVEMA NABOJEMA



$\Rightarrow$ ČE NE ZAOBJAMEMO NABOJEV

$$\oint \vec{E} \cdot d\vec{S} = 0$$

13.1. mal 3 ⇒ $\vec{E}, V$ ⇒ ZNOTRAJ IN ZUNAJ ENAKOMERNO NABITE KROGLE

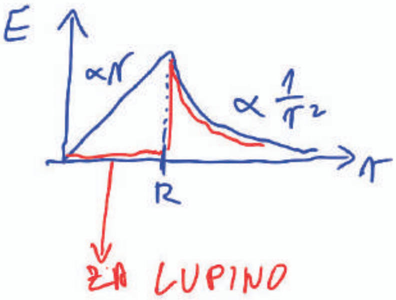
POLJE:

GOSTOTA NABOJA: $\boxed{\rho_e = \frac{e \cdot 3}{4\pi R^3}}$ POLNA KROGLA

$\vec{E} \parallel d\vec{S}$

• ZA $r<R$: $\varepsilon_0 \oint \vec{E}(\vec{r}) \cdot d\vec{S}(\vec{r}) = e(r) = e\frac{r^3}{R^3}$

$$\varepsilon_0 E(r) 4\pi \cancel{r^2} = e\frac{r^{\cancel{3}}}{R^3}$$

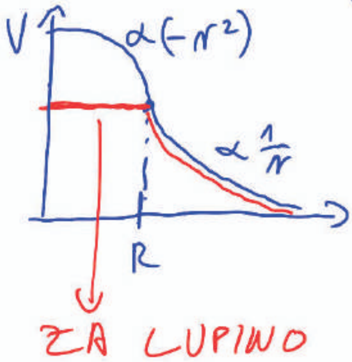


$$\boxed{E(r<R) = \frac{e \cdot r}{4\pi\varepsilon_0 R^3}}$$

• ZA $r>R$: $\varepsilon_0 E(r) 4\pi r^2 = e$

$$\boxed{E(r>R) = \frac{e}{4\pi\varepsilon_0 r^2}}$$

POTENCIAL: $\int_V^0 dV = -\int_r^\infty \vec{E} \cdot d\vec{r}$, $W_e = e \cdot V$



• ZA $r>R$: $\int_{V(r)}^{0} dV = -\int_r^\infty \vec{E} \cdot d\vec{r} = -\frac{e}{4\pi\varepsilon_0}\int_r^\infty \frac{dr}{r^2} =$

$$\boxed{V(r>R) = \frac{e}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{e}{4\pi\varepsilon_0 r}}$$

• ZA $r<R$: $\int_{V(r)}^{V(R)} dV = -\int_r^R \vec{E} \cdot d\vec{r} = -\frac{e}{4\pi\varepsilon_0 R^3}\left(\frac{R^2 - r^2}{2}\right)$

$$V(R) - V(r) = \frac{e}{8\pi\varepsilon_0 R}\left(\frac{r^2}{R^2} - 1\right)$$

$$V(r) = V(R) - \frac{e}{8\pi\varepsilon_0 R}\left(\frac{r^2}{R^2} - 1\right)$$

$$= \frac{e}{4\pi\varepsilon_0 R} - \frac{e}{8\pi\varepsilon_0 R}\left(\frac{r^2}{R^2} - 1\right)$$

$$\boxed{V(r<R) = \frac{e}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)}$$

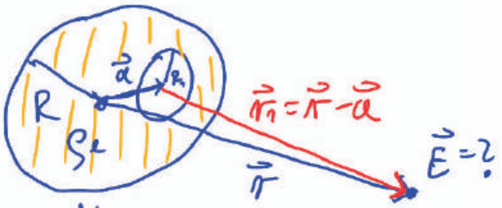
13.1 mol 4

GOSTOTA NABOJA ↓

R, R_1, a, ρ_e

$E = ?$

↳ ZNOTRAJ
ZUNAJ
V VOTLINI



$$\vec{E} = \vec{E}_R + \vec{E}_{R_1}$$

⇓

$$\vec{E}(\vec{r}) = \vec{E}_R(\vec{r}) + \vec{E}_{R_1}(\vec{r} - \vec{a})$$

⇓

RAZDELIMO NA DVE POLNI KROGLI:

• VELIKA: $R, e_R = \rho_e \cdot \frac{4\pi R^3}{3}$

• MALA: $R_1, e_{R_1} = (-)\rho_e \frac{4\pi R_1^3}{3}$

• ZUNAJ KROGLE: $\vec{E}_R = \frac{e_R \vec{r}}{4\pi\varepsilon_0 |\vec{r}|^3}$; $\vec{E}_{R_1} = \frac{e_{R_1}(\vec{r} - \vec{a})}{4\pi\varepsilon_0 |\vec{r} - \vec{a}|^3}$

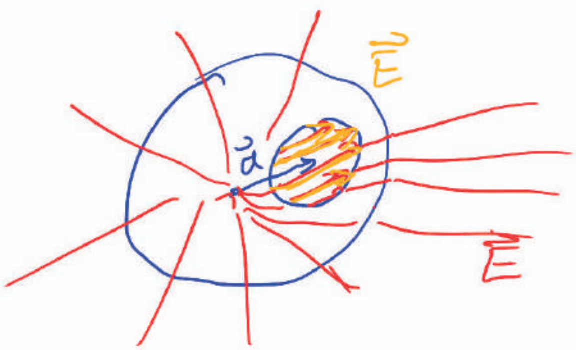
$$\vec{E} = \frac{\rho_e \cancel{4\pi} R^3 \cdot \vec{r}}{3 \cdot \cancel{4\pi}\varepsilon_0 r^3} + \frac{-\rho_e \cancel{4\pi} R_1^3 (\vec{r} - \vec{a})}{3 \cdot \cancel{4\pi}\varepsilon_0 |\vec{r} - \vec{a}|^3} = \frac{\rho_e R^3}{3\varepsilon_0}\left[\frac{\vec{r}}{r^3} - \frac{(\vec{r} - \vec{a})}{|\vec{r} - \vec{a}|^3}\left(\frac{R_1}{R}\right)^3\right]$$

• ZNOTRAJ KROGLE: $\vec{E}_R = \frac{e_R \vec{r}}{4\pi\varepsilon_0 R^3}$; $\vec{E}_{R_1} = \frac{e_{R_1}(\vec{r} - \vec{a})}{4\pi\varepsilon_0 |\vec{r} - \vec{a}|^3}$

$$\vec{E} = \frac{\rho_e \cancel{4\pi R^3}\, \vec{r}}{3 \cdot \cancel{4\pi}\varepsilon_0 \cancel{R^3}} + \frac{-\rho_e \cancel{4\pi} R_1^3 (\vec{r} - \vec{a})}{3 \cdot \cancel{4\pi}\varepsilon_0 |\vec{r} - \vec{a}|^3} = \frac{\rho_e}{3\varepsilon_0}\left[\vec{r} - \frac{(\vec{r} - \vec{a})}{|\vec{r} - \vec{a}|^3} R_1^3\right]$$

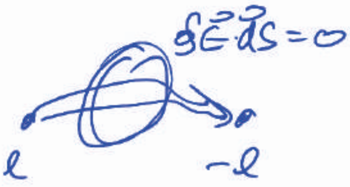
• V VOTLINI: $\vec{E}_R = \frac{e_R \vec{r}}{4\pi\varepsilon_0 R^3}$, $\vec{E}_{R_1} = \frac{e_{R_1}(\vec{r} - \vec{a})}{4\pi\varepsilon_0 R_1^3}$

$$\vec{E} = \frac{\rho_e \cancel{4\pi R^3}\, \vec{r}}{3 \cdot \cancel{4\pi}\varepsilon_0 \cancel{R^3}} + \frac{-\rho_e \cancel{4\pi R_1^3} (\vec{r} - \vec{a})}{3 \cdot \cancel{4\pi}\varepsilon_0 \cancel{R_1^3}} = \frac{\rho_e}{3\varepsilon_0}\left[\vec{r} - \vec{r} + \vec{a}\right] = \frac{\rho_e}{3\varepsilon_0} \cdot \vec{a}$$



DN: $\vec{r} = (x, 0, 0)$
$\vec{a} = (a, 0, 0)$
$E(\vec{r})$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = e$$

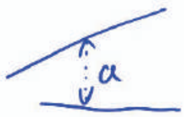


91/92 kol 3/nal 2

$a = 5\ \text{cm}$

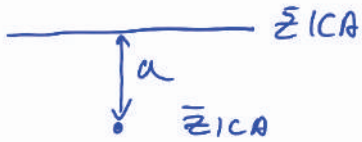
$\mu = 10^{-5}\ \text{As/m}$

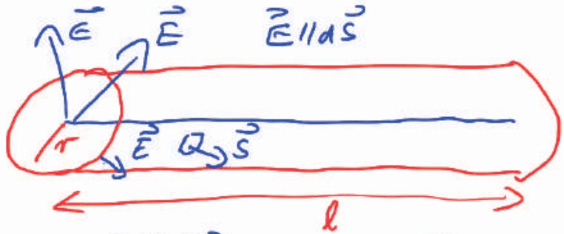
$F = ?$



$\vec{F}_e = e\vec{E}$

OD STRANI:





$\oint \vec{E}\cdot d\vec{S} = E\cdot S = E\cdot 2\pi r\cdot l$

· EL. POLJE ENE ŽICE:

$\varepsilon_0 \oint \vec{E}\cdot d\vec{S} = e$

$\varepsilon_0 E\, 2\pi r l = e$

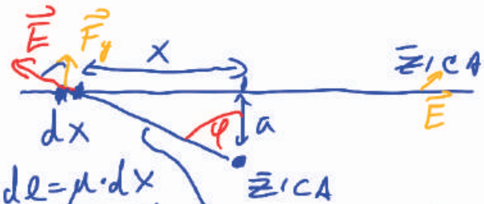
$E = \frac{e}{2\pi r \varepsilon_0 l}$; $\frac{e}{l} = \mu$

$$\boxed{E = \frac{\mu}{2\pi\varepsilon_0 r} \; ; \; \mu = \frac{e}{l}}$$

· SILA:

$d\vec{F} = \vec{E}\cdot de$

$dF_y = dF\cdot\cos\varphi$



$\underline{de = \mu\cdot dx}$

$\underline{r = \sqrt{a^2+x^2} = \frac{a}{\cos\varphi}}$

$\operatorname{tg}\varphi = \frac{x}{a} \; /'$

$\frac{d\varphi}{\cos^2\varphi} = \frac{dx}{a}$

$\underline{dx = \frac{a\, d\varphi}{\cos^2\varphi}}$

$F_y = \int E\cdot de\cos\varphi$

$= \int_{-\pi/2}^{\pi/2} \frac{\mu \cancel{\cos\varphi}\cdot\mu\cdot\cancel{a}\, d\varphi\cdot\cancel{\cos\varphi}}{2\pi\varepsilon_0 \cancel{a}\, \cancel{\cos^2\varphi}}$

$= \frac{\mu^2}{2\cancel{\pi}\varepsilon_0}\cancel{\left(\frac{\pi}{2}+\frac{\pi}{2}\right)}$

$\underline{F_y = \frac{\mu^2}{2\varepsilon_0}} = \underline{5{,}65\ \text{N}}$

95/96 kol. 3/nal 4

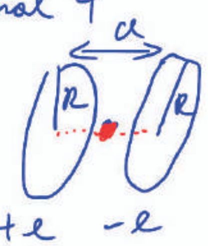
$R = 2\ \text{cm}$

$a = 5\ \text{cm}$

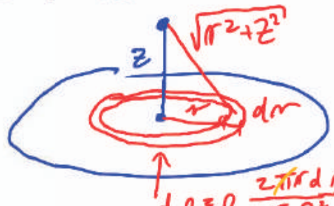
$e = 10^{-10}\ \text{As}$

E (V SREDINI) = ?

E (DALEČ NA OSI) = ?



ENA PLOŠČA



$de = e\,\dfrac{2\cancel{\pi} r\,dr}{\cancel{\pi} R^2}$

$\vec{F} = -\vec{\nabla} W_e$

$\boxed{\vec{E} = -\vec{\nabla} V}$

TOČKAST NABOJ: $V = \dfrac{e}{4\pi\varepsilon_0 r}$

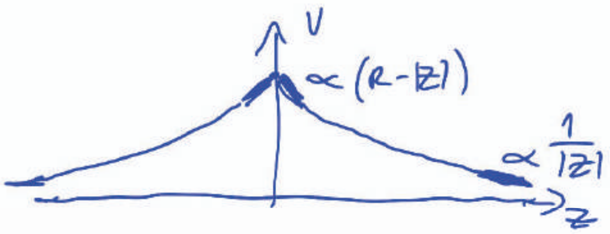
• PLOŠČA:

POTENCIAL: $$V = \int \frac{de}{4\pi\varepsilon_0\sqrt{r^2+z^2}} = \int_0^R \frac{2e\,r\,dr}{4\pi\varepsilon_0\sqrt{r^2+z^2}\,R^2}$$

$$= \frac{e}{4\pi\varepsilon_0 R^2}\int_{z^2}^{R^2+z^2}\frac{du}{\sqrt{u}} \qquad u = r^2+z^2,\quad du = 2r\,dr$$

$$= \frac{e}{\underset{2}{\cancel{4}}\pi\varepsilon_0 R^2}\,\cancel{2}\left(\sqrt{R^2+z^2}-|z|\right)$$

$$\boxed{V = \frac{e}{2\pi\varepsilon_0 R^2}\left(\sqrt{R^2+z^2}-|z|\right)}$$



• ZA $|z| \ll R$: $V = \dfrac{e}{2\pi\varepsilon_0 R^2}\left(R\sqrt{1+\cancel{\frac{z^2}{R^2}}^{\,0}}-|z|\right) = \underline{\dfrac{e}{2\pi\varepsilon_0 R^2}\left(R-|z|\right)}$

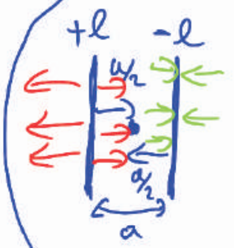
• ZA $z \gg R$: $V = \dfrac{e}{2\pi\varepsilon_0 R^2}\left(|z|\sqrt{1+\frac{R^2}{z^2}}-|z|\right) \doteq \dfrac{e}{2\pi\varepsilon_0 R^2}\left[|z|\left(\cancel{1}+\frac{1}{2}\frac{R^2}{z^2}\right)-\cancel{|z|}\right]$

$$= \frac{e}{2\pi\varepsilon_0 R^2}\left(\frac{R^2}{2|z|}\right) = \frac{e}{4\pi\varepsilon_0 |z|}$$

• POLJE:

$\vec{E} = -\vec{\nabla} V$, KER $V = V(z) \Rightarrow \vec{E} = \vec{E}(z) \Rightarrow E = E_z = -\dfrac{\partial V}{\partial z}$

$$E = -\frac{e}{2\pi\varepsilon_0 R^2}\frac{\partial}{\partial z}\left(\sqrt{R^2+z^2}-|z|\right) = -\frac{e}{2\pi\varepsilon_0 R^2}\left(\frac{1}{\cancel{2}}\frac{\cancel{2}z}{\sqrt{R^2+z^2}}-\frac{z}{|z|}\right)$$



$$E_{SKUPNI} = E_+\left(\frac{a}{2}\right)+E_-\left(-\frac{a}{2}\right) = -\frac{e}{2\pi\varepsilon_0 R^2}\left(\frac{\frac{a}{2}}{\sqrt{R^2+\left(\frac{a}{2}\right)^2}}-1\right)+\frac{-(-e)}{2\pi\varepsilon_0 R^2}\left[\frac{-\frac{a}{2}}{\sqrt{R^2+\left(\frac{a}{2}\right)^2}}-(-1)\right]$$

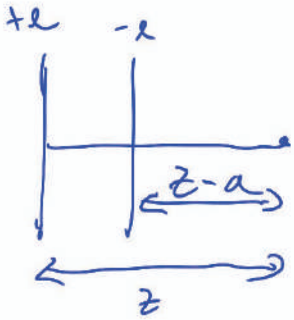
$$= \frac{-\cancel{2}e}{\cancel{2}\pi\varepsilon_0}\left(\frac{\frac{a}{2}}{\sqrt{R^2+\left(\frac{a}{2}\right)^2}}-1\right) \rightarrow \text{V SREDINI}$$

NESKONČNA PLOŠČA:

$$E(z \ll R) = -\frac{e}{2\pi\varepsilon_0 R^2}\left(\cancel{\frac{z}{\sqrt{R^2+z^2}}}^{\,0} - \cancel{\frac{z}{|z|}}^{\,1}\right) = \frac{e}{2\pi\varepsilon_0 R^2} = \frac{\sigma}{2\varepsilon_0}$$

$\sigma = \frac{e}{S} \rightarrow$ POVRŠINSKA GOSTOTA e

POLJE NAD NESKONČNO PLOŠČO



NA OSI:

$$E = E_+(z) + E_-\left(z - \frac{a}{2}\right) = \frac{e}{2\pi\varepsilon_0 R^2}\left[-\frac{z}{\sqrt{R^2+z^2}} + \cancel{1} + \frac{z-a}{\sqrt{R^2+(z-a)^2}} - \cancel{1}\right]$$

$z \gg R$

$$= \frac{e}{2\pi\varepsilon_0 R^2}\left[-\frac{\cancel{z}}{\cancel{z}\sqrt{1+\frac{R^2}{z^2}}} + \frac{z-a}{(z-a)\sqrt{1+\frac{R^2}{(z-a)^2}}}\right] =$$

$$= \frac{e}{2\pi\varepsilon_0 R^2}\left[-\left(1-\frac{1}{2}\frac{R^2}{z^2}\right) + \left(1-\frac{1}{2}\frac{R^2}{(z-a)^2}\right)\right] =$$

$$= \frac{e}{2\pi\varepsilon_0 R^2}\,R^2\,\frac{1}{2}\left[\frac{1}{z^2} - \frac{1}{(z-a)^2}\right] = \frac{e}{4\pi\varepsilon_0 z^2}\left[1 - \frac{1}{\left(1-\frac{a}{z}\right)^2}\right]$$

$$= \frac{e}{4\pi\varepsilon_0 z^2}\left[1 - \left(1 + 2\frac{a}{z}\right)\right] = \underline{\underline{\frac{-ea}{2\pi\varepsilon_0 z^3}}}$$

VPRAŠANJA:

$S = 2\,cm^2$

$\rho = 500\,kg/m^3$

$F = 30\,N$

$u(x,t)$

$\underline{u(x=0,t) = At^2 - Bt^3}$, $A = 1\,cm/s^2$, $B = 1\,cm/s^3$

$u(x=2\,m, t=0{,}6\,s)$ $\quad 0 < t < 1\,s$:

$c = \sqrt{\frac{F}{\rho S}} \Rightarrow$

$u(x,t) = u(x-ct, 0) = u(0, t-\frac{x}{c})$; $\underline{u = A\sin(kx - \omega t + \phi)}$

$u(x,t) = f(x-ct) = g(x)\cdot h(t)$

$u(0,0) = 0$

$u(0,t) = g(0)(At^2 - Bt^3)$

$\boxed{\Delta S = 0}$

SOD

$h = 1\,m$

$r = 25\,cm$

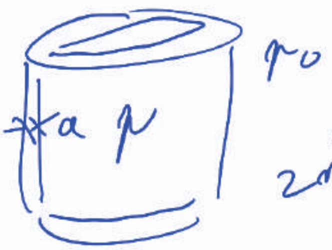
$p = 2\,bar$

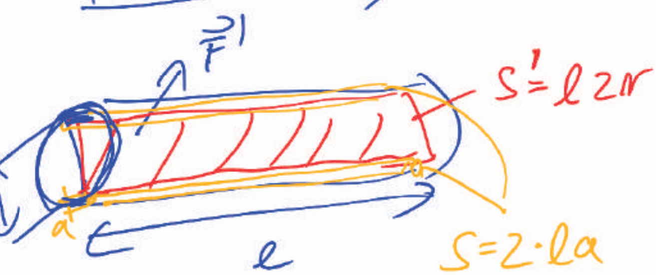
$a = 0{,}5\,mm$

$E = 100\cdot10^9\,Pa$

$\rho = 8700\,kg/m^3$

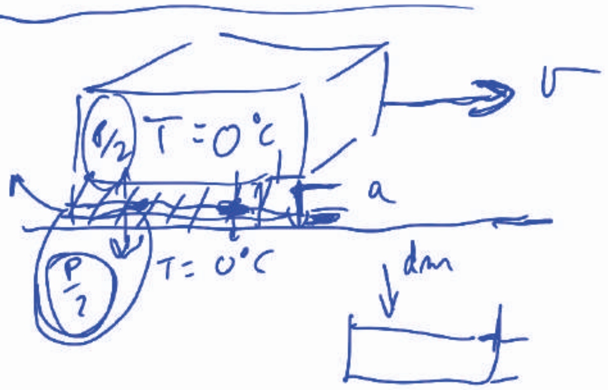
$p_0 = 1\,bar$





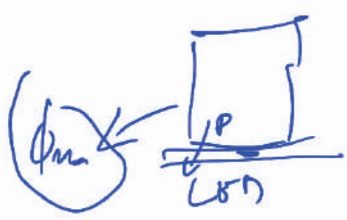
$\underline{F' = \Delta p\cdot S'} = ES$

$F' = SE\frac{\Delta(2\pi r)}{2\pi r} = SE\frac{\Delta r}{r} \Rightarrow \Delta r = r\frac{\Delta p S'}{ES}$



$F_t = \frac{\eta S v}{a}$; $\underline{P = F_t\cdot v}$

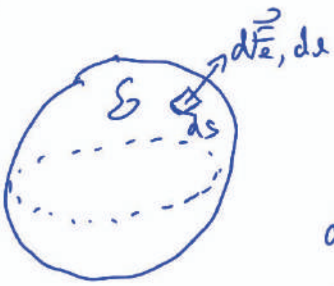
$P = \frac{dQ}{dt} =$



(13.1) nal 7

$\sigma > 0$

$p = ?$



$\vec{F}_e = e\vec{E}$; $p = \frac{dF}{dS}$

$dF = de\,E$; • EL. POLJE NAD PLOŠČO RADIJA R

$E(z << R) = \frac{\sigma}{2\varepsilon_0}$

↑ ODMIK OD PLOŠČE

↳ KER GLEDAMO E TIK OB KROGLI

$\underline{E = \frac{\sigma}{2\varepsilon_0}} \Rightarrow \vec{E} \parallel \vec{r}$

$\underline{dF = \frac{de \cdot \sigma}{2\varepsilon_0}}$

$$p = \frac{dF}{dS} = \underbrace{\frac{de}{dS}}_{=\sigma} \cdot \frac{\sigma}{2\varepsilon_0} = \underline{\underline{\frac{\sigma^2}{2\varepsilon_0}}}$$

• DRUG NAČIN → PREKO A IN W_e

$$dA = -p\,dV \Rightarrow p = -\frac{dA}{dV} = -\frac{dW_e}{dV}$$

$$\bullet\ A = \int \Delta V \cdot de = \frac{1}{4\pi\varepsilon_0 r}\int_0^e e\,de = \underline{\frac{e^2}{8\pi\varepsilon_0 r} = A = W_e} \leftarrow \text{EL. ENERGIJA KROGLE}$$

; $\Delta V = \frac{e}{4\pi\varepsilon_0 r}$

• $\underline{dV = 4\pi r^2 dr}$

TLAK: KAKO SE SPREMENI W_e, KO SPREMENIMO V:

$$p = -\frac{1}{4\pi r^2}\frac{d\left(\frac{e^2}{8\pi\varepsilon_0 r}\right)}{dr} = -\frac{1}{4\pi r^2}\frac{e^2}{8\pi\varepsilon_0}\left(-\frac{1}{r^2}\right)$$

$$= \frac{e^2}{32\pi^2\varepsilon_0 r^4} = \frac{e^2}{2 \cdot S^2 \varepsilon_0} = \underline{\underline{\frac{\sigma^2}{2\varepsilon_0}}}$$

$\frac{e^2}{S^2} = \sigma^2$

POVRŠINA $S = 4\pi r^2$

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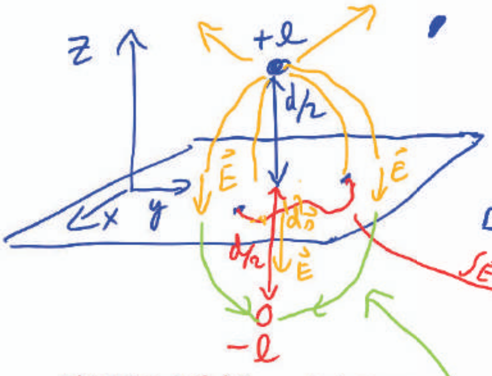
PREVODNIK:

$\vec{E}(x,y,0)=?$

$\sigma(x,y)=?$

a) NABOJ: $z=\frac{d}{2}$; $e>0$

b) ŽICA: $z=\frac{d}{2}$; $\mu>0$



POVSOD V PREVODNIKU JE EL. POTENCIAL ENAK

$\Delta V=\int \vec{E}\cdot d\vec{s}=0$ ↵

POT PO PREVODNIKU

PO PREVODNI POVRŠINI

↳ $\vec{E}\perp d\vec{s}$

↳ $\boxed{\vec{E}\perp \text{ NA PREVODNO POVRŠINO!}}$

NAVIDEZNI NABOJ

NAVIDEZNE SILNICE

⇓

ZRCALJENJE NABOJEV PREKO PREVODNE POVRŠINE

→ V RESNICI PA SE NA POVRŠINI PREVODNIKA NABERE NABOJ, KI ODGOVARJA:

- $\vec{E}\perp$ POVRŠINO TIK NAD PREVODNIKOM
- $\vec{E}=0$ V PREVODNIKU

POTENCIAL OBEH NABOJEV NAD PREVODNIKOM:

$$\hookrightarrow V(\vec{r})=V_+(\vec{r}_+)+V_-(\vec{r}_-)=\frac{e}{4\pi\varepsilon_0\sqrt{x^2+y^2+(z-\frac{d}{2})^2}}+\frac{-e}{4\pi\varepsilon_0\sqrt{x^2+y^2+(z+\frac{d}{2})^2}}$$

↳ PREVERIMO LIMITE:

$$V(0,0,0)=\frac{e}{4\pi\varepsilon_0}\left(\frac{1}{(\frac{d}{2})^2}-\frac{1}{(\frac{d}{2})^2}\right)=0 \checkmark$$

$$V(x,y,\infty)=\frac{e}{4\pi\varepsilon_0}\left(\frac{1}{\infty}-\frac{1}{\infty}\right)=0 \checkmark$$

ZVEZA MED V IN ρ_e (GOSTOTA EL. NABOJA):

$$\Rightarrow \oint \vec{E}\cdot d\vec{S}=\frac{e}{\varepsilon_0} \Rightarrow \underbrace{\vec{\nabla}\cdot\vec{E}}=\frac{\rho_e}{\varepsilon_0}$$

↳ $\mathrm{div}\,\vec{E}$ → DIVERGENCA $\vec{E}$

$$V=-\int \vec{E}\cdot d\vec{s} \Rightarrow \vec{E}=-\underbrace{\vec{\nabla}V}$$

↳ $\mathrm{grad}\,V$ → GRADIENT V

GRADIENT:

$$\vec{\nabla}=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)$$

$$\vec{\nabla}(-\vec{\nabla}V)=\frac{\rho_e}{\varepsilon_0} \Rightarrow \boxed{\nabla^2 V=-\frac{\rho_e}{\varepsilon_0}}$$ ← POISSONOVA ENAČBA

NABLA:

$$\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$$

IMA SAMO ENO REŠITEV

↳ ZATO JE NAŠA REŠITEV EDINA IN PRAVA!

• EL. POLJE NAD PREVODNO PLOŠČO:

OD ZGORAJ

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial z} = -\frac{e}{4\pi\varepsilon_0}\left[\frac{-\frac{1}{2}\cancel{2}(z-\frac{d}{2})}{\sqrt{x^2+y^2+(z-\frac{d}{2})^2}^{\,3}} - \frac{-\frac{1}{2}\cancel{2}(z+\frac{d}{2})}{\sqrt{x^2+y^2+(z+\frac{d}{2})^2}^{\,3}}\right]$$

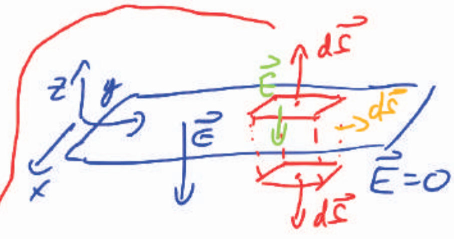
$\vec{E} \perp$ POVRŠINO $\Rightarrow \vec{E} = (0, 0, E_z)$

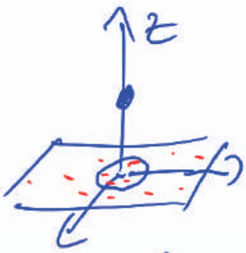
$$\bullet\; E(z=0) = -\frac{e}{4\pi\varepsilon_0}\left[\frac{d}{\sqrt{x^2+y^2+(\frac{d}{2})^2}^{\,3}}\right] = E(x,y,0) \Rightarrow \text{U SMERI } z$$

• GOSTOTA NABOJA:

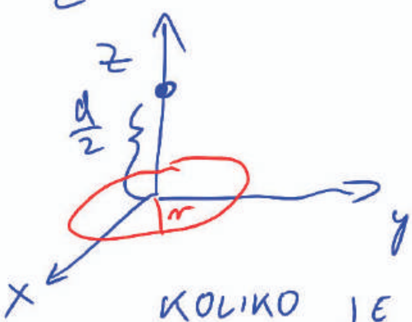
$$\oint \vec{E}\cdot d\vec{S} = -|E|S = \frac{e}{\varepsilon_0} \Rightarrow |E| = -\frac{e}{S\varepsilon_0} = -\frac{\sigma}{\varepsilon_0}$$

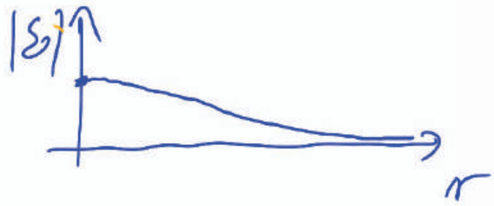
SAMO ZGORNJA POVRŠINA PRISPEVA





$$\sigma = -|E|\cdot\varepsilon_0 = -\frac{e}{4\pi}\left[\frac{d}{\sqrt{x^2+y^2+(\frac{d}{2})^2}^{\,3}}\right] = \sigma(x,y)$$

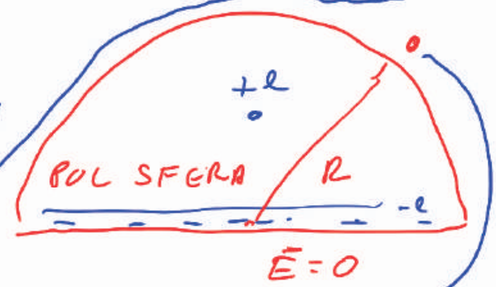




$$\varepsilon_0 \oint \vec{E}\cdot d\vec{S} = 0 = +e - e$$

KOLIKO JE VSEGA NABOJA NA POVRŠINI

$\hookrightarrow$ GA JE TOČNO ZA $-e$

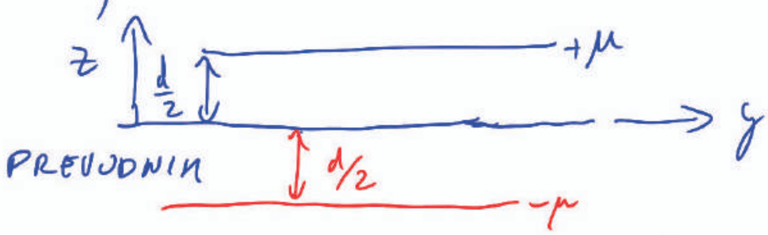


$$E(R\to\infty) \propto \frac{1}{R^3}$$

$$S(R\to\infty) \propto R^2$$

$$\hookrightarrow \int \vec{E}\cdot d\vec{S} \propto \frac{1}{R} \to 0$$

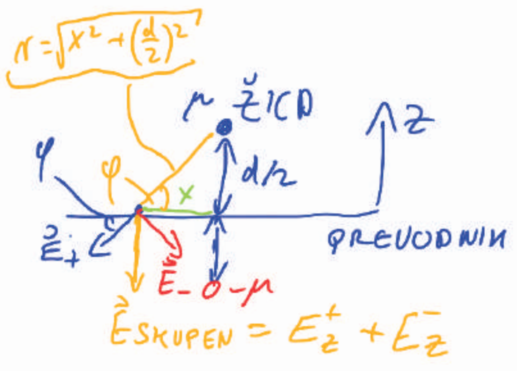
e) NABITA ŽICA



ZA ŽICO: $\vec{E} = \frac{\mu}{2\pi\varepsilon_0 r}\cdot\frac{\vec{r}}{r}$

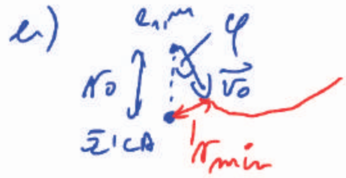
$$E_z = E\cdot\sin\varphi = E\cdot\frac{\frac{d}{2}}{\underbrace{\sqrt{x^2+(\frac{d}{2})^2}}_{=r}}$$

$$|E_z| = \frac{\mu\frac{d}{2}}{2\pi\varepsilon_0(x^2+(\frac{d}{2})^2)}$$



$$E_{SKUPEN} = E_z^+ + E_z^- = -2\cdot E_z = \frac{-\mu d}{2\pi\varepsilon_0\left[x^2+(\frac{d}{2})^2\right]}, \quad \sigma = -|E|\cdot\varepsilon_0$$

(13.1) nal 9



$\mu = 10^{-6}$ As/m

$e_1 = 10^{-10}$ As

$\varphi = 30°$

$r_0 = 10$ cm

$v_0 = 1$ m/s

$m = 0{,}01$ g

$r_{min} = ?$

a) SILE: $\vec F_e = e_1 \vec E = \frac{e_1 \mu \vec r}{2\pi\varepsilon_0 r^2}$

$m a_\perp = \frac{e_1\mu}{2\pi\varepsilon_0 r}$; $a_\perp = \frac{dv_\perp \cdot dr}{dt \cdot dr} = v_\perp \frac{dv_\perp}{dr}$ $\quad \frac{dr}{dt} = v_\perp$

$m v_\perp \frac{dv_\perp}{dr} = \frac{e_1\mu}{2\pi\varepsilon_0 r}$

$m\int_{v_0 \sin\varphi}^{0} v_\perp \, dv_\perp = \frac{e_1\mu}{2\pi\varepsilon_0}\int_{r_0}^{r_{min}} \frac{dr}{r}$ $\qquad v_{\perp 0} = v_0 \sin\varphi$

$\underbrace{-\frac{m(v_0 \sin\varphi)^2}{2}}_{\Delta W_k} = \underbrace{\frac{e_1\mu}{2\pi\varepsilon_0}\ln\frac{r_{min}}{r_0}}_{\Delta W_e}$; $\Delta W_e = e_1 \Delta V$

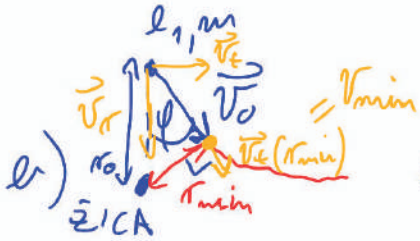
$\hookrightarrow \boxed{\Delta V = \frac{\mu}{2\pi\varepsilon_0}\ln\frac{r_1}{r_2}}$

EL. POTENCIAL NABITE ŽICE

$-\frac{m(v_0 \sin\varphi)^2 \cancel{2}\pi\varepsilon_0}{\cancel{2} e_1 \mu} = \ln\frac{r_{min}}{r_0}$

$r_{min} = r_0 \underbrace{e^{-\frac{m v_0^2 \sin^2\varphi \, \pi\varepsilon_0}{e_1\mu}}}_{0.5}$

$r_{min} = 5$ cm



ENERGIJE: $\Delta W_k + \Delta W_e = 0$

$\Delta W_k = -\Delta W_e$

$\frac{m v_{min}^2}{\cancel{2}} - \frac{m v_0^2}{\cancel{2}} = +\frac{e_1\mu}{\cancel{2}\pi\varepsilon_0}\ln\frac{r_{min}}{r_0}$

OHRANITEV Γ:

$r_0 v_0 \sin\varphi = r_{min} v_{min}$

$v_{min} = v_0 \sin\varphi \frac{r_0}{r_{min}}$

$\frac{m v_0^2 \sin^2\varphi \, r_0^2}{r_{min}^2} - m v_0^2 = +\frac{e_1\mu}{\pi\varepsilon_0}\ln\frac{r_{min}}{r_0}$

$\frac{r_{min}}{r_0} = X$

BREZ DIMENZIJSKO

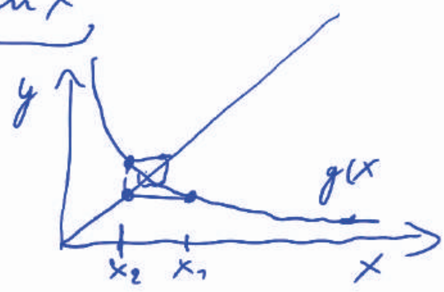
$\underbrace{\sin^2\varphi}_{=1/4} \cdot \frac{1}{X^2} = 1 + \underbrace{\frac{e_1\mu}{\pi\varepsilon_0 m v_0^2}}_{A = 0,36} \cdot \ln X$

$\frac{1}{4X^2} = 1 + A \ln X$

ITERACIJA: $X = g(X)$

$X = \underbrace{\frac{1}{2}\sqrt{\frac{1}{1 + A\ln X}}}_{g(X)}$

$X \to 0{,}56\ldots$



ZBIRKA 9 nal 14/st 49

$r_a = 2\ cm$

$r_b = 6\ cm$

$U = 6000\ V$

$e_0 = 1{,}6 \cdot 10^{-19}\ As$

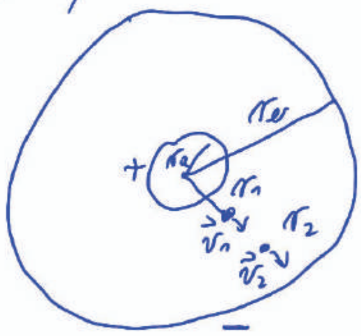
$m = 1{,}67 \cdot 10^{-27}\ kg$

$r_1 = 4\ cm$

$v_1 = 300\ km/s$

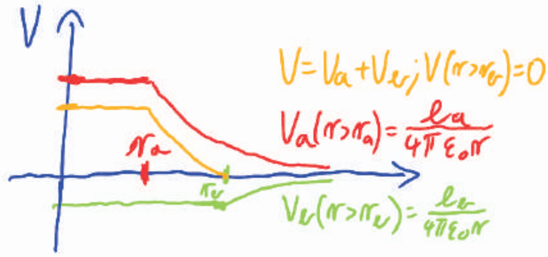
$r_2 = 5\ cm$

$v_2 = ?$



$W_e = e_0 \cdot V$

$$V = \frac{e_K}{4\pi\varepsilon_0 r} + V_{b0}$$



• ENERGIJE: $|\Delta W_k| = |\Delta W_e|$, $W = W_k + W_e$

$$W_{ZAČ} = W_{KONČNA}$$

$$\frac{m v_1^2}{2} + \left(\frac{e_K}{4\pi\varepsilon_0 r_1} + \cancel{V_{b0}}\right) e_0 = \frac{m v_2^2}{2} + \left(\frac{e_K}{4\pi\varepsilon_0 r_2} + \cancel{V_{b0}}\right) e_0$$

$$\frac{m v_1^2}{2} + \frac{\cancel{4\pi\varepsilon_0}\, U \cdot e_0}{\left(\frac{1}{r_a} - \frac{1}{r_b}\right)\cancel{4\pi\varepsilon_0}\, r_1} = \frac{m v_2^2}{2} + \frac{U e_0}{\left(\frac{1}{r_a} - \frac{1}{r_b}\right) r_2}$$

$$\boxed{v_2 = \sqrt{v_1^2 + \frac{2 U e_0}{m\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}}$$

$$v_2 = 512\ m/s$$

• NAPETOST:

$$U = V_a - V_b = \frac{e_K}{4\pi\varepsilon_0 r_a} + V_{b0} - \frac{e_K}{4\pi\varepsilon_0 r_b} - V_{b0}$$

$$U = \frac{e_K}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = e_K \frac{1}{C}$$

$$\boxed{e = C \cdot U}$$

$$\boxed{C = 4\pi\varepsilon_0 \left(\frac{1}{r_a} - \frac{1}{r_b}\right)^{-1}}$$

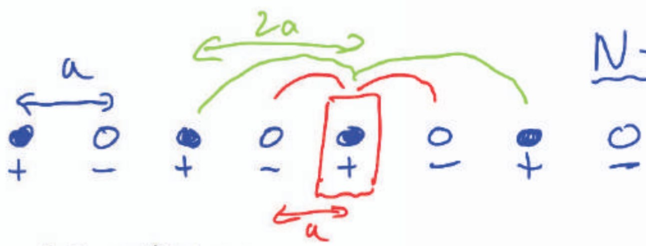
KAPACITETA KROGELNEGA KONDENZATORJA

$$e_K = \frac{4\pi\varepsilon_0 U}{\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}$$

13.1 nal 12

$a = 2{,}38 \cdot \overbrace{10^{-10}\,m}^{}$: $Å = 10^{-10}\,m$ → ANGSTROM ~ MEDATOMSKA RAZDALJA

$W_e/par = ?$



N → VSEH IONOV

$W_{SKUPNI} = W_{e_1} \cdot N$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots$

KONVERGIRA ZA:
$-1 < x \leq 1$

ZA ENEGA:

$$W_{e_1} = \left[-2\frac{e^2}{4\pi\varepsilon_0}\frac{1}{a} + 2\frac{e^2}{4\pi\varepsilon_0}\frac{1}{2a} - 2\frac{e^2}{4\pi\varepsilon_0}\frac{1}{3a} + \dots\right] \cdot \frac{1}{2}$$

KER BI DRUGAČE VSE ŠTELI DVAKRAT KO BI SEŠTELI

$W_{SKUPNI} = W_{e_1} \cdot N$

↳ $\underbrace{W_e/par = \cdot 2W_{e_1}}$ $\qquad \frac{W_e}{par} = \frac{W_{SKUPN}}{\frac{N}{2}}$

$$\frac{W_e}{par} = -\frac{e^2}{2\pi\varepsilon_0 a}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}\dots\right] = -\frac{e^2}{2\pi\varepsilon_0 a}\ln(1+1)$$

$$\underbrace{\frac{W_e}{par} = -\frac{e^2}{2\pi\varepsilon_0 a}\ln(2)}$$

↳ VEZAN PAR

$|W_{e\,VEZAN}| < |W_{e\,IZOLIRAN}|$

• IZOLIRAN PAR:

$$\underbrace{W_e = -\frac{e^2}{2\pi\varepsilon_0 a}}$$

13.1 nal 13

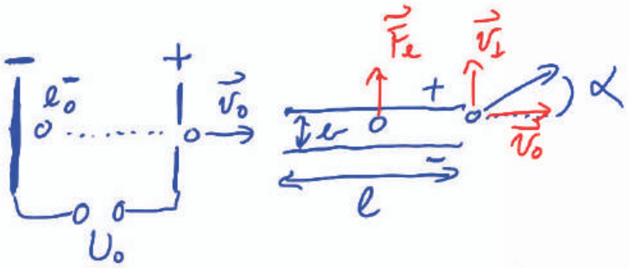
$U_0 = 3\,V$

$b = 2\,mm$

$l = 1\,cm$

$U = 0.2\,V$

$\alpha = ?$



$\operatorname{tg}\alpha = \frac{v_\perp}{v_0}$

• ZAČETNA HITROST: $\Delta W_k = \Delta W_e$ $\quad W_e = e U_0$

$$\frac{m v_0^2}{2} = e_0 U_0$$

$$\hookrightarrow \boxed{v_0 = \sqrt{\frac{2 e_0 U_0}{m}}}$$

• PRAVOKOTNA HITROST:

$$E_{PLOŠČATI} = \frac{U}{b} \longrightarrow m a_\perp = F_e = e_0 E$$

$$\boxed{a_\perp = \frac{e_0 U}{m b}}$$

$E_- = \frac{\sigma}{2\varepsilon_0}$

$E_+ = \frac{\sigma}{2\varepsilon_0}$

$E = E_+ + E_- = \frac{\sigma}{\varepsilon_0} = konst$

$$\boxed{v_\perp} = a_\perp \cdot t = \boxed{\frac{e_0 U l}{m b v_0}}$$

•ČAS LETA:

$$\boxed{t = \frac{l}{v_0}}$$

KOT: $\operatorname{tg}\alpha = \frac{v_\perp}{v_0} = \frac{e_0 U l}{m b v_0^2} = \frac{\cancel{e_0} U l \cancel{m}}{\cancel{m} b\, 2 \cancel{e_0} U_0}$

$$\boxed{\operatorname{tg}\alpha = \frac{l U}{2 b U_0}} \Rightarrow \underline{\alpha = 9{,}5^\circ}$$

13.1 nal 15

$R = 1\,m$

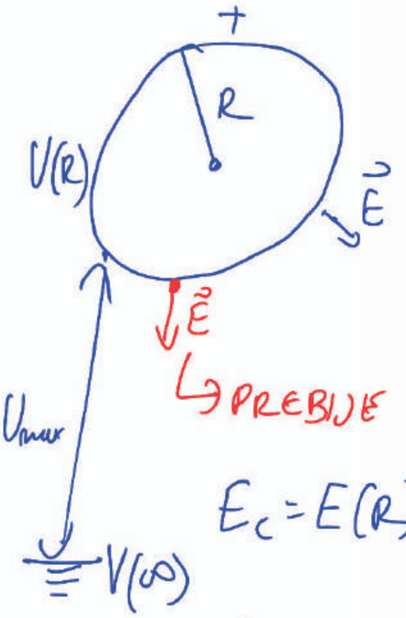
$E_c = 3\cdot 10^6\, V/m$

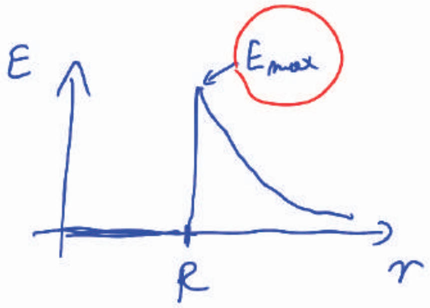
↳ PREBOJNA JAKOST E

↳ TAKRAT ZRAK IONIZIRA IN ZAČNE PREVAJATI

$e_{max} = ?$

$U_{max} = ?$





$$E(r>R) = \frac{e}{4\pi\varepsilon_0 r^2}$$

↳ PREBIJE TIK OB POVRŠINI:

$$E_c = E(R) = \frac{e_{max}}{4\pi\varepsilon_0 R^2}$$

$$e_{max} = 4\pi\varepsilon_0 R^2 E_c = \underline{\underline{3{,}4\cdot 10^{-4}\,As}}$$

$$U_{max} = \Delta V = V(R) - \underbrace{V(\infty)}_{=0} = \frac{e_{max}}{4\pi\varepsilon_0 R} = E_c \cdot R = \underline{\underline{3\cdot 10^6\,V}}$$

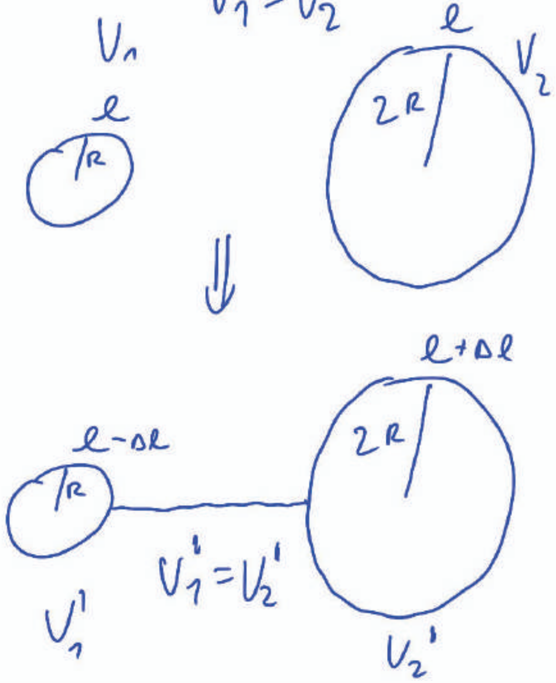
13.1 mal 16

$R, 2R$

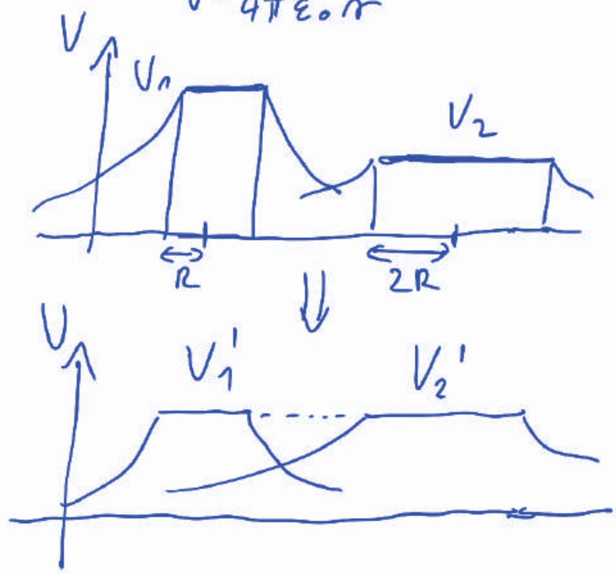
$e > 0$

Δe

$V_1 > V_2$



$V = \frac{e}{4\pi\varepsilon_0 r}$



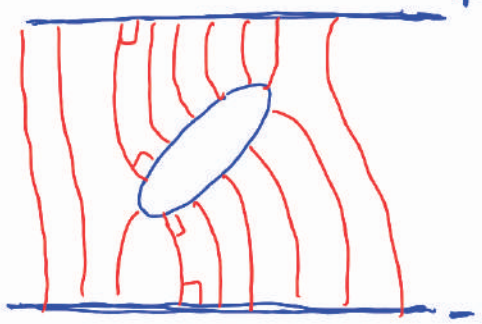
$$V_1' = V_2'$$

$$\frac{e-\Delta e}{\cancel{4\pi\varepsilon_0}\cancel{R}} = \frac{e+\Delta e}{\cancel{4\pi\varepsilon_0}2\cancel{R}}$$

$$2e - 2\Delta e = e + \Delta e$$

$$\boxed{\Delta e = \frac{e}{3}}$$

(13.1) nal 17



KOS KOVINE V KONDENZATORJU NARIŠI $\vec{E}$

$\boxed{\vec{E} \perp \text{EKVIPOTENCIALNO PLOSKEV}}$

$$\boxed{\Delta V = \int \vec{E} \cdot d\vec{s}}$$

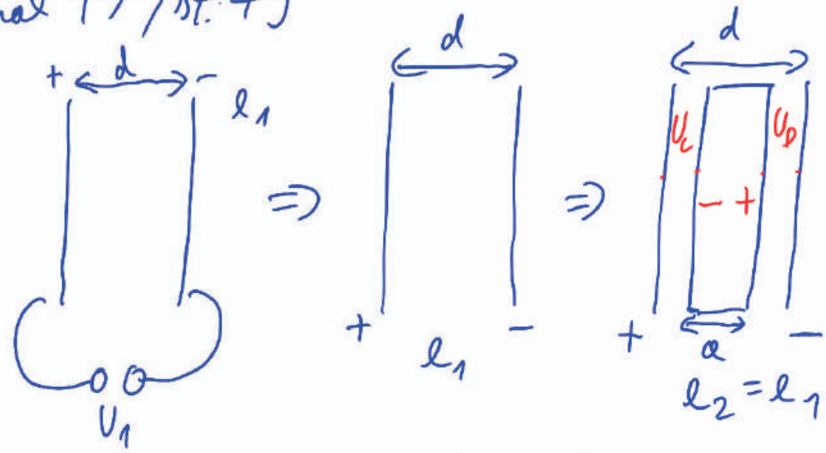
ZBIRKA 9 nal 17/st. 49

$d = 3\,\mathrm{cm}$

$U_1 = 150\,\mathrm{V}$

$a = 2\,\mathrm{cm}$

$U_2 = ?$



$U_2 = U_L + U_D$

• $e_1 = C_1 U_1$ $\qquad E_1 = 2\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{e}{S\varepsilon_0}$; $U = Ed$

$e_1 = \frac{S\varepsilon_0 U_1}{d}$ $\qquad C = \frac{e}{U} = \frac{e}{Ed} = \frac{\cancel{e}\cdot S\varepsilon_0}{\cancel{e}\, d} = \boxed{\frac{S\varepsilon_0}{d} = C}$

KAPACITETA PLOŠČATEGA KONDENZATORJA

LEVI DESNA

• $e_2 = C_2 U_2 = C_L U_L = C_D U_D$

ZAPOREDNA VEZAVA KONDENZATORJEV

$U_2 = U_L + U_D$

$\frac{\cancel{e_2}}{C_2} = \frac{\cancel{e_2}}{C_L} + \frac{\cancel{e_2}}{C_D} \Rightarrow \boxed{\frac{1}{C_2} = \frac{1}{C_L} + \frac{1}{C_D}}$

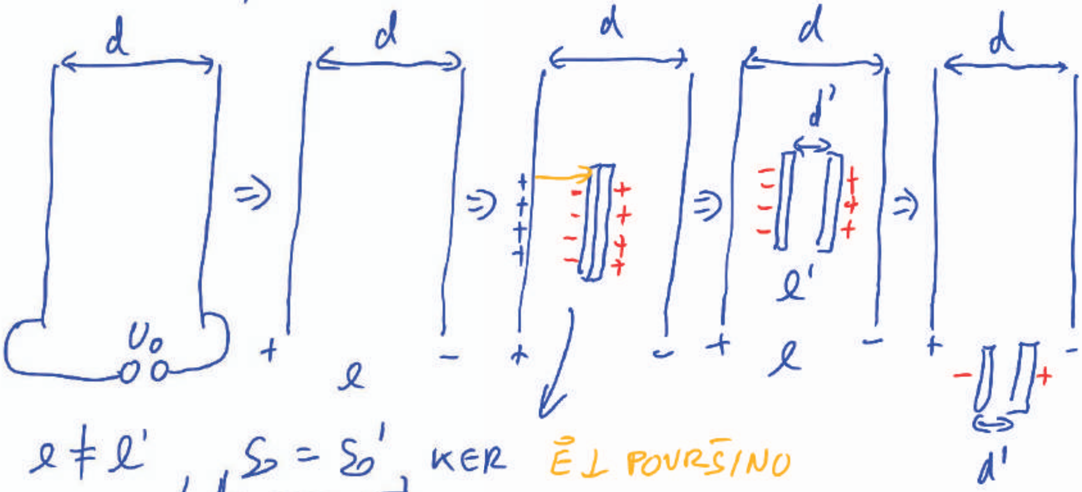
$$U_2 = \frac{e_2}{C_2} = e_2\left(\frac{1}{C_L} + \frac{1}{C_D}\right) \overset{e_2 = e_1}{=} e_1\left(\frac{d_L}{S\varepsilon_0} + \frac{d_D}{S\varepsilon_0}\right) =$$

$$= \frac{\cancel{S\varepsilon_0} U_1}{d\, \cancel{S\varepsilon_0}}\underbrace{(d_L + d_D)}_{d-a} = U_1\frac{d-a}{d} = \underline{50\,\mathrm{V}}$$

ZBIRKA 9 nal 27/st 50

$S = 200\ cm^2$
$d = 2\ cm$
$U_0 = 10\ kV$
$S' = 50\ cm^2$
$d' = 1\ cm$

$A = ?$



$e \neq e'$, $\underline{\sigma = \sigma'}$ KER $\vec{E} \perp$ POVRŠINO

$$A = \Delta W_e = W_e + W_e' - W_e = W_e' = \frac{C'U'^2}{2} = \frac{\varepsilon_0 S' U_0^2 d'^{\cancel{2}}}{2\cancel{d'} d^2} = \underline{\frac{\varepsilon_0 S' U_0^2 d'}{2d^2}}$$

$$= \underline{5{,}5 \cdot 10^{-9}\ J}$$

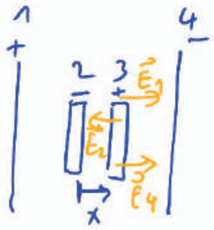
$\boxed{W = \frac{CU^2}{2}}$ → ENERGIJA KONDENZATORJA

$$\underline{C' = \frac{\varepsilon_0 S'}{d'}} \qquad U' = \frac{e'}{C'} = \frac{U_0 \cancel{\varepsilon_0} \cancel{S'} d'}{d \cancel{\varepsilon_0} \cancel{S'}} = \underline{U_0 \frac{d'}{d}}$$

$$\frac{e'}{S'} = \frac{e}{S} \Rightarrow e' = e\frac{S'}{S} = \underline{U_0 \frac{\varepsilon_0 \cancel{S} S'}{d \cancel{S}}}$$

$$e = U_0 C = U_0 \frac{\varepsilon_0 S}{d}$$

S SILAMI: $A = \int \vec{F} \cdot d\vec{s}$



GLEDAMO SILE NA PLOŠČO 3: $\vec{F}_e = e\vec{E}$

$$\vec{E}_{NA3} = \vec{E}_1 + \vec{E}_2 + \vec{E}_4 \Rightarrow \underline{E_{NA3}} = \frac{\sigma}{2\varepsilon_0}(1 - 1 + 1) = \underline{\frac{\sigma}{2\varepsilon_0}}$$

$$|E_1| = \frac{\sigma}{2\varepsilon_0}$$

$$A = \int \vec{F} \cdot d\vec{s} = \int_0^{d'} e' \cdot E\, dx = \frac{e'e'}{S' 2\varepsilon_0} \int_0^{d'} dx = \frac{U_0^2 \varepsilon_0^{\cancel{2}} S'^{\cancel{2}}}{\cancel{S'} 2 \cancel{\varepsilon_0} d^2} d' = \underline{\frac{\varepsilon_0 U_0^2 S' d'}{2d^2}}$$

$\sigma = \sigma'$

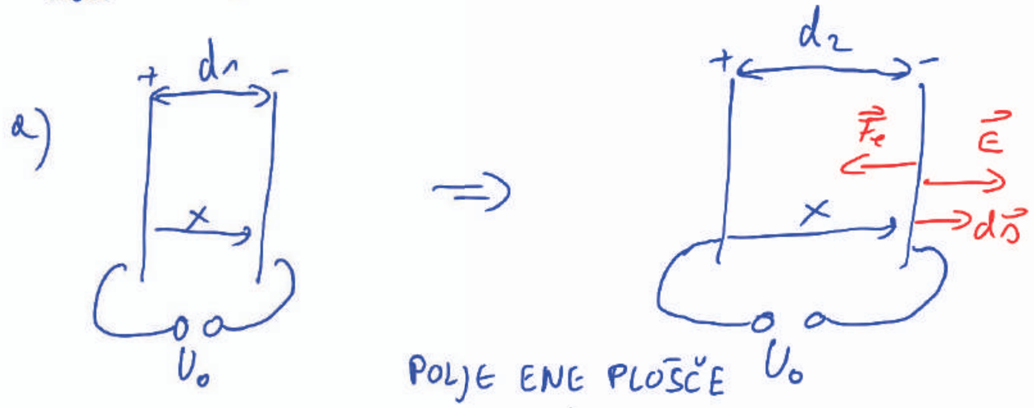
ENAKO

ZBIRKA 9 nal 28/str 51

$S = 100\ cm^2$
$d_1 = 3\ cm$
$d_2 = 5\ cm$
$U_0 = 1000\ V$

$A = ?$

a) $U_0 = konst$
b) $e = konst$



POLJE ENE PLOŠČE

• PREKO SIL: $\vec{F}_e = e\vec{E}$; $E = \frac{U_0}{2x}$; $e = U_0 C = U_0 \frac{\varepsilon_0 S}{x}$

$$A = \int \vec{F} d\vec{s} = -\int \frac{U_0 \varepsilon_0 S U_0}{2x \cdot x} \cdot dx = -\frac{U_0^2 \varepsilon_0 S}{2} \int_{d_1}^{d_2} \frac{dx}{x^2} = \frac{U_0 \varepsilon_0 S}{2}\left(\frac{1}{d_2} - \frac{1}{d_1}\right)$$

$$\left(-\frac{1}{x}\right)\Big|_{d_1}^{d_2}$$

$= -5{,}9 \cdot 10^{-7}\ J$

• PREKO W:

$$A = W_{ek} - W_{ez} = \frac{C_2 U_0^2}{2} - \frac{C_1 U_0^2}{2} = \frac{\varepsilon_0 S U_0^2}{2}\left(\frac{1}{d_2} - \frac{1}{d_1}\right)$$

$C = \frac{\varepsilon_0 S}{d}$

< 0

KER $W_{ek} < W_{ez}$

KER SE NABOJ ZMANJŠA

b) $$A = W_{ek} - W_{ez} = \frac{C_2 U_2^2}{2} - \frac{C_1 U_0^2}{2} = \frac{1}{2}\left[\frac{\cancel{C_2} U_0^2 C_1^2}{C_2^{\cancel{2}}} - C_1 U_0^2\right]$$

$U_2 = \frac{e_2}{C_2} = U_0 \frac{C_1}{C_2}$

$e_2 = e_1 = C_1 U_0$

$$= \frac{C_1 U_0^2}{2}\left(\frac{C_1}{C_2} - 1\right)$$

$$A = \frac{\varepsilon_0 S U_0^2}{2 d_1}\left(\frac{d_2}{d_1} - 1\right) > 0$$

$= 9{,}8 \cdot 10^{-7}\ J$

ZBIRKA 9 nal 19 / str 49

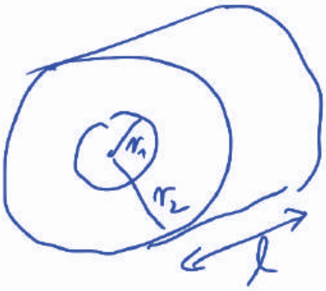
$l = 1\,m$

$2r_1 = 4\,mm$

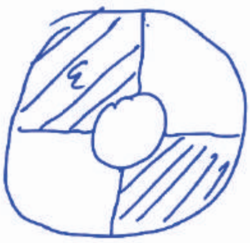
$2r_2 = 8\,mm$

$\varepsilon = 5$

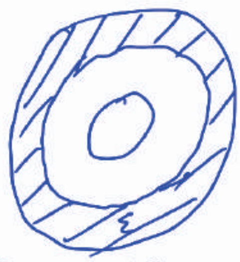
$C = ?$



a)



b)



GOSTA EL. POLJA: $\underline{\vec{D} = \varepsilon\varepsilon_0\vec{E}}$

$\hookrightarrow \varepsilon\varepsilon_0 \oint \vec{E}\cdot d\vec{s} = e \Rightarrow \underline{\oint \vec{D}\cdot d\vec{s} = e}$

a)

$E = E(r)$

$\vec{D}_A = \varepsilon\varepsilon_0\vec{E}$, $\vec{D}_B = \varepsilon_0\vec{E}$

- $\oint \vec{D}\cdot d\vec{s} = e$

$D_A \int_{S_A} d\vec{s} + D_B \int_{S_B} d\vec{s} = e \quad ; S_A = S_B = 2\cdot\frac{1}{4}2\pi r\cdot l = \pi r l$

$(D_A + D_B)\pi r l = e$

$(\varepsilon + 1)\varepsilon_0 E \pi r l = e$

EL. POLJE $\underline{E = \frac{e}{\pi r l \varepsilon_0(\varepsilon+1)}}$

- NAPETOST:

$U = -\int_{r_1}^{r_2} E\,dr = \frac{-e}{\pi l \varepsilon_0(\varepsilon-1)}\int_{r_1}^{r_2}\frac{dr}{r}$

$\underline{U = \frac{-e}{\pi l \varepsilon_0(\varepsilon+1)} \ln\frac{r_2}{r_1}}$

- KAPACITETA: $e = C\cdot U$

$C = \left|\frac{e}{U}\right| = \frac{\cancel{e}\pi l \varepsilon_0(\varepsilon+1)}{\cancel{e}\ln\frac{r_2}{r_1}} \Rightarrow \boxed{C = \frac{\pi l \varepsilon_0(\varepsilon+1)}{\ln\frac{r_2}{r_1}}}$

$\Rightarrow C = C_A + C_B$

b)

$\vec{D}_A = \varepsilon\varepsilon_0\vec{E}_A$

$\vec{D}_B = \varepsilon_0\vec{E}_B$

$\oint \vec{D}\cdot d\vec{s} = e$

$\varepsilon\varepsilon_0 E 2\pi r l = e$

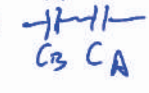
$\underline{E = \frac{e}{2\pi r l \varepsilon\varepsilon_0}}$

$E_A(r_0 < r < r_2) = \frac{e}{2\pi r l \varepsilon\varepsilon_0}$

$E_B(r_1 < r < r_0) = \frac{e}{2\pi r l \varepsilon_0}$

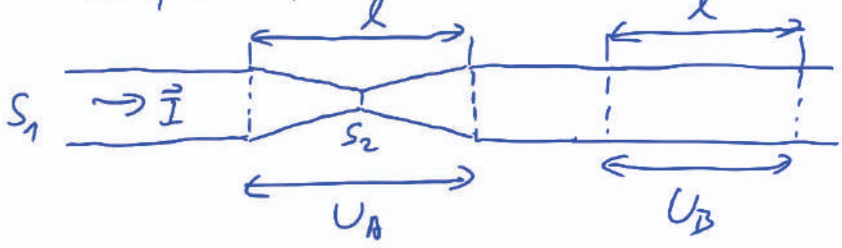
$U = -\int_{r_1}^{r_2} E\,dr = -\int_{r_1}^{r_0}\frac{e}{2\pi\varepsilon_0 l}\frac{dr}{r} - \int_{r_0}^{r_2}\frac{e}{2\pi\varepsilon_0\varepsilon l}\frac{dr}{r} = \frac{-e}{2\pi\varepsilon_0 l}\left[\ln\frac{r_0}{r_1} + \frac{1}{\varepsilon}\ln\frac{r_2}{r_0}\right]$

$\boxed{C = \left|\frac{e}{U}\right| = \frac{2\pi\varepsilon_0 l}{\left[\ln\frac{r_0}{r_1} + \frac{1}{\varepsilon}\ln\frac{r_2}{r_0}\right]}}$ $\quad \frac{1}{C} = \frac{1}{2\pi\varepsilon_0 l}\left[\ln\frac{r_0}{r_1} + \frac{1}{\varepsilon}\ln\frac{r_2}{r_0}\right] = \frac{1}{C_B} + \frac{1}{C_A}$

$\Rightarrow$ 

13.2 Elektricni tok

ZBIRKA 9 vol 1/ st 44



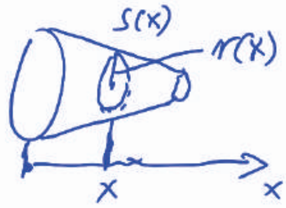
$S_1 = 3\ mm^2$

$S_2 = 1\ mm^2$

$\underline{l = 15\ cm}$

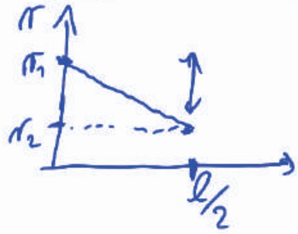
$\frac{U_A}{U_B} = ?$

- $I = I_A = I_B$; $\underline{U = R \cdot I \quad \text{Ohmov z.}}$
- $\underline{\frac{U_A}{U_B} = \frac{R_A}{R_B}}$
- UPORNOST: $R = \frac{\xi l}{S} \Rightarrow dR = \frac{\xi\, dx}{S(x)}$



$$\int_0^{\frac{R_A}{2}} dR = \int_0^{\frac{l}{2}} \xi \frac{dx}{\pi (r_1 + k x)^2}$$

$\underline{S(x) = \pi r^2(x)}$



$\underline{r(x) = r_1 + \underbrace{\frac{r_2 - r_1}{l/2}}_{k} \cdot x}$

$u = r_1 + kx$

$du = k\, dx$

$dx = \frac{1}{k} du$

$$\frac{R_A}{2} = \frac{\xi}{\pi k} \int_{r_1}^{r_2} \frac{du}{u^2}$$

$$\frac{R_A}{2} = \frac{\xi}{\pi k} \left(-\frac{1}{u}\right)\Big|_{r_1}^{r_2}$$

$$\frac{R_A}{2} = \frac{\xi\, l/2}{\pi (r_2 - r_1)} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{R_A}{\cancel{2}} = \frac{\xi\, l}{\cancel{2}\pi \cancel{(r_2 - r_1)}} \left(\frac{\cancel{r_2 - r_1}}{r_2 \cdot r_1}\right)$$

$$\boxed{R_A = \frac{\xi\, l}{\pi r_1 \cdot r_2}} \qquad \underline{R_B = \frac{\xi\, l}{\pi r_1^2}}$$

$$\boxed{\frac{U_A}{U_B} = \frac{R_A}{R_B} = \frac{r_2}{r_1}} = \sqrt{\frac{S_2}{S_1}}$$

ZBIRKA 9 nal 7/st 45

$l = 1\,cm$

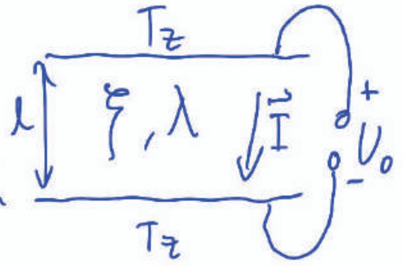
$U_0 = 1V$

$\xi = 110\,\Omega \cdot mm^2/m$

$\lambda = 110\,W/mK$

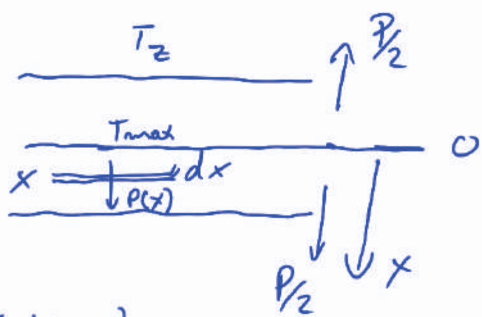
$T_z = 20°C$

$T_{max} = ?$



$\Rightarrow$

TOPLOTNI TOK



$P = U \cdot I$

$P(x) = U(x) \cdot I_0$

$P(x) = I_0^2 \cdot R(x)$

$P(x) = \frac{U_0^2 \cancel{S^2}}{\xi^{\cancel{2}} l^2} \cdot \frac{\cancel{\xi} \cdot x}{\cancel{S}}$

$$\boxed{P(x) = \frac{U_0^2 S x}{\xi l^2}}$$

$I = I_0 \neq I(x)$

$U = RI \Rightarrow I_0 = \frac{U_0}{R_0} = \boxed{\frac{U_0 \cdot S}{\xi l}}$

$R_0 = \frac{\xi \cdot l}{S}$

$$\boxed{R(x) = \frac{\xi x}{S}}$$

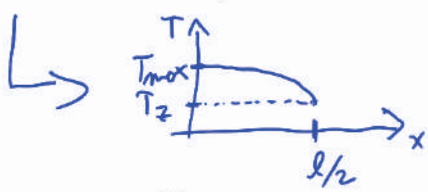
TOPLOTNI TOK: $\boxed{P = \lambda S\left(-\frac{dT}{dx}\right)}$

→ GLEAMO SAMO ENO POLOVICO:

$$P_{EL.} = P_{TOP.}$$

$$\frac{U_0^2 \cancel{S} x}{\xi l^2} = \lambda \cancel{S}\left(-\frac{dT}{dx}\right)$$

$$\frac{U_0^2}{\xi l^2 \lambda}\int_0^{l/2} x\,dx = -\int_{T_{max}}^{T_z} dT \quad \rightarrow T(x) = T_z + \frac{U_0^2 x^2}{8\xi\lambda l^2}$$



$$\frac{U_0^2 \cancel{l^2}}{8\xi\lambda \cancel{l^2}} = T_{max} - T_z$$

$$\boxed{T_{max} = T_z + \frac{U_0^2}{8\xi\lambda}} = \underline{30{,}3\,°C}$$

ZBIRKA 9 nal 8/st 45

$2r_0 = 1\,mm$

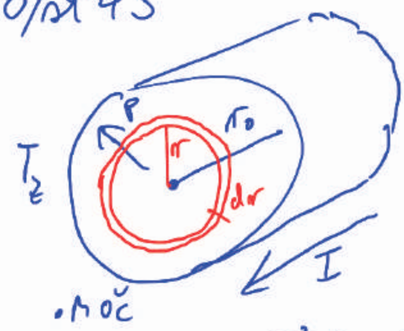
$T_{max} = 50°C$

$T_z = 20°C$

$\lambda = 380\,W/mK$

$\rho = 0{,}017\,\Omega\,mm^2/m$

$I_0 = ?$



$R = \frac{\rho \cdot l}{S} = \frac{\rho\, l}{\pi r^2} = R(r)$

$I = I_0 \frac{r^2}{r_0^2}$

$P = U \cdot I = I^2 \cdot R = I_0^2 \frac{r^{\cancel{4}\,2}}{r_0^4} \frac{\rho\, l}{\pi \cancel{r^2}} = \frac{I_0^2 \rho\, l\, r^2}{r_0^4 \pi} = P(r)$

• TOPLOTNI TOK : $P = \lambda S' \left(-\frac{dT}{dr}\right)$; $S' = 2\pi r l$

$P = \lambda 2\pi r l \left(-\frac{dT}{dr}\right)$

• RAVNOVESJE:

$P_{EL} = P_{TOP}$

ŠE VEDNO KVADRATNI PROFIL T.

$\frac{I_0^2 \rho \cancel{l} r^{\cancel{2}}}{r_0^4 \pi} = \lambda 2\pi \cancel{r} \cancel{l} \left(-\frac{dT}{dr}\right)$

$\frac{I_0^2 \rho}{2 r_0^4 \pi^2 \lambda} \int_0^{r_0} r\,dr = -\int_{T_{max}}^{T_z} dT \quad \rightarrow \quad T(r) = T_z + \frac{I_0^2 \rho\, r^2}{4\pi^2 \lambda\, r_0^4}$

$\frac{I_0^2 \rho \cancel{r_0^2}}{2 r_0^{\cancel{4}\,2} \pi^2 \lambda\, 2} = T_{max} - T_z \quad \Rightarrow \quad T_{max} = T_z + \frac{I_0^2 \rho}{4\pi^2 \lambda r_0^2}$

$I_0 = 2\pi r_0 \sqrt{\frac{(T_{max} - T_z)\lambda}{\rho}} = 2573\,A$

DN ZBIRKA 9 nal 9/st 45

ZBIRKA 9 nal 11/str 45

$l = 5\ m$

$S = 0{,}1\ mm^2$

$\zeta = 0{,}5\ \Omega\ mm^2/m$

$U_0 = 10\ V$

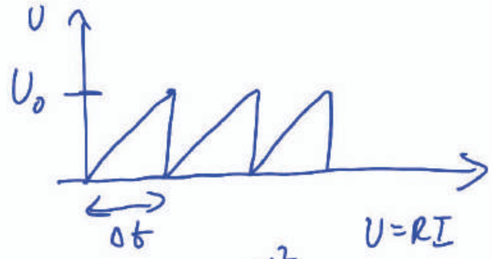
$\Delta t = 1/100\ s$

$t = 1\ s$

$c = 450\ J/kg\,K$

$\rho = 8\ g/cm^3$

$\Delta T = ?$



$U = RI$

$$\boxed{R = \frac{\zeta l}{S}}$$

$$U(0<t<\Delta t) = U_0 \frac{t}{\Delta t}$$

$$P = UI = \frac{U^2}{R}$$

• POVPREČNA MOČ: $\bar{P} = \frac{1}{\Delta t}\int_0^{\Delta t} P(t)\,dt$

$$\bar{P} = \frac{1}{\Delta t}\int_0^{\Delta t} \frac{U_0^2 t^2}{R \cdot \Delta t^2}\cdot dt$$

$$= \frac{U_0^2}{R\,\cancel{\Delta t^3}}\cdot\frac{\cancel{\Delta t^3}}{3} = \boxed{\frac{U_0^2}{3R} = \bar{P}}$$

• TOPLOTA

$$Q = \bar{P}\cdot t = mc\Delta T$$

$$\Delta T = \frac{\bar{P}\cdot t}{mc} = \frac{U_0^2 \cdot t}{3R\cdot Sl\cdot \rho c}$$

$$= \frac{U_0^2\cdot \cancel{S}\cdot t}{\zeta l^2 3 \cancel{S} \rho c}$$

$$\boxed{\Delta T = \frac{U_0^2\cdot t}{3\zeta\rho c l^2}} = \underline{\underline{0{,}74\ K}}$$

• POVPREČNA MOČ: EFEKTIVNA U, I

$$\bar{P} = U_{eff}\cdot I_{eff} = \frac{U_{eff}^2}{R}$$

$$\frac{U_0^2}{3R} = \frac{U_{eff}^2}{R} \longrightarrow \underline{\underline{U_{eff} = \frac{U_0}{\sqrt{3}}}}$$

DN: ZBIRKA 9 nal 10/str 45

ZBIRKA 9 nal 19/str 46

$U_1 = 1V$

$U_2 = 2V$

$U_3 = 3V$

$R_{1m} = 15\Omega$

$R_{2m} = 20\Omega$

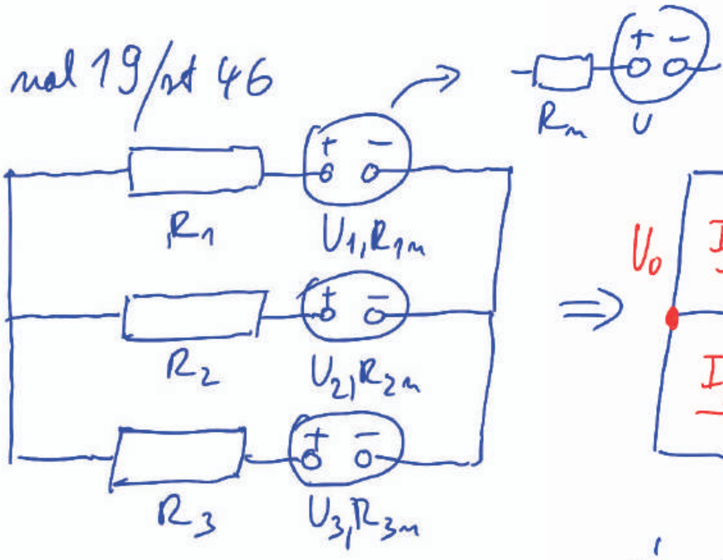
$R_{3m} = 30\Omega$

$R_1 = 85\Omega$

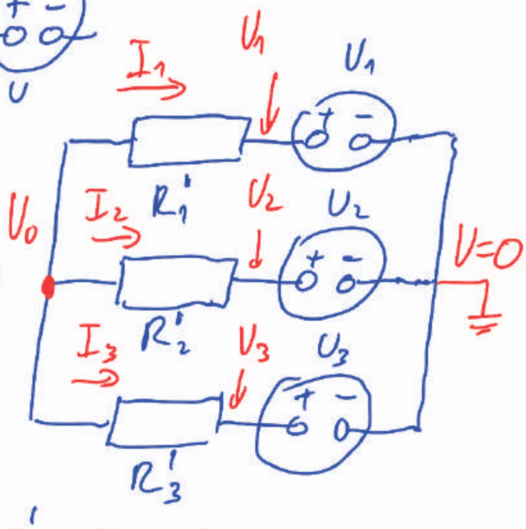
$R_2 = 180\Omega$

$R_3 = 270\Omega$

$I_1, I_2, I_3 = ?$



$\Rightarrow$



$\underline{R_i' = R_i + R_{im}}$

• REŠEVANJE S POTENCIALI:

$\bullet\ I_1 + I_2 + I_3 = 0$

$$I_1 = \frac{U_0 - U_1}{R_1'} = 6{,}4\,mA$$

$$I_2 = \frac{U_0 - U_2}{R_2'} = -1{,}8\,mA$$

$$I_3 = \frac{U_0 - U_3}{R_3'} = -4{,}55\,mA$$

$$\frac{U_0 - U_1}{R_1'} + \frac{U_0 - U_2}{R_2'} + \frac{U_0 - U_3}{R_3'} = 0$$

$$U_0\left(\frac{1}{R_1'} + \frac{1}{R_2'} + \frac{1}{R_3'}\right) = \frac{U_1}{R_1'} + \frac{U_2}{R_2'} + \frac{U_3}{R_3'}$$

$$U_0 = \frac{\frac{U_1}{R_1'} + \frac{U_2}{R_2'} + \frac{U_3}{R_3'}}{\frac{1}{R_1'} + \frac{1}{R_2'} + \frac{1}{R_3'}} = \frac{18}{11}\,V$$

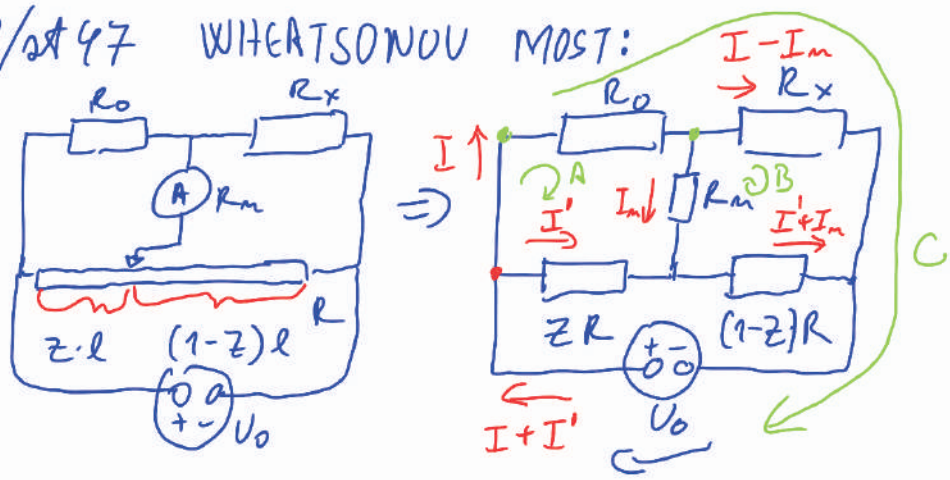
ZBIRKA 9 nal 21/st 47 WHEATSONOV MOST:

$U_0 = 4{,}5\,V$

$R = 1000\,\Omega$

$R_0 = 400\,\Omega$

$R_m = 25\,\Omega$



V RAVNOVESJU ($I_m = 0$):

$(1-z) : z = 4 : 5 \Rightarrow z = \frac{5}{9}$

• $I_m \left[(1-z) : z = 3 : 7\right] = ?$

$\hookrightarrow z = \frac{7}{10}$

A: $-IR_0 - I_m R_m + I' z R = 0 \Rightarrow I' = \dfrac{IR_0 + I_m R_m}{zR}$

B: $-(I - I_m) R_x + (I' + I_m)(1-z) R + I_m R_m = 0$

$$\hookrightarrow -(I - I_m) R_x + \left(\frac{IR_0 + I_m R_m}{zR} + I_m\right)(1-z) R + I_m R_m = 0$$

$$I_m \left[R_x + R_m \frac{(1-\cancel{z})}{z} + (1-z) R + \cancel{R_m}\right] - I\left[R_x - R_0 \frac{1-z}{z}\right] = 0$$

$$\underbrace{I_m = I\, \frac{R_x - R_0 \frac{1-z}{z}}{R_x + R_m \frac{1}{z} + (1-z) R}}$$

• RAVNOVESJE: $(z = \frac{5}{9})$ $I_m = 0 \Rightarrow \underbrace{R_x = R_0 \frac{1-z}{z}} = \underline{\underline{320\,\Omega}}$

• I_m PRI $z = \frac{7}{10}$: C: $-IR_0 - (I - I_m) R_x + U_0 = 0$

$$\hookrightarrow \underbrace{I = \frac{U_0 + I_m R_x}{R_0 + R_x}}$$

$$I_m = \frac{U_0 + I_m R_x}{R_0 + R_x} \cdot \frac{R_x - R_0 \frac{1-z}{z}}{R_x + R_m \frac{1}{z} + (1-z) R}$$

$$I_m = \frac{U_0}{R_0 + R_x} \cdot \frac{R_x - R_0 \frac{1-z}{z}}{R_x + R_m \frac{1}{z} + (1-z) R} + \frac{I_m R_x}{R_0 + R_x} \cdot \frac{R_x - R_0 \frac{1-z}{z}}{R_x + R_m \frac{1}{z} + (1-z) R}$$

$$I_m = \frac{U_0}{R_0 + R_x} \cdot \frac{R_x - R_0 \frac{1-z}{z}}{R_x + R_m \frac{1}{z} + (1-z) R} \cdot \frac{1}{1 - \frac{R_x}{R_0 + R_x} \cdot \frac{R_x - R_0 \frac{1-z}{z}}{R_x + R_m \frac{1}{z} + (1-z) R}}$$

$$\underbrace{I_m = \frac{U_0 \left(R_x - R_0 \frac{1-z}{z}\right)}{(R_0 + R_x)\left(R_x + R_m \frac{1}{z} + (1-z) R\right) - R_x \left(R_x - R_0 \frac{1-z}{z}\right)}} = \underline{\underline{-1{,}6\,mA}}$$

K WHITSONOVEM MOSTIČKU

• KDAJ JE NAJBOLJ OBČUTLJIV?

(OD ZADNJIČ $I_A = 0$:)

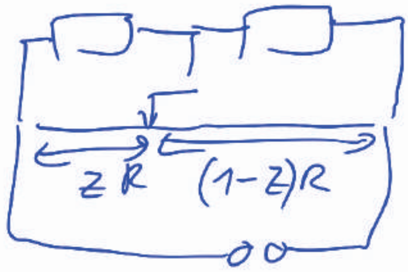
$$\boxed{R_x = R_0 \frac{(1-z)}{z}}$$

$$dR_x = R_0 \frac{-1\cdot\cancel{z} - (1-\cancel{z})\cdot 1}{z^2}\cdot dz$$

$$dR_x = -R_0 \frac{dz}{z^2}$$

$$\Downarrow$$

$$\boxed{\Delta R_x = -R_0 \frac{\Delta z}{z^2}} \rightarrow \text{ZVEZA MED NAPAKO V } R_x \text{ IN } z$$



$\Downarrow$ KJE JE NAJBOLJ OBČUTLJIV

$$\frac{d}{dz}\left(\frac{\Delta R_x}{R_x}\right) = \frac{d}{dz}\left(\frac{\Delta z}{z^{\cancel{2}}}\cdot\frac{\cancel{z}}{(1-z)}\right) = \frac{d}{dz}\left(\frac{\Delta z}{z(1-z)}\right)$$

$$\overline{\Delta z = \text{konst}}$$

↑ DOLOČEN Z NATANČNOSTJO MERITVE

$$= \Delta z \frac{-1+2z}{z^2(1-z)^2} = 0$$

↑ MINIMUM:

$$-1+2z = 0 \Rightarrow \underline{\underline{z = \tfrac{1}{2}}}$$

ZBIRKA 9 nal 20/str 46

$U_1 = 6V$

$R_1 = 5\Omega$

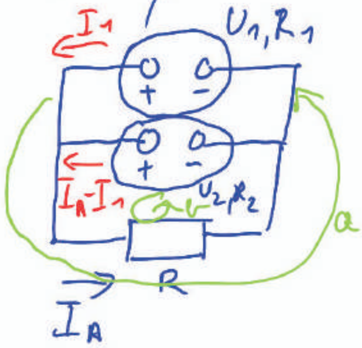
$R = 10\Omega$

$U_2 = 8V$

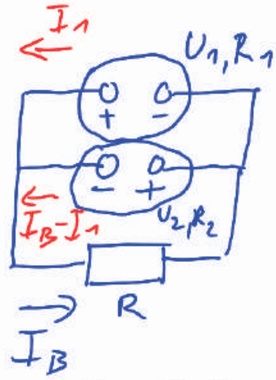
$R_2 = 7\Omega$

$\frac{I_A}{I_B}$

A)



B



A) a) $-I_A R - I_1 R_1 + U_1 = 0 \Longrightarrow I_1 = \frac{U_1 - I_A R}{R_1}$

b) $-I_A R - (I_A - I_1) R_2 + U_2 = 0$

$$-I_A R - I_A R_2 + \frac{U_1 - I_A R}{R_1} \cdot R_2 + U_2 = 0$$

$$-I_A\left(R + R_2 + R\frac{R_2}{R_1}\right) + U_1\frac{R_2}{R_1} + U_2 = 0 / \cdot R_1$$

$$\underline{I_A = \frac{U_1 R_2 + U_2 R_1}{R R_1 + R_2 R_1 + R R_2}}$$

B) $\Rightarrow$ SE SPREMENI PREDZNAK PRED U_2!

$$\underline{\Rightarrow I_B = \frac{U_1 R_2 - U_2 R_1}{R R_1 + R_2 R_1 + R R_2}}$$

$$\underline{\frac{I_A}{I_B} = \frac{U_1 R_2 + U_2 R_1}{U_1 R_2 - U_2 R_1}} = \underline{-41}$$

88/89 3. kol, 1. nal

$R = 50\,\Omega$

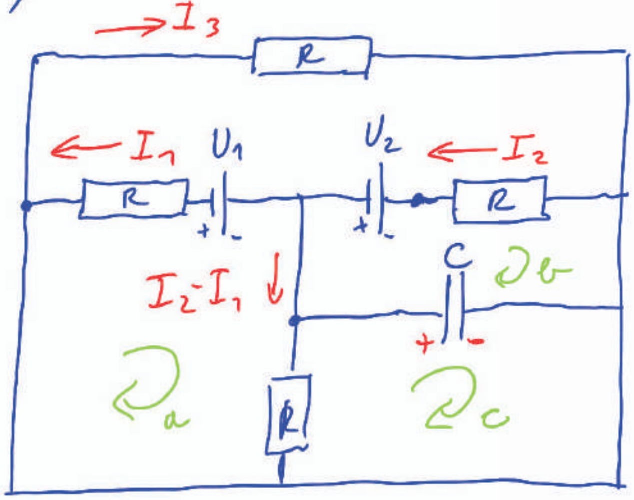
$U_1 = 9\,V$

$U_2 = 5\,V$

$C = 3\,\mu F$

$e = ?$

$U_c = \frac{e}{C}$



$\Rightarrow I_3 = 0$

a) $I_1 R - U_1 - (I_2 - I_1) R = 0$

b) $I_2 R + U_c - U_2 = 0$

c) $-U_c + (I_2 - I_1) R = 0$

b) $I_2 R = U_2 - U_c$, 2

a) $I_1 R - U_1 - (U_2 - U_c) + I_1 R = 0$

$$I_1 R = \frac{U_1 + U_2 - U_c}{2}$$

c) $-U_c + U_2 - U_c - \frac{U_1 + U_2 - U_c}{2} = 0 \quad / \cdot 2$

$-4U_c + 2U_2 - U_1 - U_2 + U_c = 0$

$$U_c = \frac{U_2 - U_1}{3} = -\frac{4}{3}\,V$$

$$e = |C\,U_c| = 4\,\mu As$$

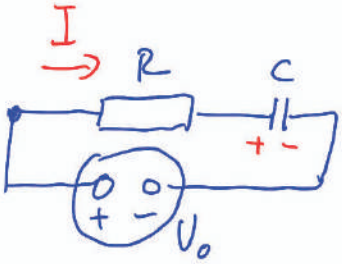
13.3 Prehodni pojavi z upori in kondenzatorji

13.3. nal 1

$C = 50\,\mu F$

$R = 0{,}2\,M\Omega$

$U_0 = 200\,V$



$I = \frac{de}{dt}$

$U_C = \frac{e}{C}$

$-IR - U_c + U_0 = 0$

$-\frac{de}{dt}R - \frac{e}{C} + U_0 = 0$

$\frac{de}{dt}R = U_0 - \frac{e}{C} \quad /\cdot C$

$\int_0^e \frac{de}{U_0 \cdot C - e} = \int_0^t \frac{dt}{RC}$; $RC = \tau$

$U_0C - e = u$

$-de = du$

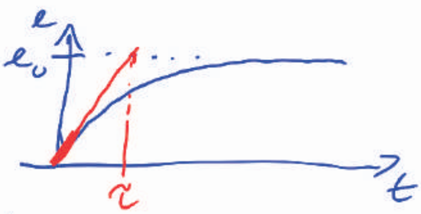
$-\int_{U_0C}^{U_0C-e} \frac{du}{u} = \frac{t}{\tau}$

$-\ln\frac{U_0C-e}{U_0C} = \frac{t}{\tau}$

$\frac{U_0C-e}{U_0C} = e^{-t/\tau}$

$e = U_0C\left(1-e^{-t/\tau}\right)$

$e = e_0\left(1-e^{-t/\tau}\right) \Rightarrow I = \frac{e_0}{\tau}e^{-t/\tau}$



a) $t\left(\frac{e}{e_0} = 0{,}9\right) = ?$

$\frac{e}{e_0} = \left(1-e^{-t/\tau}\right)$

$e^{-t/\tau} = 1 - \frac{e}{e_0} \Rightarrow t = -\tau\ln\left(1-\frac{e}{e_0}\right) = 23\,s$

b) $W_c(t_1 = RC) = ?$

$\hookrightarrow t_1 = \tau$

$W_c = \frac{CU_c^2}{2} = \frac{e^2}{2C} = \frac{e_0^2}{2C}\left(1-e^{-t/\tau}\right) = \frac{U_0^2C}{2}\left(1-e^{-1}\right) = 0{,}4\,J$

c) $P_R(t_1 = RC) = ?$

$P_R = RI^2 = R\left(\frac{e_0}{\tau}\right)^2 e^{-2t/\tau} = \cancel{R}\frac{U_0^2\cancel{C^2}}{R^{\cancel{2}}\cancel{C^2}}e^{-2t/\tau} = \frac{U_0^2}{R}e^{-2t/\tau} = \frac{U_0^2}{R}e^{-2} = 27\,mW$

d) $A_{CELOTNO} = ?$ $A = \int_0^\infty P_{CEL}\,dt = \int_0^\infty U_0 I\,dt = U_0\int_0^\infty \frac{de}{\cancel{dt}}\cancel{dt} = U_0\int_0^{e_0} de = U_0 e_0 = \underbrace{U_0^2 C}_{=2W_c(t=\infty)}$

$P_{CEL} = U_0 I$ $\quad I = \frac{de}{dt}$

e) $W_c(t=\infty) = ?$ $\quad W_c(t=\infty) = \frac{U_0^2C}{2}$

f) $A_{UPOR} = A_{CEL} - W_c = W_c(t=\infty)$

ZBIRKA 9 nal 33/ot 51

$C_1 = 0{,}1\,\mu F$

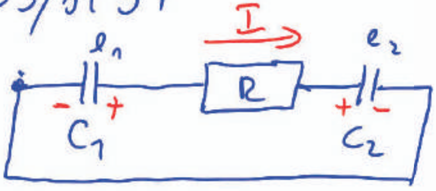
$U_{10} = 1000\,V$

$C_2 = 0{,}4\,\mu F$

$R = 1600\,\Omega$

$U_\infty = ?$

$t\left(U_2 = \frac{U_\infty}{2}\right) = ?$



$\underline{e_1 + e_2 = e_{10}}$

$e_1 = C_1 \cdot U_1$

$e_2 = C_2 \cdot U_2$

• NA ZAČETKU:

$C_1, U_{10} \Rightarrow \underline{e_{10} = C_1 U_{10}}$

• NA KONCU:

$\underline{e_1 + e_2 = e_{10}}$

$U_1 = U_2 = U_\infty$

$C_1 U_\infty + C_2 U_\infty = C_1 U_{10}$

$$\underline{U_\infty = \frac{C_1}{C_1 + C_2} U_{10}}$$

• VMES: $e_1 + e_2 = e_{10} \,/\, d$

$de_1 + de_2 = 0$

$\underline{de_1 = -de_2}$

$$\underline{I = -\frac{de_1}{dt} = \frac{de_2}{dt}}$$

• ZANKA: $U_1 - IR - U_2 = 0$

$\underline{de_2 = C_2\, dU_2}$

$$\frac{de_2}{dt} R = U_1 - U_2$$

$e_1 + e_2 = e_{10}$

$C_1 U_1 + C_2 U_2 = C_1 U_{10}$

$$U_1 = \frac{U_{10} C_1 - C_2 U_2}{C_1}$$

$$\underline{U_1 = U_{10} - U_2 \frac{C_2}{C_1}}$$

$$\frac{dU_2}{dt} R C_2 = U_{10} - U_2\left(1 + \frac{C_2}{C_1}\right)$$

$$\int_0^{U_2} \frac{dU_2}{U_{10} - U_2\left(1 + \frac{C_2}{C_1}\right)} = \int_0^t \frac{dt}{RC_2}$$

$U_{10} - U_2\left(1 + \frac{C_2}{C_1}\right) = u$

$-\left(1 + \frac{C_2}{C_1}\right) dU_2 = du$

$$-\frac{1}{\left(1 + \frac{C_2}{C_1}\right)} \int_{U_{10}}^{U_{10} - U_2\left(1 + \frac{C_2}{C_1}\right)} \frac{du}{u} = \frac{t}{RC_2}$$

$$-\frac{C_1}{C_1 + C_2} \ln \frac{U_{10} - U_2\left(1 + \frac{C_2}{C_1}\right)}{U_{10}} = \frac{t}{RC_2}$$

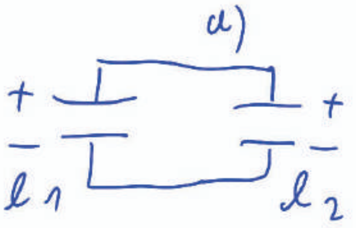
$$\underline{t = R\frac{C_1 C_2}{C_1 + C_2} \ln \frac{U_{10}}{U_{10} - U_2\left(1 + \frac{C_2}{C_1}\right)}}$$

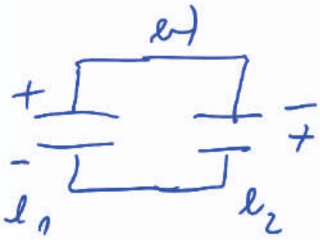
$\parallel$

$$C \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 \cdot C_2}$$

$$t\left(U_2 = \frac{U_\infty}{2}\right) = \underline{\underline{8{,}9 \cdot 10^{-5}\,s}}$$

DN: ZBIRKA 9 nal 25/st 50





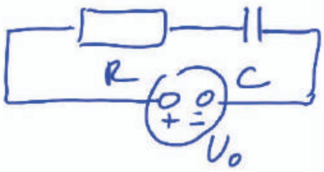
ZBIRKA 9 nal 32/st 51

$R = 10^5\,\Omega$

$C = 10\,\mu F$

$U_0 = k t$

$\frac{U_R}{U_C}(t = 1s) = ?$



$$\boxed{U_0 = U_R + U_C}$$

$$\boxed{I = \frac{de}{dt}}$$

$$kt = IR + \frac{e}{C}$$

$$kt = \frac{de}{dt}R + \frac{e}{C} \quad / \frac{d}{dt}$$

$$k = \frac{dI}{dt}R + \frac{I}{C} \quad / \cdot C$$

$$kC - I = \frac{dI}{dt} \cdot RC$$

$$\int_0^t \frac{dt}{RC} = \int_0^I \frac{dI}{kC - I}$$

$$kC - I = u$$
$$-dI = du$$

$$\frac{t}{\tau} = -\int_{kC}^{kC-I} \frac{du}{u}$$

$$\boxed{RC = \tau}$$

$$-\frac{t}{\tau} = \ln \frac{kC - I}{kC}$$

$$\boxed{I = kC\left(1 - e^{-t/\tau}\right)}$$

$$\frac{U_R}{U_C} = \frac{RI}{U_0 - RI} = \frac{R\cancel{k}C\left(1 - e^{-t/\tau}\right)}{\cancel{k}t - RkC\left(1 - e^{-t/\tau}\right)} = \boxed{\frac{\tau\left(1 - e^{-t/\tau}\right)}{t - \tau\left(1 - e^{-t/\tau}\right)}}$$

$$\frac{U_R}{U_C}(t = 1s) = \underline{\underline{e - 1}}$$

$$\tau = RC = 1s$$

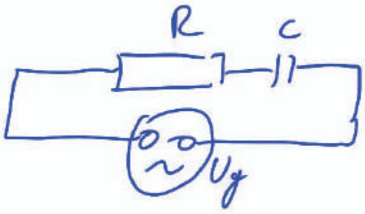
13.3 nal 5

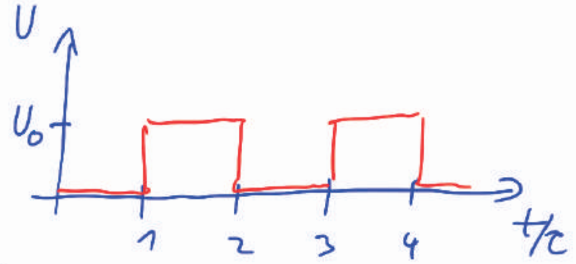
$U_0 = 100\,V$

$\tau = RC = 10_n$

$U_{C0} = U_C(t=0) = ?$

↳ DA BO NIHANJE U_C PERIODIČNO





- $U_C(t=0) = U_C(t=2n\tau)$ ← PERIODIČNOST PERIODA 2τ

- $U_g = U_R + U_C$

$$\underline{U_g = \tau \frac{dU_C}{dt} + U_C}$$

$U_R = IR = \frac{de}{dt}R = \frac{dU_C}{dt}\underbrace{RC}_{\tau}$

$e = CU_C$

- $\underline{0 < t < \tau:}$

$$0 = \tau \frac{dU_C}{dt} + U_C$$

$$-\int_0^t \frac{dt}{\tau} = \int_{U_{CMAX}}^{U_C} \frac{dU_C}{U_C}$$

$$-\frac{t}{\tau} = \ln \frac{U_C}{U_{CMAX}} \Rightarrow \underline{U_C(0<t<\tau) = U_{CMAX}\, e^{-t/\tau}}$$

- $\underline{\tau < t < 2\tau:}$

$$U_0 = \tau \frac{dU_C}{dt} + U_C$$

$$-\tau \frac{dU_C}{dt} = U_C - U_0$$

$$\int_{U_{CMIN}}^{U_C} \frac{dU_C}{U_C - U_0} = -\int_\tau^t \frac{dt}{\tau}$$

$$\ln \frac{U_C - U_0}{U_{CMIN} - U_0} = -\frac{t-\tau}{\tau}$$

$$U_C - U_0 = (U_{CMIN} - U_0)e^{-\frac{t-\tau}{\tau}} \Rightarrow \underline{U_C(\tau<t<2\tau) = U_0 + (U_{CMIN} - U_0)e^{-\frac{t-\tau}{\tau}}}$$

ROBNI POGOJI:

- $t = \tau$: $U_C(0<t<\tau)\big|_{t=\tau} = U_C(\tau<t<2\tau)\big|_{t=\tau}$

$$U_{CMAX}\, e^{-1} = \cancel{U_0} + (U_{CMIN} - \cancel{U_0})\cdot 1 \Rightarrow \underline{U_{CMAX} = U_{CMIN}\cdot e}$$

- $t = 0, 2\tau$: $U_C(0<t<\tau)\big|_{t=0} = U_C(\tau<t<2\tau)\big|_{t=2\tau}$

$$\underline{U_{CMAX} = U_0 + (U_{CMIN} - U_0)e^{-1}}$$

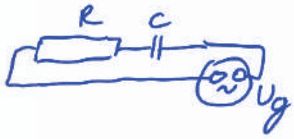
$$\Rightarrow U_{C\,MAX} = U_0 + \left(U_{C\,MAX}' e^{-1} - U_0\right) e^{-1}$$

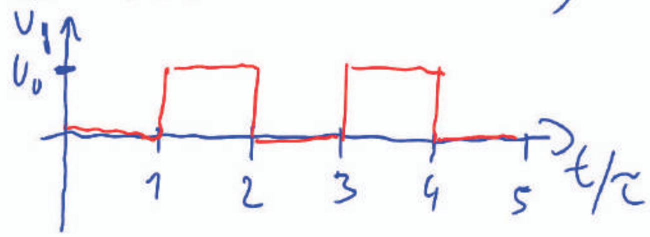
$$U_{C\,MAX}\left(1 - e^{-2}\right) = U_0\left(1 - e^{-1}\right) \qquad \left(1 - e^{-2}\right) = \left(1 - e^{-1}\right)\left(1 + e^{-1}\right)$$

$$U_{C\,MAX} = U_0 \frac{\left(1 - e^{-1}\right)}{\left(1 - e^{-2}\right)} = U_0 \frac{\cancel{\left(1 - e^{-1}\right)}}{\cancel{\left(1 - e^{-1}\right)}\left(1 + e^{-1}\right)}$$

$$\underline{U_{C\,MAX} = U_0 \frac{e}{e+1}} \; ; \; \underline{U_{C\,MIN} = U_0 \frac{1}{e+1}}$$

13.3 nal 5 POVZETEK a)





U_C: $\underline{U_C(0<t<\tau) = U_{C_{MAX}}\, e^{-t/\tau}}$

$\underline{U_C(\tau<t<2\tau) = U_0 + (U_{C_{MIN}} - U_0)\, e^{-\frac{t-\tau}{\tau}}}$

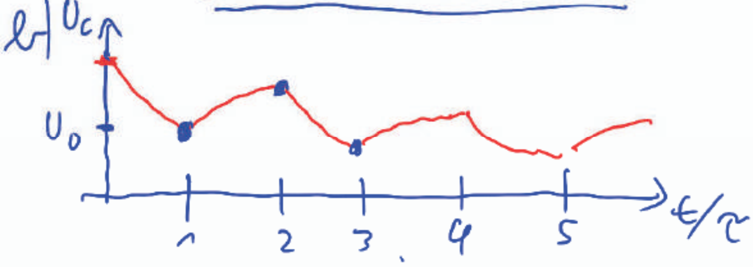
a)

$\rightarrow$ PERIODIČNO: $\underline{U_{CMAX} = U_0 \frac{e}{e+1}}$

$\underline{U_{CMIN} = U_0 \frac{1}{e+1}}$

$\rightarrow$ NOVO: $\underline{\underline{U_C(t=0) = U_{C0} = 200\,V}}$

b)



$$U_C(0) = U_{C0}$$

$$U_C(\tau) = U(0)\, e^{-1} = U_{C0}\, e^{-1}$$

$$U_C(2\tau) = U_0 + (U_C(\tau) - U_0)\, e^{-1} = U_0(1-e^{-1}) + U_{C0}\, e^{-2}$$

$$U_C(3\tau) = U_C(2\tau)\, e^{-1} = U_0(1-e^{-1})\, e^{-1} + U_{C0}\, e^{-3}$$

$$\underbrace{U_C(4\tau)}_{n=2} = U_0 + (U_C(3\tau) - U_0)\, e^{-1} = \underbrace{U_0(1-e^{-1}) + U_0(1-e^{-1})\, e^{-2}}_{U_0(1-e^{-1})(1+e^{-2})} + U_{C0}\, e^{-4}$$

$$U_C(2n\tau) = U_0(1-e^{-1}) \cdot \underbrace{(1+e^{-2}+e^{-4}+e^{-6}+\dots e^{-2(n-1)})} + U_{C0}\, e^{-2n}$$

$$1+q+q^2+\dots q^n = \frac{1-q^{n+1}}{1-q} \rightarrow \underline{= \frac{1-e^{-2n}}{1-e^{-2}}}$$

$$U_C(2n\tau) = U_0(1-e^{-1}) \cdot \frac{1-e^{-2n}}{1-e^{-2}} + U_{C0}\, e^{-2n}$$

$$\underline{U_C(2n\tau) = U_0 \frac{1-e^{-2n}}{1+e^{-1}} + U_{C0}\, e^{-2n}} \quad \longleftarrow \quad 1-e^{-2} = (1+e^{-1})(1-e^{-1})$$

LIMITA $n \to \infty$: $U_C(n\to\infty) = \frac{U_0}{1+e^{-1}} = \underline{\underline{U_0 \frac{e}{e+1}}} = U_{CMAX\,a)}$

13.4 Magnetna sila in navor

13.4 nal 1

^{20}Ne, ^{22}Ne

$e = +|e_0|$

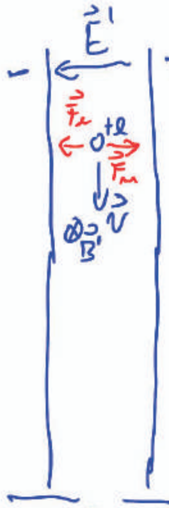
$B = 0{,}08\,T$

$v = 10^5\,m/s$

$B' = 0{,}01\,T$

$E' = ?$

$X = ?$



$$\vec{F} = \vec{F}_e + \vec{F}_m = e\vec{E} + e(\vec{v}\times\vec{B})$$

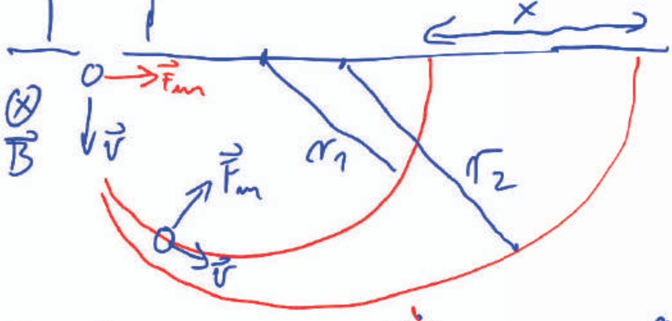
TESLA $T = \frac{Vs}{m^2}$

$\Rightarrow |F_e| = |F_m|$ POGOJ DA SE NE ZALETIJO

$$eE' = evB'$$

$$\underline{E' = vB'} = \underline{10^3\,V/m}$$

$$X = 2r_2 - 2r_1$$



$$F_m = e\cancel{v}B = F_c = m\frac{v^{\cancel{2}}}{r} \Rightarrow \underline{r = \frac{mv}{eB}}$$

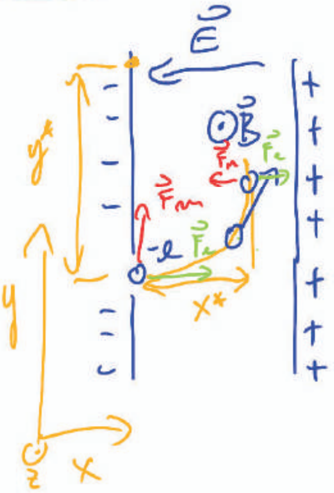
$$\underline{X = 2\frac{v}{eB}}\underbrace{(m_2 - m_1)}_{2M_0} = \underline{5{,}1\,cm}$$

$2M_0$ → MASA NEVTRONA ≈ PROTONA

$$M_0 = 1{,}66\cdot 10^{-27}\,kg = \frac{1}{12}m(^{12}C)$$

$$B_{ZEMELJSKI} \sim \underline{10^{-5}\,T} \sim \underline{0{,}1\,GAUSS}$$

13.4 nal 2 (ZBIRKA 9 nal 31/st 73)



$$\vec{F} = e(\vec{E} + \vec{v}\times\vec{B}) \Rightarrow m\begin{pmatrix}\dot{v}_x\\ \dot{v}_y\\ \dot{v}_z\end{pmatrix} = e\vec{E} + e(\vec{v}\times\vec{B})$$

$$\begin{pmatrix}\cdot\\0\\0\end{pmatrix} \qquad \begin{pmatrix}\cdot\\ \cdot\\0\end{pmatrix}$$

$$\vec{v} = (v_x, v_y, 0)$$

$$\vec{E} = (-E, 0, 0)$$

$$\vec{B} = (0, 0, B)$$

DOBITE ENAČBE:

$m\dot{v}_x = \ldots$

$m\dot{v}_y = \ldots$

$\underline{v_z = 0}$

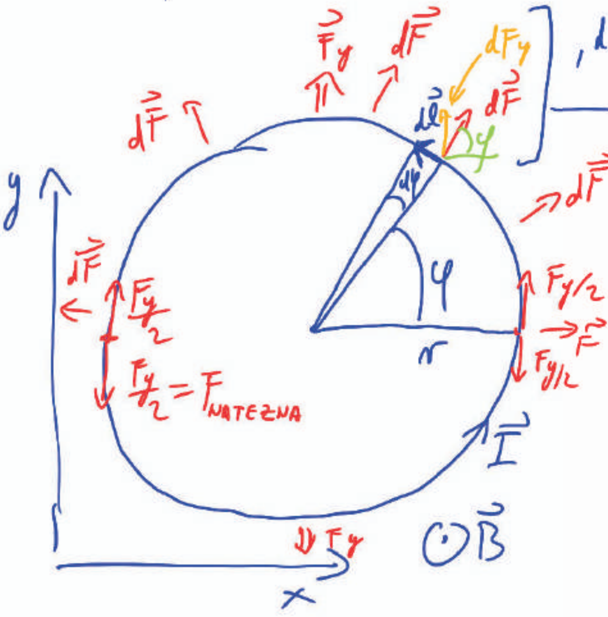
ZBIRKA 9 nal 10/st 52

$I = 100\,A$

$2r = 10\,cm$

$B = 0{,}2\,T$

$F_{NATEZNA} = ?$



$, \; dl = r \cdot d\varphi$

$\vec{F} = I\,\vec{l} \times \vec{B}$

$dF_y = dF \cdot \sin\varphi$

$dF_y = I\,dl\,B \cdot \sin\varphi$

$dF_y = I\,r\,B \sin\varphi\,d\varphi$

$F_y = I r B \underbrace{\int_0^{\pi} \sin\varphi\,d\varphi}_{=2}$

$\boxed{F_y = 2IrB}$

$$\underline{F_{NATEZNA} = \frac{F_y}{2} = IrB} = \underline{\underline{1N}}$$

DW: ZBIRKA 9 nal 12/st 53

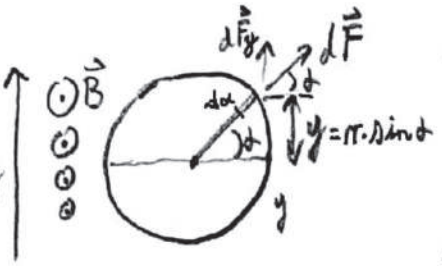
12/sl 53 (9)

$\frac{dB}{dy} = 0{,}05\ T/m$

$r = 2\ cm$

$I = 10A$

$F_R = ?$



$dF_y = I \cdot r \cdot B \cdot d\alpha \cdot \sin\alpha$

$B = B_0 + \frac{dB}{dy} \cdot y = B_0 + \frac{dB}{dy} \cdot r \cdot \sin\alpha$

$dF_y = I r \left(B_0 + \frac{dB}{dy} \cdot r \cdot \sin\alpha\right) \sin\alpha\, d\alpha$

$$F_R = \int_0^{2\pi} dF_y = I r \left[B_0 \int_0^{2\pi} \sin\alpha\, d\alpha + \frac{dB}{dy} \cdot r \int_0^{2\pi} \sin^2\alpha\, d\alpha\right]$$

$$= I r B_0 \cdot 0 + I r^2 \frac{dB}{dy} \cdot \int_0^{2\pi} \frac{1}{2}(1 - \cos 2\alpha)\, d\alpha = \underline{I r^2 \frac{dB}{dy} \cdot \pi} = \underline{\underline{6 \cdot 10^{-4} N}}$$

PO DELIH:

$$\Rightarrow F_{y\uparrow} = \int_0^{\pi} dF_y = I r \left[\int_0^{\pi} B_0 \cdot \sin\alpha\, d\alpha + \frac{dB}{dy} \cdot r \int_0^{\pi} \sin^2\alpha\, d\alpha\right]$$

$$= 2 I r B_0 + I r^2 \frac{dB}{dy} \cdot \int_0^{\pi} \frac{1}{2}(1 - \cos 2\alpha)\, d\alpha =$$

$$= \underline{2 I r B_0 + I r^2 \frac{dB}{dy} \cdot \frac{1}{2}\pi}$$

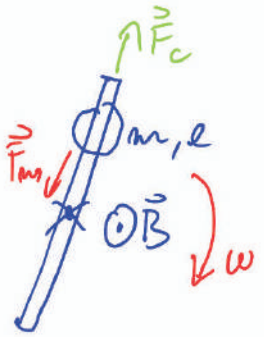
$$F_{y\downarrow} = \int_{\pi}^{2\pi} dF_y = \underline{-2 I r B_0 + I r^2 \frac{dB}{dy} \cdot \frac{1}{2}\pi}$$

$$F_R = F_{y\uparrow} + F_{y\downarrow} = I r^2 \frac{dB}{dy} \cdot \pi = \underline{\underline{6 \cdot 10^{-4} N}}$$ ← EKVIVALENTNO

13.4 nal 5

m
$e>0$
ω
B

SMER = ?



• V RAVNOVESJU:

$$F_m = F_c$$
$$e(\vec{v}\times\vec{B}) = m\frac{v^2}{r}$$
$$e\cancel{v}B = m\frac{v^{\cancel{2}}}{r}$$
$$\underbrace{\frac{v}{r} = \frac{eB}{m} = \omega_0}$$

• ČE IZMAKNEMO KROGLO:

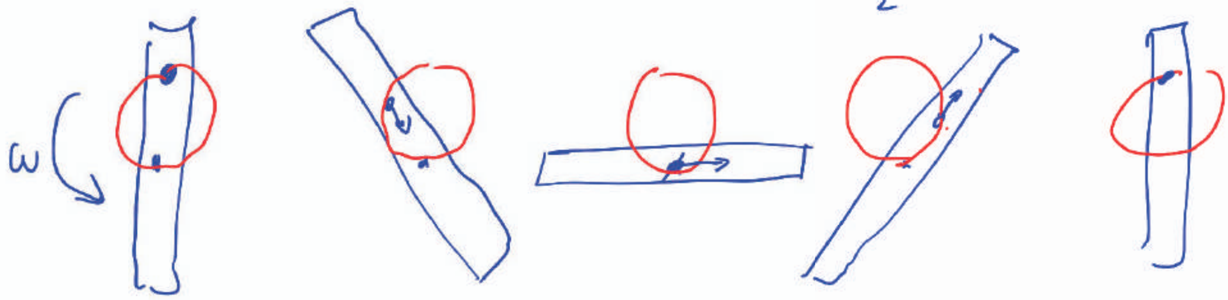
$$m a_r = F_c - F_m$$
$$m\ddot{r} = m\frac{v^2}{r} - evB \quad / \cdot \frac{1}{m}$$
$$\ddot{r} = \omega^2 r - \frac{eB}{m}\omega r \qquad \omega = \frac{v}{r}$$
$$\ddot{r} + \underbrace{\omega(\omega_0 - \omega)}r = 0$$
$$\Omega^2 = \omega(\omega_0 - \omega)$$
$$\underbrace{\Omega = \omega\sqrt{\frac{\omega_0}{\omega} - 1}} \; ; \; \underbrace{r = r_0\cos(\Omega t + \varphi)}$$

d

→ POGOJ ZA NIHANJE: $\underline{\underline{\omega_0 > \omega}}$

Ω = MAKSIMALEN DN

$$\Omega_{max} \Rightarrow \omega^* = \frac{\omega_0}{2}$$



ZBIRKA 9 nal 18/str 53

$\omega_0 = 0{,}8\,s^{-1}$

$B_z = 0{,}2 \cdot 10^{-4}\,T$

$\omega_z = 0{,}02\,s^{-1}$

$B_0 = ?$

NAVOR: $\vec{M} = \vec{p}_m \times \vec{B}$; $\vec{p}_m = N I \vec{S}$

$$\begin{cases} M = p_m B \sin\varphi \\ M = -J \cdot \alpha = -J\ddot{\varphi} \end{cases}$$

$\Rightarrow -J\ddot{\varphi} = p_m B \sin\varphi$

MAJHNI φ: $\sin\varphi \to \varphi$

$\ddot{\varphi} = -\underbrace{\frac{p_m B}{J}}_{\omega^2} \varphi$

$\omega^2 \Rightarrow \boxed{\omega = \sqrt{\frac{p_m B}{J}}}$

$\omega_z = \sqrt{\frac{p_m B_z}{J}} \Rightarrow B \propto \omega^2$

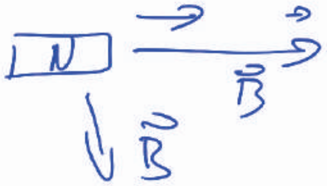
$B = \left(\frac{p_m}{J}\right)^2 \omega^2$

$\Downarrow$

$\frac{B_0}{B_z} = \frac{\omega_0^2}{\omega_z^2} \Rightarrow \boxed{B_0 = B_z \frac{\omega_0^2}{\omega_z^2}} = \underline{\underline{3{,}2 \cdot 10^{-2}\,T}}$

(13.4) nal 4

MONOPOL



DIPOL V MAG. POLJU:

- NAVOR:

$\vec{M} = \vec{p}_m \times \vec{B}$

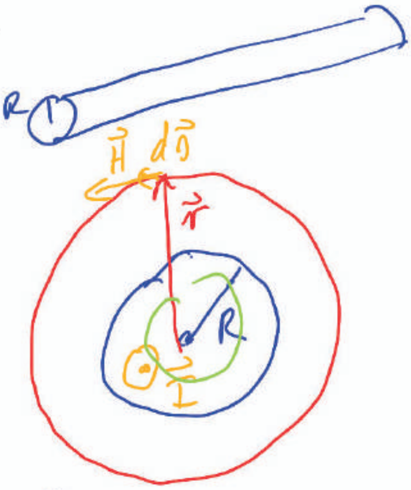
- ENERGIJA:

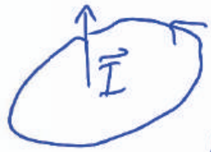
$W = -\vec{p}_m \cdot \vec{B}$

[W] S

13.4 nal 7

I, R





AMPEROV Z.

$$\boxed{\oint \vec H \cdot d\vec s = I}$$

KER NI MAGNETNIH MONOPOLOV POLJE KAŽE TANGENTNO

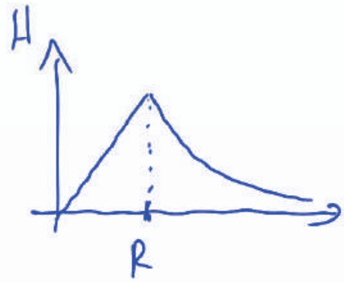
↓

$\vec H \parallel d\vec s$

• $r > R$: $\oint \vec H \cdot d\vec s = I = I_0$ ← VES TOK

$H \cdot 2\pi r = I_0$

$$\boxed{H = \frac{I_0}{2\pi r}} \Rightarrow \boxed{\vec H = \frac{\vec I \times \vec r}{2\pi r^2}} \Rightarrow \boxed{\vec B = \frac{\mu_0 \vec I \times \vec r}{2\pi r^2}}$$



$$\vec B = \mu \mu_0 \vec H$$

GOSTOTA ↑   ↑ JAKOST

• $r < R$: $\oint \vec H \cdot d\vec s = I = I_0 \frac{r^2}{R^2}$ ← $I = I_0 \frac{\pi r^2}{\pi R^2}$

$H \cdot 2\pi r = I_0 \frac{r^2}{R^2}$

$$\boxed{H = \frac{I_0 r}{2\pi R^2}} \Rightarrow \boxed{\vec B = \frac{\mu_0 \vec I_0 \times \vec r}{2\pi R^2}}$$

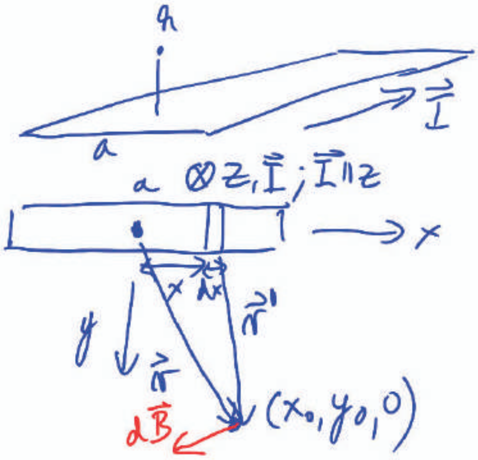
(13.4) nal 8

$a = 2\ \text{cm}$

$I = 1\ \text{A}$

$r = 1\ \text{cm}$

$B = ?$



ZA ŽICO:

$$\vec{B} = \frac{\mu_0\, \vec{I}\times\vec{r}}{2\pi r^2}$$

$$d\vec{B} = \frac{\mu_0\, d\vec{I}\times\vec{r}\,'}{2\pi r'^2}$$

$$\vec{r} = (x_0, y_0, 0)$$
$$\vec{r}\,' = (x_0 - x, y_0, 0)$$
$$d\vec{I} = (0, 0, dI)$$
$$dI = I_0 \frac{dx}{a}$$

$$d\vec{I}\times\vec{r}\,' = \begin{vmatrix} i & j & k \\ 0 & 0 & dI \\ x_0 - x & y_0 & 0 \end{vmatrix}\begin{matrix} i & j \\ 0 & 0 \\ x_0 - x & y_0 \end{matrix} = \left(-dI\, y_0,\ dI(x_0 - x),\ 0\right)$$

$$r'^2 = (x_0 - x)^2 + y_0^2$$

$$d\vec{B} = \frac{-\mu_0 I_0}{2\pi a}\left(\frac{y_0\,dx}{(x-x_0)^2 + y_0^2},\ \frac{dx\,(x - x_0)}{(x-x_0)^2 + y_0^2}\right)$$

$$\vec{B} = -\frac{\mu_0 I_0}{2\pi a}\left(\int_{-a/2}^{a/2}\frac{y_0}{(x-x_0)^2 + y_0^2}dx,\ \int_{-a/2}^{a/2}\frac{(x-x_0)}{(x-x_0)^2 + y_0^2}dx\right)$$

$u = x - x_0 \Rightarrow du = dx$

$$\vec{B} = -\frac{\mu_0 I_0}{2\pi a}\left(\int_{-\frac{a}{2}-x_0}^{\frac{a}{2}-x_0}\frac{y_0}{u^2 + y_0^2}du,\ \int_{-\frac{a}{2}-x_0}^{a/2 - x_0}\frac{u}{u^2 + y_0^2}du\right)$$

$z = u^2 + y_0^2$

$dz = 2u\,du$

$u^2 + y_0^2 = z^2$

$u = z \sin\varphi$

$y_0 = z\cos\varphi$

$\frac{u}{y_0} = \operatorname{tg}\varphi$

$du = \frac{y_0\, d\varphi}{\cos^2\varphi}$

$$\vec{B} = -\frac{\mu_0 I_0}{2\pi a}\left(\operatorname{arctg}\frac{u}{y_0}\Big|_{-\frac{a}{2}-x_0}^{\frac{a}{2}-x_0},\ \frac{1}{2}\ln\left(u^2 + y_0^2\right)\Big|_{-a/2 - x_0}^{a/2 - x_0}\right)$$

$$\vec{B} = -\frac{\mu_0 I_0}{2\pi a}\left(\operatorname{arctg}\left(\frac{\frac{a}{2}-x_0}{y_0}\right) - \operatorname{arctg}\left(\frac{-\frac{a}{2}-x_0}{y_0}\right),\ \frac{1}{2}\ln\frac{\left(\frac{a}{2}-x_0\right)^2 + y_0^2}{\left(\frac{a}{2}+x_0\right)^2 + y_0^2}\right)$$

$$\vec{B}(0, y_0) = -\frac{\mu_0 I_0}{2\pi a}\left(2\operatorname{arctg}\left(\frac{a}{2y_0}\right),\ 0\right)$$

⊗$\vec{I}$ $\vec{B}$

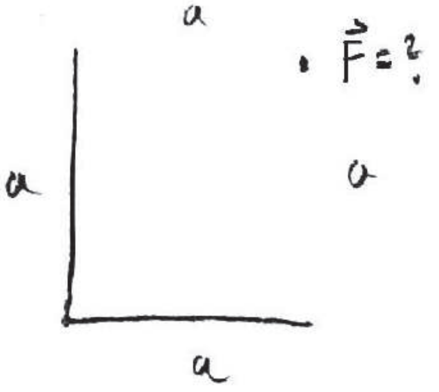
$$\vec{B}\left(x_0 > \tfrac{a}{2}, 0\right) = -\frac{\mu_0 I_0}{2\pi a}\left(0,\ \ln\frac{\frac{a}{2}-x_0}{\frac{a}{2}+x_0}\right)$$

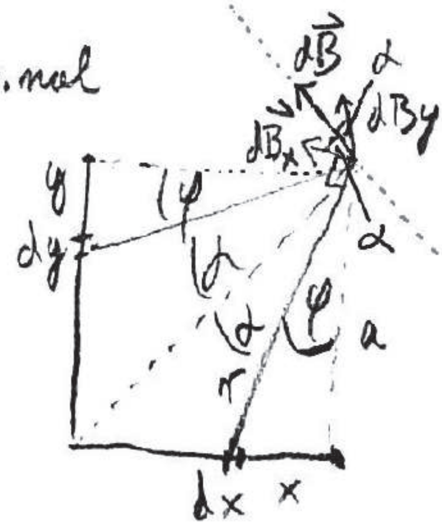
⊗$\vec{I}$ $\vec{B}$

ZA $a \to 0$: $\vec{B} = -\frac{\mu_0 I_0}{2\pi a}\,2\,\frac{a}{2y_0} = -\frac{\mu_0 I_0}{2\pi y_0}$ ← ZA ŽICO

$\operatorname{arctg}\varphi \to \varphi$

13.4.{9} 89/90/3.rol/4.nal





$$d\vec{B} = d\vec{B}_x + d\vec{B}_y$$

$$|d\vec{B}_x| = |d\vec{B}_y|$$

$$|d\vec{B}| = 2 \cdot |d\vec{B}_x| \cdot \cos\alpha$$

$$\alpha = \frac{\pi}{4} - \varphi$$

$$|d\vec{B}| = 2 \cdot |d\vec{B}_x| \cdot \cos\left(\frac{\pi}{4} - \varphi\right)$$

$$dB_x = \frac{\mu_0 dI}{2\pi r} \; ; \; r = \frac{a}{\cos\varphi} \; ; \; dI = \frac{I_0}{2} \cdot \frac{dx}{a} \; ; \; x = a \cdot tg\,\varphi$$

$$dI = \frac{I_0}{2} \cdot \frac{d\varphi}{\cos^2\varphi} \qquad dx = \frac{a \cdot d\varphi}{\cos^2\varphi}$$

$$dB_x = \frac{\mu_0 I_0 \cdot d\varphi \cdot \cos\varphi}{2\pi \cdot 2 \cdot \cos^2\varphi \, a}$$

$$dB_x = \frac{\mu_0 I_0}{4\pi a} \cdot \frac{d\varphi}{\cos\varphi}$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\Rightarrow |d\vec{B}| = 2 \cdot |d\vec{B}_x| \cdot \cos\left(\frac{\pi}{4} - \varphi\right)$$

$$dB = \frac{\mu_0 I_0}{2\pi a} \cdot \frac{d\varphi}{\cos\varphi} \cdot \cos\left(\frac{\pi}{4} - \varphi\right) = \frac{\mu_0 I_0}{2\pi a} \frac{\frac{\sqrt{2}}{2}(\cos\varphi + \sin\varphi)}{\cos\varphi} =$$

$$= \frac{\sqrt{2}\mu_0 I_0}{4\pi a} \cdot (1 + tg\,\varphi)\, d\varphi$$

$$u = \cos\varphi \qquad du = -\sin\varphi \, d\varphi \Rightarrow tg\,\varphi \, d\varphi = -\frac{du}{u}$$

$$B = \frac{\sqrt{2}\mu_0 I_0}{4\pi a} \cdot \int_0^{\frac{\pi}{4}} (1 + tg\,\varphi)\, d\varphi = \frac{\sqrt{2}\mu_0 I_0}{4\pi a} \cdot \left(\frac{\pi}{4} - \ln\frac{\cos\frac{\pi}{4}}{\cos 0}\right) =$$

$$= \frac{\sqrt{2}\mu_0 I_0}{4\pi a} \left(\frac{\pi}{4} + \ln\sqrt{2}\right)$$

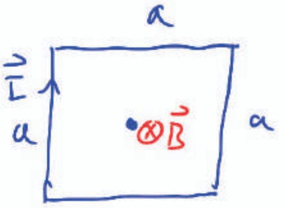
$$\Rightarrow F = I_0 \vec{l} \times \vec{B} = I_0 l B = \frac{\sqrt{2}\mu_0 I_0^2 l}{4\pi a} \cdot \left(\frac{\pi}{4} + \ln\sqrt{2}\right)$$

ZBIRKA 9   nal 1/st 52

$a = 5\ cm$

$I = 1\ A$

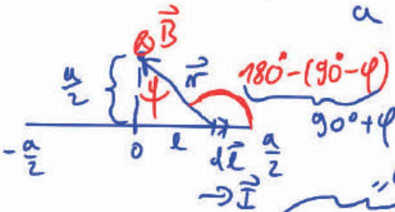
$B(\text{v sredini}) = ?$



BIOT–SAVART

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{l} \times \vec{r}}{r^3}$$

• POLJE ENE STRANICE:



$$|dl \times r| = dl \cdot r \sin(90° + \varphi) = dl \cdot r \cos\varphi$$

$$l = \frac{a}{2} \cdot tg\varphi \Rightarrow dl = \frac{a}{2}\frac{d\varphi}{\cos^2\varphi}$$

$$r = \frac{a}{2}\frac{1}{\cos\varphi}$$

$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{dl\, \cancel{r} \cos\varphi}{r^{\cancel{3}2}}$$

$$= \frac{\mu_0 I}{\cancel{4}\pi} \frac{\cancel{a}\, d\varphi \cos\varphi \cdot \cancel{4\cos^2\varphi}}{2\cancel{\cos^2\varphi}\, a^{\cancel{2}}}$$

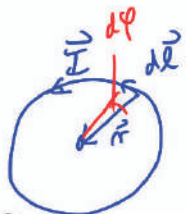
$$\underline{dB = \frac{\mu_0 I}{2\pi a} \cos\varphi\, d\varphi}$$

$$B = \frac{\mu_0 I}{2\pi a} \overbrace{\int_{-\pi/4}^{\pi/4} \cos\varphi\, d\varphi}^{=2\cdot\frac{\sqrt{2}}{2}}$$

$$\underline{B = \frac{\mu_0 I}{2\pi a} \cdot \sqrt{2}} \leftarrow \text{ENA STRANICA}$$

$$\Rightarrow \text{SKUPNO POLJE } \underline{B_\square = 4 \cdot B = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}}$$

• ZA OBROČ:



$$|d\vec{l} \times \vec{r}| = dl\, r$$

$$dl = r\, d\varphi$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0 I}{4\pi} \cdot \frac{dl\, \cancel{r}}{r^{\cancel{3}2}} = \frac{\mu_0 I}{4\pi} \frac{\cancel{r}\, d\varphi}{r^{\cancel{2}}}$$

$$B = \frac{\mu_0 I}{4\pi r}\int_0^{2\pi} d\varphi = \underline{B_\circ = \frac{\mu_0 I}{2r}}$$

ZA ENAK DOLG OVOJ:

$$l_\square = l_\circ = l$$

$$l_\square = 4a \Rightarrow a = \frac{l}{4}$$

$$l_\circ = 2\pi r \Rightarrow r = \frac{l}{2\pi}$$

$$\frac{B_\square}{B_\circ} = \frac{\frac{2\sqrt{2}\cdot 4}{\pi l}}{\frac{1\cdot \cancel{2}\pi}{\cancel{2} l}} = \frac{8\sqrt{2}}{\pi^2} = \underline{1{,}15}$$

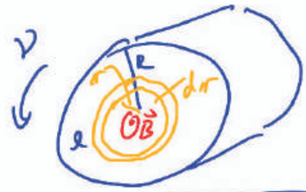
ZBIRKA 9 nal 3/str 52

$R = 5\,cm$

$e = 10^{-5}\,As$

$\nu = 80\,s^{-1}$

$B(r=0) = ?$



POSAMEZEN OBROČ
↳ IZ PREJŠNJE NALOGE

$$\boxed{dB = \frac{\mu_0\, dI}{2r}}$$

$$dI = I_0 \frac{2\cancel{\pi} r\,dr}{\cancel{\pi} R^2}$$

$$\boxed{dI = \frac{2e\nu}{R^2}\, r\,dr}$$

$$; \quad \boxed{I_0} = \frac{de}{dt}\cdot\frac{d\varphi}{d\varphi} = \omega \frac{de}{d\varphi} = 2\pi\nu \frac{e}{2\pi} = \boxed{e\nu}$$

($\frac{d\varphi}{dt} = \omega$, $\frac{de}{d\varphi} = \frac{e}{2\pi}$)

$$= \frac{e}{t_0} \leftarrow \text{CEL NABOJ V ENEM OBHODNEM ČASU}$$

$$\boxed{B} = \int_0^R \frac{\mu_0 \cancel{2} e\nu \cancel{r}\,dr}{\cancel{2} R^2 \cancel{r}} = \frac{\mu_0 e\nu R}{R^2} = \boxed{\frac{\mu_0 e\nu}{R}} = \underline{20{,}1\,mT}$$

ZBIRKA 9 nal 19/str 54

$N_1 = 100$

$l_1 = 1\,m$

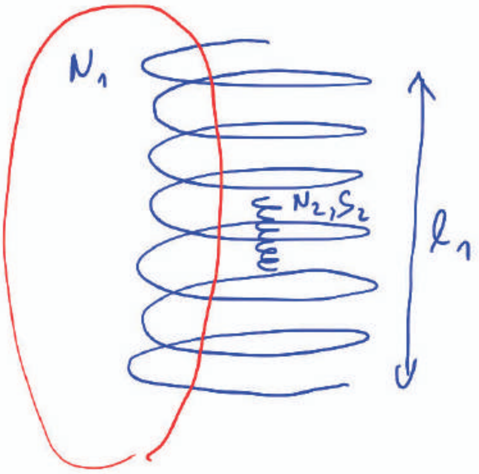
$I_1 = 25\,A$

$N_2 = 500$

$S_2 = 20\,cm^2$

$I_2 = 0{,}1\,A$

$A = ?$



$\Rightarrow$

$B_1 = \frac{\mu_0 N_1 I_1}{l_1}$ ← DOLGA TULJAVA

$\vec{p}_{m_2} = N_2 I_2 \vec{S}_2$ ← PLOŠČATA TULJAVA

· Z NAVOROM: $A = \int_0^{\pi} M\,d\varphi$ $\qquad \vec{M} = \vec{p}_m \times \vec{B}$

$= p_m B \int_0^{\pi} \sin\varphi\,d\varphi = 2 \cdot p_m B$ $\qquad \mu_0 = 4\pi \cdot 10^{-7}$

$A = \frac{N_2 I_2 S_2 \mu_0 N_1 I_1}{l_1} \cdot 2 = 2\pi \cdot 10^{-4}\,J$

· Z ENERGIJAMI: $A = \Delta W$ $\qquad W = -\vec{p}_m \cdot \vec{B}$

$= p_m B - (-p_m B) = 2 p_m B$ $\qquad = -p_m \cdot B \cos\varphi$

NA KONCU $\varphi = \pi$ $\quad$ NA ZAČETKU $\varphi = 0$

ZBIRKA 9 zad 8/str 52

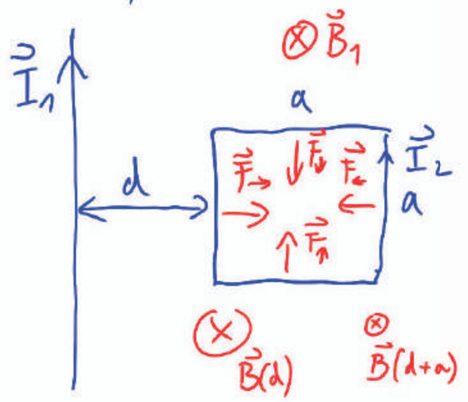
$I_1 = 30\,A$

$a = 5\,cm$

$I_2 = 10\,A$

$d = 3\,cm$

$F = ?$



$\vec{F} = I\vec{\ell} \times \vec{B}$

$|F_{\downarrow}| = |F_{\uparrow}|$

$|F_{\rightarrow}| > |F_{\leftarrow}|$

$\Rightarrow$ SKUPNA SILA KAŽE DESNO

$$F_{\rightarrow} = I_2\, a\, B_1(d) = \frac{I_2 a \mu_0 I_1}{2\pi d}$$

$$F_{\leftarrow} = I_2\, a\, B_1(d+a) = \frac{I_2 a \mu_0 I_1}{2\pi (d+a)}$$

$$F = F_{\rightarrow} - F_{\leftarrow} = \frac{I_2 a \mu_0 I_1}{2\pi}\left(\frac{1}{d} - \frac{1}{d+a}\right) = \frac{I_2 a^2 \mu_0 I_1}{2\pi d(d+a)}$$

$$= 6{,}25 \cdot 10^{-5}\,N$$

ZBIRKA 9 nal 5/st 52

$\mu = 300$

$N = 200$

$r = 10\,\text{cm}$

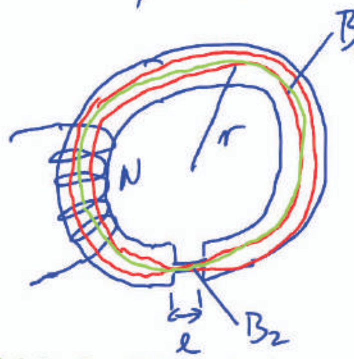
$S_1 = 3\,\text{cm}^2$

$l = 2\,\text{cm}$

$S_2 = 0{,}3\,\text{cm}^2$

$I = 1\,\text{A}$

$B_1, B_2 = ?$



FEROMAGNETNO JEDRO ZADRŽI SILNICE!

$\hookrightarrow B_2 > B_1$

$\Rightarrow$ OHRANJA SE $\phi_m = \vec{B}\cdot\vec{S}$

$$\phi_{m_1} = \phi_{m_2}$$

$$B_1 S_1 = B_2 \cdot S_2 \Rightarrow \underline{B_2 = B_1 \frac{S_1}{S_2}}$$

$$\mu\mu_0 H_2 = \mu\mu_0 H_1 \frac{S_1}{S_2}$$

$$\underline{H_2 = H_1 \frac{S_1}{S_2}}$$

GREMO VZPOREDNO S SILNICAMI DA $\vec{H} \parallel d\vec{s}$

$$\oint \vec{H}\cdot d\vec{s} = NI$$

$$H_1(2\pi r - l) + H_2 l = N\cdot I$$

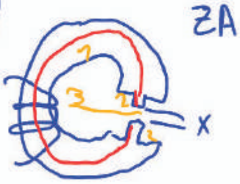
$$H_1\left(2\pi r - l + l\frac{S_1}{S_2}\right) = N\cdot I$$

$$\underline{H_1 = \frac{NI}{2\pi r + l\left(\frac{S_1}{S_2} - 1\right)}} \Rightarrow \underline{B_1 = \frac{\mu\mu_0 N I}{2\pi r + l\left(\frac{S_1}{S_2} - 1\right)}}$$

$$\underline{B_1 = 0{,}093\,\text{T}}\ ;\ \underline{B_2 = 0{,}93\,\text{T}}$$

b) REŽA $x = 0.5\,\text{mm}$:

b-1)



ZA OZKO REŽO:

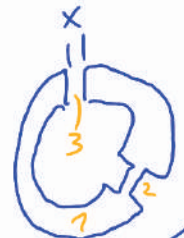
$$\phi_{m_2} = \phi_{m_3}$$

$$B_2 S_2 = B_3 S_3 \Rightarrow B_2 = B_3$$

$$\mu\mu_0 H_2 = \mu_0 H_3 \Rightarrow \underline{H_3 = \mu H_2}$$

$$\underline{H_3 = \mu \frac{S_1}{S_2} H_1}$$

b-2)



$\hookrightarrow \oint \vec{H}\, d\vec{s} = NI$

$$H_1(2\pi r - l) + H_2(l - x) + H_3 x = NI$$

$$H_1\left[(2\pi r - l) + \frac{S_1}{S_2}(l - x) + \mu\frac{S_1}{S_2}x\right] = NI$$

$$\underline{H_1 = \frac{NI}{2\pi r + l\left(\frac{S_1}{S_2} - 1\right) + x\frac{S_1}{S_2}(\mu - 1)}}\ ;\ \underline{B_1 = \mu\mu_0 H_1}$$

b-2) $\phi_{m_3} = \phi_{m_1}$

$$B_3 = B_1 \Rightarrow H_3 = \mu H_1$$

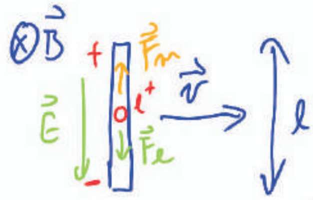
$$H_1(2\pi r - l - x) + H_2 l + H_3 x = NI$$

$$H_1\left[2\pi r - l - x + \frac{S_1}{S_2} l + \mu x\right] = NI$$

$$\underline{H_1 = \frac{NI}{2\pi r + l\left(\frac{S_1}{S_2} - 1\right) + x(\mu - 1)}}$$

13.5 Indukcija

INDUKCIJA



$\vec{F}_m = e(\vec{v}\times\vec{B})$

$\vec{F}_e = -e\vec{E}$

$\vec{E} = -\vec{v}\times B$

ĆE $v \perp B \perp \vec{E}$

$U_i = lE = lvB$

$$\underline{U_i = \vec{l}\cdot(\vec{v}\times\vec{B})} = -\frac{d(\vec{B}\cdot\vec{S})}{dt} = \underline{-\frac{d\phi_m}{dt}}$$

ZBIRKA 9 zad 2 / str 54

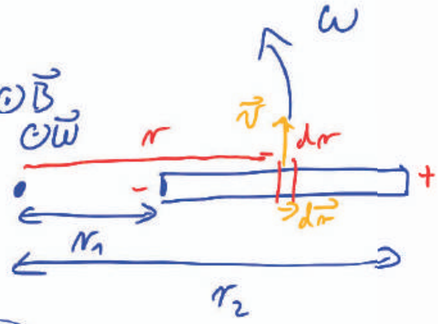
$B = 0.5\,T$

$\nu = 2\,s^{-1}$

$r_1 = 25\,cm$

$r_2 = 75\,cm$

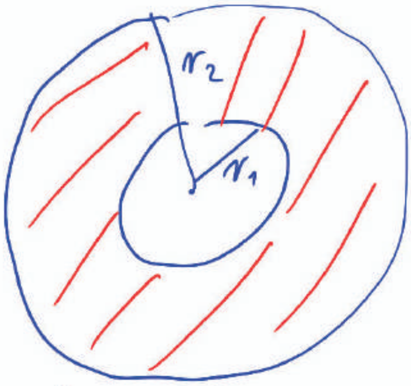
$U_i = ?$



$$dU_i = d\vec{r}\cdot(\vec{v}\times\vec{B}) = dr\,vB = dr\,\omega r B$$

$$U_i = 2\pi\nu B\int_{r_1}^{r_2} r\,dr = 2\pi\nu B\,\frac{r_2^2 - r_1^2}{2}$$

$$\underline{U_i = \pi\nu B(r_2^2 - r_1^2)}$$



$\phi_m = \vec{B}\cdot\vec{S}$

$$U_i = -\frac{d\phi_m}{dt} = -\frac{B\cdot(\pi r_2^2 - \pi r_1^2)}{t_0} = \underline{-B\pi\nu(r_2^2 - r_1^2)}$$

ZBIRKA 9 nal 4/st54

$R = 20\,cm$

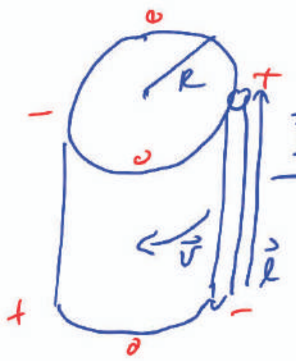
$t_0 = 0.2\,s$

$B = 0.4\,T$

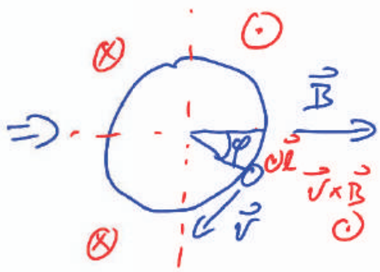
$U_i(t) = ?$

$U_{i0} = ?$

$l = 0.5\,m$



$U_i = \vec{l} \cdot (\vec{v} \times \vec{B})$

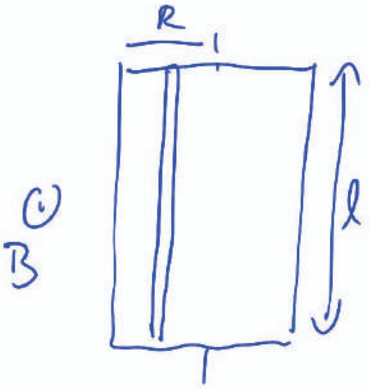


$|\vec{v} \times \vec{B}| = vB \cdot \sin(\frac{\pi}{2} + \varphi) = vB\cos\varphi$

$$U_i = lvB\cos\varphi = lvB\cos\omega t = \underbrace{lvB}_{U_{i0}}\cos\left(\frac{2\pi}{t_0} t\right)$$

$$U_i = \underbrace{\frac{l\,2\pi R B}{t_0}}_{U_{i0}} \cos\left(\frac{2\pi}{t_0} t\right)$$

$$U_{i0} = 0.4\pi\ V$$



$$\frac{d\Phi_m}{dt} = \frac{d\,\vec{B}\cdot\vec{S}}{dt} = \omega B l R \cos\omega t$$

$S = lR$

$\vec{B}\cdot\vec{S} = B \cdot S \sin\varphi$

$= BlR\sin\omega t$

ZBIRKA 9   nal 13/st 55

$I = 300A$

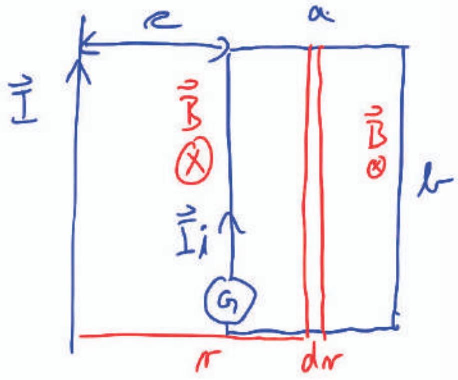
$a = 5$ cm

$b = 10$ cm

$c = 3$ cm

$R = 10\Omega$

$\int I_i dt = e = ?$



$I_i = \frac{U_i}{R} \rightarrow$ ŽELI OHRANJATI $\vec{B}$

$U_i = -\frac{d\phi_m}{dt}$

$d\phi_m = B \cdot dS = B \cdot b \cdot dr$

$dS = b\,dr$

$B = \frac{\mu_0 I}{2\pi r}$

$\hookrightarrow \underbrace{d\phi_m = \frac{\mu_0 I b}{2\pi} \frac{dr}{r}}$

$$\int I_i dt = \int \frac{U_i}{R} dt = -\int \frac{d\phi_m}{\cancel{dt}} \frac{1}{R} \cancel{dt}$$

$$= -\frac{1}{R} \int_{\phi_{m0}}^{0} \phi_m = +\frac{1}{R} \underbrace{\frac{\mu_0 I b}{2\pi} \int_{c}^{c+a} \frac{dr}{r}}_{= \phi_{m0}}$$

$$\underline{\int I_i dt} = \underline{+\frac{\mu_0 I b}{2\pi R} \ln \frac{c+a}{c}} = \underline{\underline{5{,}9 \cdot 10^{-7} As}}$$

ZBIRKA 9 nal 11/str 55

$S = 1\,mm^2$

$\xi = 0{,}017\,\Omega\,mm^2/m$

$a = 10\,cm$

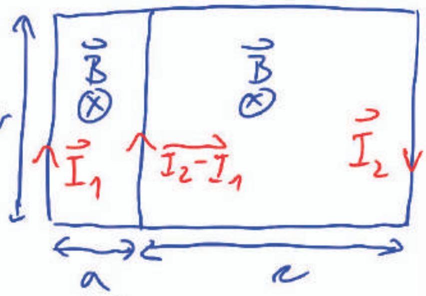
$b = 20\,cm$

$c = 30\,cm$

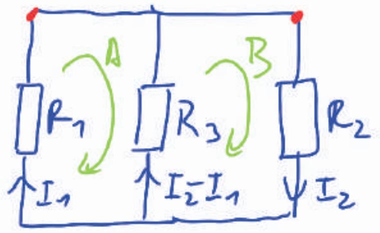
$B = 0{,}4\,T$

$t_0 = 10\,s$

$I = ?$



$\Rightarrow$



$$R_1 = \frac{(2a+b)\cdot\xi}{S} = 6{,}8\,m\Omega$$

$$R_2 = \frac{(2c+b)\xi}{S} = 13{,}6\,m\Omega$$

$$R_3 = \frac{b\xi}{S} = 3{,}4\,m\Omega$$

$$B(t) = B - \frac{B}{t_0}t$$

$$U_{iA} = -\frac{d\Phi_{mA}}{dt} = -S_A\left(-\frac{B}{t_0}\right) = \frac{abB}{t_0} = 0{,}8\,mV$$

$$U_{iB} = \frac{bc}{t_0}B = 2{,}4\,mV$$

$$A:\ (I_2 - I_1)R_3 - I_1R_1 + U_{iA} = 0 \Rightarrow \underbrace{I_2 = \frac{I_1(R_1+R_3) - U_{iA}}{R_3}}$$

$$B:\ -I_2R_2 - (I_2 - I_1)R_3 + U_{iB} = 0$$

$$-I_2(R_2+R_3) + I_1R_3 + U_{iB} = 0$$

$$-\frac{[I_1(R_1+R_3) - U_{iA}](R_2+R_3)}{R_3} + I_1R_3 + U_{iB} = 0$$

$$I_1\left(R_3 - \frac{(R_1+R_3)(R_2+R_3)}{R_3}\right) = -U_{iB} - U_{iA}\frac{R_2+R_3}{R_3}$$

$$I_1 = \frac{-U_{iB}R_3 - U_{iA}(R_2+R_3)}{R_3^2 - (R_1+R_3)(R_2+R_3)}$$

$$= -\frac{U_{iB}R_3 + U_{iA}(R_2+R_3)}{-R_1R_2 - R_1R_3 - R_3R_2 - \cancel{R_3^2} + \cancel{R_3^2}}$$

$$\underbrace{I_1 = \frac{U_{iB}R_3 + U_{iA}(R_2+R_3)}{R_1(R_2+R_3) + R_2R_3}} = \underline{\underline{0{,}13\,A}}$$

$$\underbrace{I_2 = 0{,}17\,A}$$

ZBIRKA 9 nal 15/str 56

$N = 10$

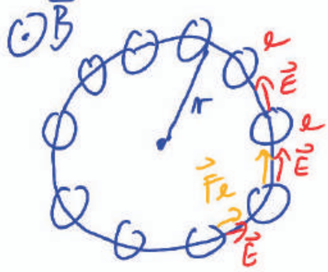
$e = +10^{-3}\,As$

$r = 10\,cm$

$J = 10^{-4}\,kg\,m^2$

$B = 0{,}5\,T$

$\omega_k = ?$



$$\oint \vec{E}\cdot d\vec{s} = 0 + U_i = -\frac{d\phi_m}{dt}$$

$$E\,2\pi r = -\frac{d\phi_m}{dt}$$

$$E = -\frac{d\phi_m}{dt}\cdot\frac{1}{2\pi r}$$

$$M = J\alpha$$

$$M_e = F_e\cdot N\cdot r = eENr = -\frac{d\phi_m}{dt}\frac{eN\cancel{r}}{2\pi\cancel{r}}$$

$$J\alpha = -\frac{d\phi_m}{dt}\frac{eN}{2\pi}$$

$$\int_0^{t_k}\alpha\,dt = -\frac{eN}{2\pi J}\int_{\phi_{m_0}}^{0} d\phi_m$$

$$\phi_{m_0} = \vec{B}\cdot\vec{S} = B\pi r^2$$

$$\omega_k = \frac{eN}{2\cancel{\pi}J}\cdot B\cancel{\pi}r^2$$

$$\omega_k = \frac{eNB}{2J}\cdot r^2 = \underline{\underline{0{,}25\,s^{-1}}}$$

ZBIRKA 9 nal 7/st 55

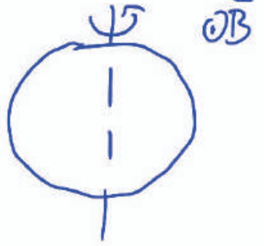
$\varrho = 0{,}05\ \Omega\ mm^2/m$

$r = 5\ cm$

$S_0 = 1\ mm^2$

$B = 0{,}7\ T$

$\overline{M}(\nu = 10\ Hz) = ?$



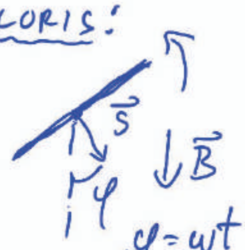
$\vec{M} = \vec{p}_m \times \vec{B}$

$\vec{p}_m = I \cdot \vec{S} \quad ; S = \pi r^2$

$I = \frac{U_i}{R}$

$R = \frac{\varrho \cdot l}{S_0} = \frac{\varrho \cdot 2\pi r}{S_0} = 0{,}016\ \Omega$

TLORIS:



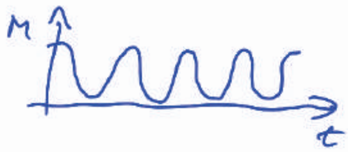
$\varphi = \omega t$

$U_i = -\frac{d\phi_m}{dt} = +BS\omega \sin \omega t$

$\phi_m = \vec{B} \cdot \vec{S} = BS \cdot \cos \omega t$

$p_m = I \cdot S = \frac{U_i}{R} \cdot S = \frac{BS^2 \omega \sin \omega t}{R}$

$M = p_m B \sin \omega t = \frac{B^2 S^2 \omega \sin^2 \omega t}{R}$ →



$\overline{M} = \frac{\int_0^{t_0} M\,dt}{t_0} = \frac{B^2 S^2 \omega}{R} \underbrace{\frac{1}{t_0} \int_0^{t_0} \sin^2 \omega t\, dt}_{1/2} = \frac{B^2 S^2 \omega}{2R} = 23{,}7\ Nm$

ZBIRKA 9 nal 17/st 56

$B = 0{,}9\,T$

$\nu_B = 60\,s^{-1}$

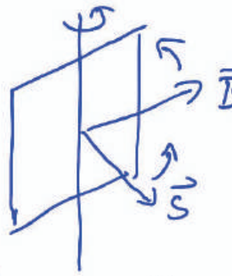
$N = 10$

$S = 10\,cm^2$

$R = 1\,\Omega$

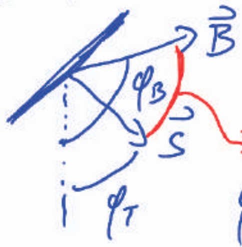
$\nu_T = 20\,s^{-1}$

$P = ?$



$\Rightarrow$ TLORIS

$\omega = 2\pi\nu$



$\varphi_B = \omega_B t$

$\varphi_B - \varphi_T = (\omega_B - \omega_T)\cdot t$

$\varphi_T = \omega_T t$

$$\vec{M} = \vec{p}_m \times \vec{B}$$

$$M = p_m B \sin(\varphi_B - \varphi_T) = p_m B \sin[(\omega_B - \omega_T)\cdot t]$$

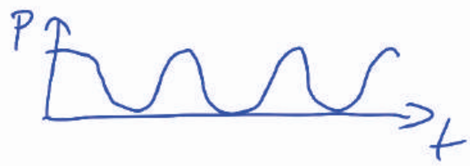
$$p_m = \frac{NBS^2(\omega_B - \omega_T)\sin[(\omega_B - \omega_T)\cdot t]}{R}$$ → IZ PREJŠNJE NALOGE

$$M = \frac{NB^2S^2(\omega_B - \omega_T)\sin^2[(\omega_B - \omega_T)t]}{R}$$ → $M \neq 0$ TUDI KO SE TULJAVA ŠE NE VRTI!

- MOČ, KI SE IZGUBLJA → GRE V TOPLOTO

- $$P = \frac{dA}{dt} = \frac{M\,d(\varphi_B - \varphi_T)}{dt} = M\,\frac{d\left([\omega_B - \omega_T]t\right)}{dt} = M(\omega_B - \omega_T)$$

$$P = \frac{NB^2S^2(\omega_B - \omega_T)^2\sin^2[(\omega_B - \omega_T)t]}{R}$$



$$\overline{P} = \frac{NB^2S^2(\omega_B - \omega_T)^2}{2R} = 1{,}28\,W$$

- MEHANSKA MOČ MOTORJA: $P = M\cdot\omega_T$

VRTENJE TULJAVE DOLOČA MOČ NA OSI → POGON

13.3 nal 6

$C = 0{,}01\,\mu F$

$R = 1\,M\Omega$

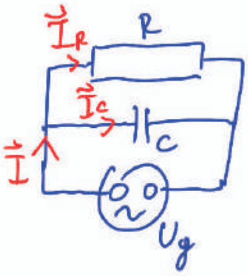
$U_g(t) = U_0 \cos\omega t$

$I(t) = ?$

$P(t), \bar{P} = ?$

REALNO

KOMPLEKSNO



$$I_R = \frac{U_g}{R} = \frac{U_0}{R}\cos\omega t$$

$$I_C = \frac{de}{dt} = C\frac{dU_C}{dt} = C\frac{dU_g}{dt} = -CU_0\omega\sin\omega t$$

$$I = I_R + I_C$$

AMPLITUDA FAZA

$$I = \frac{U_0}{R}\cos\omega t - U_0 C\omega\sin\omega t = I_0\cos(\omega t - \delta)$$

$$= I_0(\cos\omega t\cos\delta + \sin\omega t\sin\delta)$$

$$\frac{U_0}{R} = I_0\cos\delta$$

$$-U_0 C\omega = I_0\sin\delta$$

$$\operatorname{tg}\delta = -\frac{U_0 C\omega}{\frac{U_0}{R}} = -RC\omega$$

$$I_0^2\cos^2\delta + I_0^2\sin^2\delta = I_0^2\cdot 1$$

$$\frac{U_0^2}{R^2} + U_0^2C^2\omega^2 = I_0^2 \Rightarrow I_0 = U_0\cdot\sqrt{\frac{1}{R^2} + C^2\omega^2}$$

$$\Downarrow$$

$$\boxed{I = I_0\cos(\omega t - \delta)\,;\; I_0 = U_0\sqrt{\frac{1}{R^2} + C^2\omega^2}\,;\; \delta = \operatorname{arctg}(-RC\omega)}$$

MOČ: $P(t) = U_g\cdot I = U_0\cos(\omega t)\, I_0\cos(\omega t - \delta) = U_0 I_0\left(\cos^2\omega t\cos\delta + \frac{1}{2}\sin 2\omega t\sin\delta\right)$

POVPREČJE $= \frac{1}{2}$ POVPREČJE $= 0$

$$\bar{P} = \frac{U_0 I_0}{2}\cos\delta = \frac{U_0^2}{2R} = \frac{U_{eff}^2}{R}$$

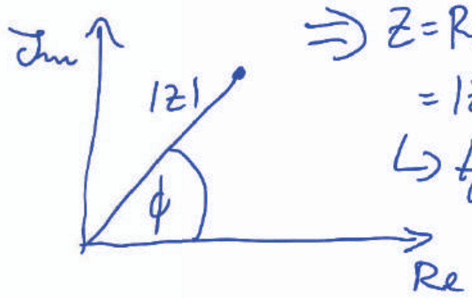
$$U_{eff} = \frac{U_0}{\sqrt{2}}$$

→ ZA OMREŽJE:

$U_{eff} = 220\,V$, $U_0 = 310\,V$

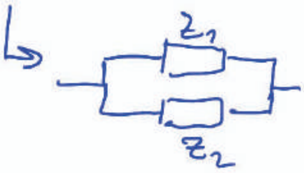
KOMPLEKSEN ZAPIS:

$U_g = U_0 \cdot \cos \omega t \Rightarrow U_g = U_0 e^{i\omega t}$



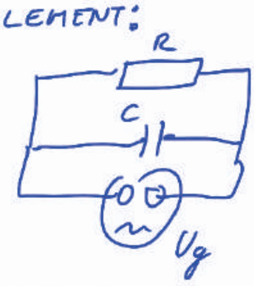
$\Rightarrow z = Re(z) + i\, Im(z)$
$= |z| \cdot e^{i\phi}$
$\hookrightarrow tg\, \phi = \frac{Im(z)}{Re(z)}$

→ IMPEDNCA: IMA VLOGO UPORA ZA POSAMEZEN ELEMENT:



$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$Z = Z_1 + Z_2$



- ZA UPOR: $Z_R = R$
- ZA KONDENZATOR: $Z_C = \frac{1}{i\omega C}$

MOJE VEZJE: $U = ZI$; $Z \to$ NADOMESTNA IMPEDANCA

$\hookrightarrow \frac{1}{Z} = \frac{1}{Z_R} + \frac{1}{Z_C} = \frac{1}{R} + i\omega C$

$I = \frac{U}{Z} = U \cdot \left(\frac{1}{R} + i\omega C\right)$

KOMPLEKSEN $\Rightarrow I = |I| e^{i\varphi}$

$U = U_0 e^{i\omega t} \Rightarrow |U| = U_0$

$|I| = |U| \sqrt{\frac{1}{R^2} + \omega^2 C^2}$; $\varphi_U = 0$; $\varphi_{(I)} = \operatorname{arctg} \frac{\omega C}{\frac{1}{R}} = \operatorname{arctg} RC\omega$

$I = \underbrace{U_0 \sqrt{\frac{1}{R^2} + \omega^2 C^2}}_{I_0}\, e^{i\omega t} e^{i\varphi_{(I)}}$

$\hookrightarrow$ NAZAJ V REALNO: $I = I_0 \cdot \cos(\omega t + \varphi_{(I)})$

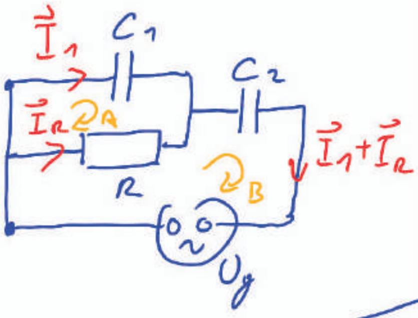
(13.3) nal 8

$U_g(t) = U_0 \cos\omega t$

R, C_1, C_2

$I_R = ?$

$U_{C_2} = ?$



$\xrightarrow{Z}$ ⇒ PADEC NAPETOSTI $-Z \cdot I$, $\rightarrow \vec{I}$

$$A: \; -I_1 Z_{C_1} + I_R Z_R = 0$$

$$B: \; -I_R Z_R - (I_1 + I_R) Z_{C_2} + U_g = 0$$

$$A \Rightarrow \; I_1 = I_R \frac{Z_R}{Z_{C_1}}$$

$$-I_R Z_R - I_R\left(\frac{Z_R}{Z_{C_1}} + 1\right) Z_{C_2} + U_g = 0$$

$$Z_R = R \qquad Z_{C_1} = \frac{1}{i\omega C_1} \qquad Z_{C_2} = \frac{1}{i\omega C_2}$$

$$I_R = \frac{U_g}{Z_R + Z_R \frac{Z_{C_2}}{Z_{C_1}} + Z_{C_2}} = \frac{U_g}{Z}$$

$$Z = R + R\frac{C_1}{C_2} + \frac{1}{i\omega C_2} = \underbrace{R\left(1 + \frac{C_1}{C_2}\right)}_{Re} - i \underbrace{\frac{1}{\omega C_2}}_{Im}$$

$$= |Z| e^{i\varphi}$$

$$|Z| = \sqrt{R^2\left(1 + \frac{C_1}{C_2}\right)^2 + \frac{1}{\omega^2 C_2^2}} = R\sqrt{\left(1 + \frac{C_1}{C_2}\right)^2 + \frac{1}{R^2 C_2^2 \omega^2}}$$

$$\tan\varphi = \frac{-1}{\omega C_2 R\left(1 + \frac{C_1}{C_2}\right)} = \frac{-1}{R(C_2 + C_1)\omega}$$

$$I_R = \frac{U_0 e^{i\omega t}}{|Z| e^{i\varphi}} = \frac{U_0}{|Z|} e^{i(\omega t - \varphi)}$$

↳ KONČNO $Re(I_R) \rightarrow$ $\boxed{I_R = \frac{U_0}{R\sqrt{\left(1 + \frac{C_1}{C_2}\right)^2 + \frac{1}{R^2 C_2^2 \omega^2}}} \cdot \cos(\omega t - \varphi)}$

$U_{C_2} = ?$

$$U_{C_2} = U_g - U_R = U_0 e^{i\omega t} - I_R R = U_0 e^{i\omega t} - \frac{U_0 \cdot R}{|Z|} e^{i(\omega t - \varphi)}$$

$$= U_0 e^{i\omega t}\left[1 - \frac{R}{|Z|} e^{-i\varphi}\right] = U_0\left(e^{i\omega t} - \frac{R}{|Z|} e^{i(\omega t - \varphi)}\right)$$

→ V REALNO: $U_{C_2} = U_0\left(\cos\omega t - \frac{R}{|Z|}\cos(\omega t - \varphi)\right)$

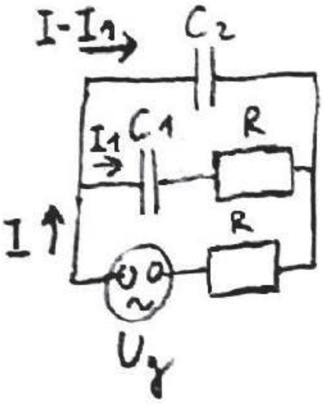
↳ DN (13.3) nal 7

13.3. {7}

$U_g = U_0 \cdot \sin \omega t$

R, C_1, C_2

$I_1(t) = ?$



A) ZGORNJI KROG:

$$-(I-I_1)\cdot Z_{C_2} + I_1 R + I_1 Z_{C_1} = 0$$

B) VELIK KROG:

$$-(I-I_1)\cdot Z_{C_2} - IR + U_g = 0$$

IZ A)

$$I = \frac{I_1}{Z_{C_2}}\left(Z_{C_2} + R + Z_{C_1}\right) = I_1\left(1 + \frac{R}{Z_{C_2}} + \frac{Z_{C_1}}{Z_{C_2}}\right)$$

V B)

$$-I_1\left(1 + \frac{R}{Z_{C_2}} + \frac{Z_{C_1}}{Z_{C_2}} - 1\right) Z_{C_2} - I_1\left(1 + \frac{R}{Z_{C_2}} + \frac{Z_{C_1}}{Z_{C_2}}\right) R + U_g = 0$$

$$I_1\left(R + Z_{C_1} + R + \frac{R^2}{Z_{C_2}} + \frac{R \cdot Z_{C_1}}{Z_{C_2}}\right) = U_g$$

$$I_1 = \frac{U_g}{R}\left(2 + \frac{Z_{C_1}}{R} + \frac{R + Z_{C_1}}{Z_{C_2}}\right)^{-1}$$

$\Rightarrow U_g = U_0 \cdot \sin \omega t = U_0 \cdot \operatorname{Im}\left(e^{i\omega t}\right) \rightarrow$ GLEDAMO Im DEL !!

$$I_1 = \frac{U_0 e^{i\omega t}}{R}\left(2 + \frac{1}{i\omega R C_1} + \frac{R + \frac{1}{i\omega C_1}}{\frac{1}{i\omega C_2}}\right)^{-1} =$$

$$= \frac{U_0 e^{i\omega t}}{R} \cdot \left(2 - \frac{i}{\omega R C_1} + i\omega R C_2 + \frac{C_2}{C_1}\right)^{-1} =$$

$$I_1 = \frac{U_0 e^{i\omega t}}{R \cdot \left[2 + \frac{C_2}{C_1} + i\left(\omega R C_2 - \frac{1}{\omega R C_1}\right)\right]} = \frac{U_0 e^{i\omega t}}{Z}$$

$$Z = |Z| \cdot e^{i\varphi} \Rightarrow |Z| = R \cdot \sqrt{\left(2 + \frac{C_2}{C_1}\right)^2 + \left(\omega R C_2 - \frac{1}{\omega R C_1}\right)^2}$$

$$\operatorname{tg} \varphi = \frac{\left(\omega R C_2 - \frac{1}{\omega R C_1}\right)}{2 + \frac{C_2}{C_1}}$$

$$I_1 = \frac{U_0 e^{i\omega t}}{|Z| \cdot e^{i\varphi}} = \frac{U_0}{|Z|} \cdot e^{i(\omega t - \varphi)}$$

GLEDAMO Im DEL:

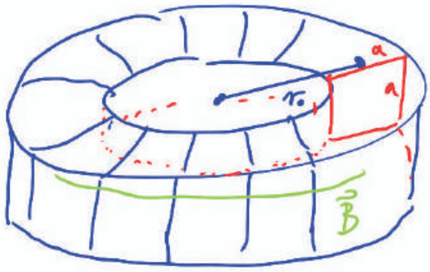
$$\underbrace{I_1 = \frac{U_0 \cdot \sin(\omega t - \varphi)}{R\sqrt{\left(2 + \frac{C_2}{C_1}\right)^2 + \left(\omega R C_2 - \frac{1}{\omega R C_1}\right)^2}} \; ; \; \varphi = \operatorname{arctg} \frac{\left(\omega R C_2 - \frac{1}{\omega R C_1}\right)}{2 + \frac{C_2}{C_1}}}$$

MOČ NA UPORU ZRAVEN C_1: $P = \frac{1}{2}|I_1|^2 R$

13.6 Vezave tuljave, upora in kondenzatorja

13.6 nal 1

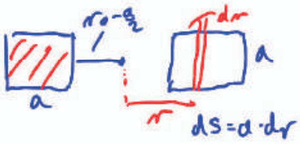
r_0, a, N



- INDUKTIVNOST:
  - $L = \frac{\phi_m}{I} \Leftrightarrow \phi_m = LI$
  - LASTNOST TULJAVE

- TLORIS:

STRANSKI RIS:



$$\oint \vec{H} \cdot d\vec{l} = NI$$

$$H \cdot 2\pi r = NI \quad B = \mu_0 H$$

$$\underline{B(r) = \frac{\mu_0 N I}{2\pi r}}$$

$$\underline{d\phi_m = N \cdot B\, dS = NBa\,dr = \frac{\mu_0 N^2 I a}{2\pi} \frac{dr}{r}}$$

$$\underline{\phi_m = \frac{\mu_0 N^2 I a}{2\pi} \int_{r_0-\frac{a}{2}}^{r_0+\frac{a}{2}} \frac{dr}{r} = \frac{\mu_0 N^2 I a}{2\pi} \ln \frac{r_0+\frac{a}{2}}{r_0-\frac{a}{2}}}$$

$$\underline{L = \frac{\phi_m}{I} = \frac{\mu_0 N^2 a}{2\pi} \ln \frac{2r_0+a}{2r_0-a}}$$

- LIMITA: $a \ll r_0$

$$L = \frac{\mu_0 N^2 a}{2\pi} \underbrace{\left[\ln \frac{1+\frac{a}{2r_0}}{1-\frac{a}{2r_0}}\right]}$$

$$\left[\underbrace{\ln\left(1+\frac{a}{2r_0}\right)}_{\simeq \frac{a}{2r_0}} - \underbrace{\ln\left(1-\frac{a}{2r_0}\right)}_{\simeq -\frac{a}{2r_0}}\right] \simeq \frac{a}{r_0}$$

$$\underline{L = \frac{\mu_0 N^2 a^2}{2\pi r_0} \; ; \; a \ll r_0}$$

$a^2 \rightarrow$ PRESEK

ZBIRKA 9 nal 21/st 56

$L = 0.01\,H$

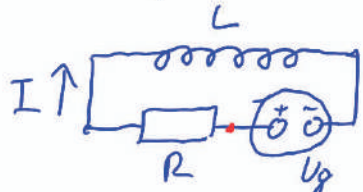
$U_g = 2V$

$R = 0{,}1\,\Omega$

$t\left(W_L = \frac{W_{L\,max}}{2}\right) = ?$

$U_L, U_R(t' = 0{,}01\,s) = ?$

→ DW



$$W_L = \frac{L I^2}{2}$$

KADAR GREMO V SMERI TOKA:

$$U_L = -L\frac{dI}{dt} = -\frac{d\Phi_m}{dt}$$

$$-IR - L\frac{dI}{dt} + U_g = 0 \,/ : R \quad ; \quad \underset{\parallel}{U_R} + \underset{\parallel}{U_L} = U_g$$

$IR \quad L\frac{dI}{dt}$

$$I + \frac{L}{R}\frac{dI}{dt} = \frac{U_g}{R}$$

$$\tau\frac{dI}{dt} = \frac{U_g}{R} - I$$

$$\int_0^I \frac{dI}{\frac{U_g}{R} - I} = \int_0^t \frac{dt}{\tau}$$

$$\frac{U_g}{R} - I = u$$

$$-dI = du$$

$$-\int_{\frac{U_g}{R}}^{\frac{U_g}{R} - I} \frac{du}{u} = \frac{t}{\tau}$$

$$\ln\frac{\frac{U_g}{R} - I}{\frac{U_g}{R}} = -\frac{t}{\tau}$$

$$\frac{U_g}{R} - I = \frac{U_g}{R} e^{-t/\tau}$$

$$I = \frac{U_g}{R}\left(1 - e^{-t/\tau}\right)$$

$$I + \tau\dot{I} = \frac{U_g}{R} \; ; \; \tau = \frac{L}{R}$$

$$\to I_H = A e^{-t/\tau} \; ; \; I_p = \frac{U_g}{R}$$

$$I = I_H + I_p \quad I(t=0) = 0$$

$$A = -\frac{U_g}{R}$$

$t = 0{,}12\,s$

$U_R = 0{,}2\,V \Rightarrow U_L = U_g - U_R = 1{,}8\,V$

ZBIRKA 9 nal 22/st 57

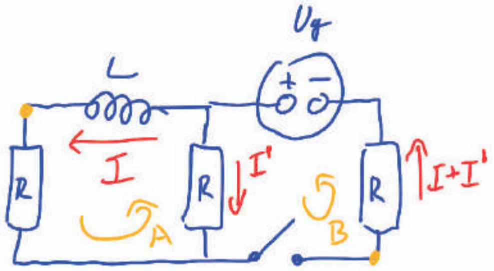
$R = 5\,\Omega$

$L = 0.1\,H$

$t = 0.1\,s$

$U_g = 10\,V$

$I(t) = ?$



$A: -IR + I'R - L\frac{dI}{dt} = 0$

$B: -(I+I')R + U_g - I'R = 0$

$\hookrightarrow \underline{I' = \frac{U_g - IR}{2R}}$

$$-IR + \frac{U_g - IR}{2R}\cdot R - L\frac{dI}{dt} = 0$$

$$-I\left(R + \frac{R}{2}\right) + \frac{U_g}{2} - L\frac{dI}{dt} = 0$$

$$I\frac{3R}{2} + L\frac{dI}{dt} = \frac{U_g}{2} \quad \Big/ \frac{2}{3R}$$

$$\underline{I + \frac{2}{3}\frac{L}{R}\frac{dI}{dt} = \frac{U_g}{3R}} \Rightarrow \underline{I + \tau \dot{I} = \frac{U_g}{3R};\ \tau = \frac{2}{3}\frac{L}{R}}$$

$\Downarrow$

$$I_H = Ae^{-t/\tau},\quad I_P = \frac{U_g}{3R}$$

$$I = I_H + I_P;\quad I(t=0) = 0$$

$\Downarrow$

$$\underline{I = Ae^{-t/\tau} + \frac{U_g}{3R}}$$

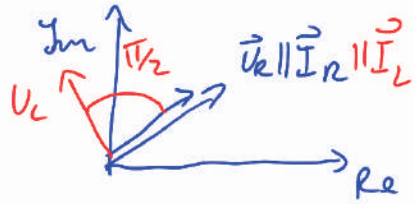
$$A + \frac{U_g}{3R} = 0$$

$$\underline{A = -\frac{U_g}{3R}}$$

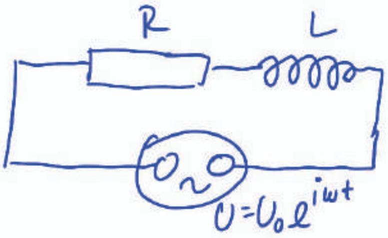
$$\underline{I = \frac{U_g}{3R}\left(1 - e^{-t/\tau}\right);\ \tau = \frac{2}{3}\frac{L}{R}}$$

$$\underline{\underline{I(0.1s) = \frac{2}{3}A}}$$

13.6 mal 4 $\quad f(\omega) = \left| \frac{U_R(\omega)}{U(\omega)} \right| = ?$



a)



$U_R = R\, I(\omega)$

$I = \frac{U}{Z_N}$

$U_L = L \frac{dI}{dt} = i\omega L\, I_L$, $\quad i = e^{i\frac{\pi}{2}}$

$U_L = Z_L \cdot I_L$

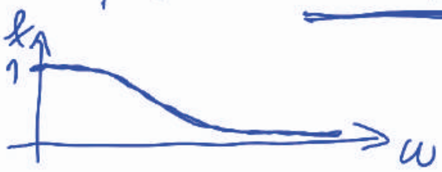
$\hookrightarrow Z_L = i\omega L$

• $Z_N = Z_R + Z_L = R + i\omega L$
$= |Z_N| \cdot e^{i\varphi}$; $|Z_N| = \sqrt{R^2 + \omega^2 L^2}$; $\operatorname{tg}\varphi = \frac{\omega L}{R}$

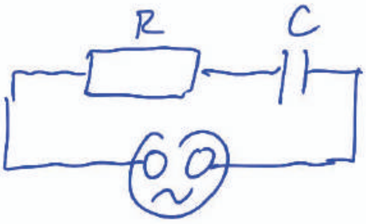
• $I = \frac{U}{Z_N} = \frac{U_0 e^{i\omega t}}{|Z_N| e^{i\varphi}} = \frac{U_0}{|Z_N|} \cdot e^{i(\omega t - \varphi)}$

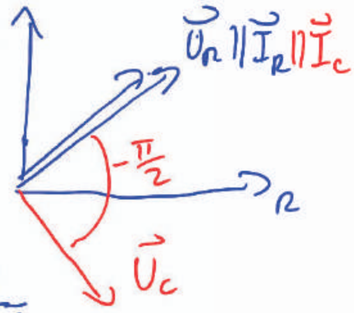
$$f(\omega) = \left| \frac{U_R(\omega)}{U(\omega)} \right| = \left| \frac{R\, I(\omega)}{U(\omega)} \right| = \left| \frac{R \cancel{U_0} e^{i(\cancel{\omega t} - \varphi)}}{|Z_N| \cancel{U_0} \cancel{e^{i\omega t}}} \right| = \frac{R}{|Z_N|} = \frac{1}{\sqrt{1 + \omega^2 \frac{L^2}{R^2}}}$$

$f(\omega) \xrightarrow{\omega \to \infty} 0$, $f(\omega) \xrightarrow{0} 1$



b)





$U_C = Z_C I_C$

$\hookrightarrow Z_C = \frac{1}{i\omega C} = \frac{-i}{\omega C}$

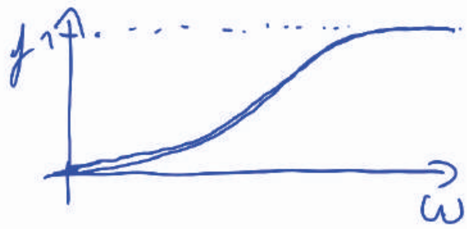
$-i = e^{-i\frac{\pi}{2}}$

$Z_N = Z_R + Z_C = R + \frac{1}{i\omega C}$
$= \underbrace{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}_{|Z_N|} e^{i\varphi}$; $\operatorname{tg}\varphi = \frac{-1}{\omega R C} \longrightarrow I = \frac{U(\omega)}{Z_N(\omega)}$

$$f(\omega) = \left| \frac{U_R(\omega)}{U(\omega)} \right| = \left| \frac{R\, I(\omega)}{U(\omega)} \right| = \left| \frac{R \cancel{U(\omega)}}{\cancel{U(\omega)} Z_N(\omega)} \right| = \frac{R}{|Z_N|} = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$f(\omega) = \frac{\omega R C}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$f(\omega \to \infty) = 1$, $f(\omega \to 0) = 0$



ZBIRKA 9 nal 25/str 57

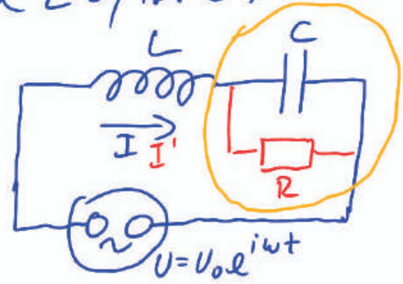
$\nu = 10^4 s^{-1}$

$L = 10^{-5} H$

$C = 50 \mu F$

$R = 1 \Omega$

$\left|\frac{I'}{I}\right| = ?$



• BREZ UPORA:

$$U = Z_N I \Rightarrow I = \frac{U}{Z_N}$$

$$Z_N = Z_L + Z_C = i\omega L + \frac{1}{i\omega C} = i\left(\omega L - \frac{1}{\omega C}\right) = \left(\omega L - \frac{1}{\omega C}\right)e^{i\frac{\pi}{2}}$$

$$I = \frac{U_0 e^{i\omega t}}{|Z_N|e^{i\varphi}} = \frac{U_0}{\left(\omega L - \frac{1}{\omega C}\right)} \cdot e^{i(\omega t - \frac{\pi}{2})}$$

• Z UPOROM: $\tilde{Z} = \left(\frac{1}{Z_R} + \frac{1}{Z_C}\right)^{-1}$

$$Z'_N = Z_L + \tilde{Z} = Z_L + \left(\frac{1}{Z_R} + \frac{1}{Z_C}\right)^{-1} = i\omega L + \left(\frac{1}{R} + i\omega C\right)^{-1} =$$

$$= \frac{\left[i\omega L\left(\frac{1}{R} + i\omega C\right) + 1\right]\left(\frac{1}{R} - i\omega C\right)}{\left[\frac{1}{R} + i\omega C\right]\left(\frac{1}{R} - i\omega C\right)} = \frac{i\omega L\left(\frac{1}{R^2} + \omega^2 C^2\right) + \frac{1}{R} - i\omega C}{\frac{1}{R^2} + \omega^2 C^2} \begin{matrix} /\cdot R^2 \\ /\cdot R^2 \end{matrix}$$

$$\frac{|A|e^{i\varphi_A}}{|B|e^{i\varphi_B}}$$

$$= \frac{R + i\left[\omega L\left(1 + \omega^2 R^2 C^2\right) - \omega R^2 C\right]}{1 + \omega^2 R^2 C^2}$$

$$= \underbrace{\frac{R\sqrt{1 + \left[\omega \frac{L}{R}\left(1 + \omega^2 R^2 C^2\right) - \omega R C\right]^2}}{1 + \omega^2 R^2 C^2}}_{|Z'_N|} \cdot e^{i\varphi'}$$

$$\operatorname{tg} \varphi' = \left[\omega \frac{L}{R}\left(1 + \omega^2 R^2 C^2\right) - \omega R C\right]$$

$$I' = \frac{U}{Z'_N} = \frac{U_0 e^{i\omega t}}{|Z'_N| e^{i\varphi'}}$$

$$\left|\frac{I'}{I}\right| = \left|\frac{\cancel{U_0 e^{i\omega t}}}{|Z'_N| e^{i\varphi'}} \cdot \frac{|Z_N| e^{i\varphi}}{\cancel{U_0 e^{i\omega t}}}\right| = \frac{|Z_N|}{|Z'_N|} = \frac{\left(\omega L - \frac{1}{\omega C}\right)\left(1 + \omega^2 R^2 C^2\right)}{R\sqrt{1 + \left[\omega \frac{L}{R}\left(1 + \omega^2 R^2 C^2\right) - \omega R C\right]^2}}$$

$$= 0{,}88$$

ZBIRKA 9 nal 25/str 57

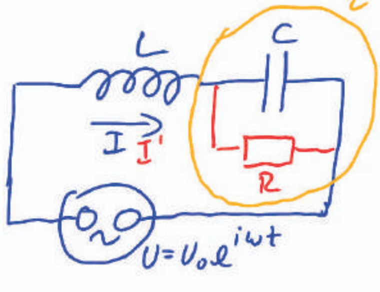
$\nu = 10^4 s^{-1}$

$L = 10^{-5} H$

$C = 50 \mu F$

$R = 1 \Omega$

$\left|\frac{I'}{I}\right| = 0{,}88$



$\tilde{Z}$ $\quad Z'_N = Z_L + \tilde{Z}$

$$= \frac{R + i\left[\omega L(1+\omega^2R^2C^2) - \omega R^2 C\right]}{1+\omega^2R^2C^2}$$

$$= \underbrace{\frac{R\sqrt{1+\left[\omega \frac{L}{R}(1+\omega^2R^2C^2) - \omega RC\right]^2}}{1+\omega^2R^2C^2}}_{|Z_N'|} \cdot e^{i\varphi'}$$

$$\operatorname{tg}\varphi' = \left[\omega\frac{L}{R}(1+\omega^2R^2C^2) - \omega RC\right]$$

13.6 nal 6

$U_0 = 100 V$

$\overline{P} = ?$

$$\overline{P} = \frac{1}{2} U_0 \cdot I_0' \cdot \cos\varphi'$$

$$= U_{ef} \cdot I_{ef} \cos\varphi'$$

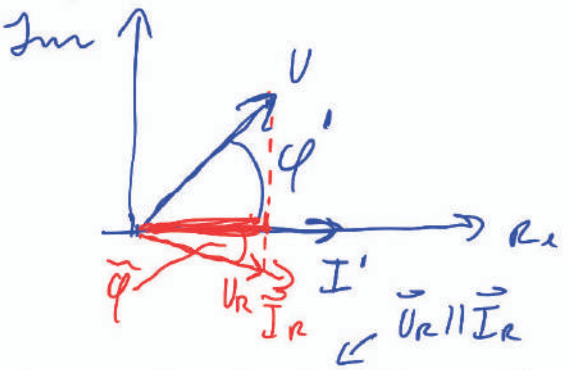
$$\underbrace{\overline{P} = \vec{U}_{ef} \cdot \vec{I}_{ef}}_{\text{V KOMPLEKSNI RAVNINI}}$$

$$p = U \cdot I' = U_0 \cos\omega t \cdot I_0' \cos(\omega t - \varphi')$$

$$= U_0 I_0' \left[\underbrace{\cos^2(\omega t)}_{1/2}\cos\varphi' + \underbrace{\tfrac{1}{2}\sin 2\omega t}_{0}\sin\varphi'\right]$$

POVPREČJE

$$\overline{P} = \frac{1}{2}\frac{U_0^2}{|Z_N'|}\cdot\cos\varphi'$$



$\vec{U}_R \parallel \vec{I}_R$

$$U = Z_N' \cdot I'$$

$$I' = \frac{U_0}{|Z_N'|}$$

$$\cos\varphi' = \frac{Re(Z_N')}{|Z_N'|}$$

$$\overline{P} = \frac{1}{2}|U_R|\cdot|I_R| = \frac{1}{2}|I_R|^2 \cdot R = \frac{1}{2}|I'|^2\left(\frac{\sqrt{1+\omega^2R^2C^2}}{1+\omega^2R^2C^2}\right)^2 \cdot R = \frac{1}{2}\frac{U_0^2}{|Z_N'|^2}\frac{R}{1+\omega^2R^2C^2}$$

$U_R = U_C \qquad I' = I_R + I_C$

$I_R R = I_C \cdot Z_C \qquad \hookrightarrow I_C = I' - I_R$

$I_R R = (I' - I_R) Z_C$

$$I_R = \frac{I' Z_C}{R + Z_C} = I'\frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} = I'\frac{1}{1+i\omega RC} = I'\frac{1 - i\omega RC}{1+\omega^2R^2C^2}$$

$\tilde{\varphi}$

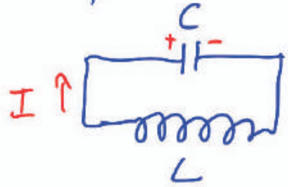
$$\cos\varphi' = \frac{R}{|Z_N|(1+\omega^2R^2C^2)} = \frac{R\cdot\cancel{(1+\omega^2R^2C^2)}}{\cancel{(1+\omega^2R^2C^2)}\sqrt{R^2+[\quad]^2}} = \frac{R}{\sqrt{R^2+[\quad]^2}} = \frac{Re(Z_N)}{|Z_N|}$$

ZBIRKA 9 nal 5/str 58

$C = 1\mu F$

$L = 10^{-3} H$

$t\left(W_L = \frac{W_{c0}}{2}\right) = ?$



$W_C = \frac{CU_c^2}{2}$; $W_L = \frac{LI^2}{2}$; $I = \frac{de}{dt}$

$U_L = -L\frac{dI}{dt} = -L\frac{d^2e}{dt^2} = \underline{-LC\frac{d^2U_c}{dt^2}}$

$e = CU_c$

$U_c = U_L$

$U_c = -LC\ddot{U}_c$

$U_c + LC\ddot{U}_c = 0$

$\ddot{U}_c + \underbrace{\frac{1}{LC}}_{=\omega^2} U_c = 0 \leftarrow$ NIHAJNA ENAČBA

$\boxed{U_c = U_{c0} \cdot \cos\omega t \qquad \omega = \frac{1}{\sqrt{LC}}} \leftarrow$ KER $U_c(t=0) = U_{c0}$

$W_L = \frac{W_{c0}}{2}$

$\underline{W_C = \frac{CU_c^2}{2} = \frac{CU_{c0}^2}{2}\cos^2\omega t}$

$W_L = \frac{LI^2}{2} = \frac{L(\dot{e})^2}{2} = \frac{LC^2(\dot{U}_c)^2}{2} = \frac{LC^2\omega^2U_{c0}^2}{2}\cdot\sin^2\omega t$

$\underline{\underline{W_L = \frac{CU_{c0}^2}{2}\cdot\sin^2\omega t}}$

$\underline{W_c + W_L = \frac{CU_{c0}^2}{2}\cdot 1}$

$\Rightarrow \sin^2\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{4} + n\pi \Rightarrow \underline{t = \frac{\pi}{4\omega} + \frac{n\pi}{\omega}}$

ZBIRKA 9 nal 4/st 58

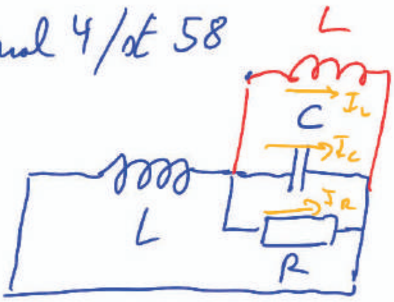
$L = 0{,}001\,H$

$C = 0.5\,\mu F$

$R = 2000\,\Omega$

$t_0 = ?$

$\beta = ?$



$U_L = U_C = U_R \quad ; \quad U_L = Z_L \cdot I_L = i\omega L I_L$

$i\omega L I_L = \frac{I_C}{i\omega C} = R I_R \quad ; \quad I_R + I_L + I_C = 0 \rightarrow I_C = -I_R - I_L$

$-\frac{I_R + I_L}{i\omega C} = R I_R \quad / \cdot i\omega C$

$I_R = \frac{-I_L}{1 + i\omega RC}$

$i\omega L \cancel{I_L} = R \cancel{I_L} \frac{-1}{1 + i\omega RC}$

$i\omega L (1 + i\omega RC) = -R$

$i\omega L - \omega^2 LRC + R = 0 \quad / \cdot \left(-\frac{1}{R}\right)$

$\omega^2 LC - i\omega \frac{L}{R} - 1 = 0 \Rightarrow \omega = \frac{i\frac{L}{R} \pm \sqrt{-\frac{L^2}{R^2} + 4LC}}{2LC}$

$\omega = i\underbrace{\frac{1}{2RC}}_{\beta} \pm \sqrt{-\underbrace{\frac{1}{4R^2C^2}}_{\beta^2} + \underbrace{\frac{1}{LC}}_{\omega_0^2}} = i\beta \pm \omega'$

$\omega' = \sqrt{\omega_0^2 - \beta^2}$

$U = U_0 e^{i\omega t} = \underbrace{U_0 e^{-\beta t}}_{\text{DUŠENJE}} \underbrace{\left[A e^{i\omega' t} + B e^{-i\omega' t}\right]}_{\text{NIHANJE}}$

$t_0 = \frac{2\pi}{\omega'}$

$\beta = \frac{1}{2RC}$

$\beta < \omega_0$

$\beta > \omega_0$

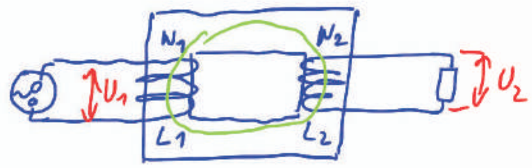
13.7 Transformator

ZBIRKA 9 nal 29/57

$\mu = 500$
$S = 10\ \text{cm}^2$
$l = 40\ \text{cm}$
$N_1 = 1000$
$N_2 = 400$
$R = 500\,\Omega$
$\nu = 50\ \text{Hz}$

$\varphi(\text{MED } I_1 \text{ IN } U_1) = ?$

$\frac{I_1}{I_{1\parallel}} = ?$



$$\frac{\phi_{m_1}}{\phi_{m_2}} = \frac{N_1}{N_2} = \frac{U_1}{U_2}$$

$$\phi_{m_{1,2}} = N_{1,2} B S = \mu\mu_0 H \cdot S \cdot N_{1,2}$$

JEDRO ZADRŽI SILNICE:

$$\oint \vec{H}\cdot d\vec{s} = N_1 I_1 + N_2 I_2$$

$$H\cdot l = N_1 I_1 + N_2 I_2$$

$$L_{1,2} = \frac{\mu\mu_0 N_{1,2}^2 S}{l}; \quad L_{12} = \frac{\mu\mu_0 N_1 N_2 S}{l}$$

$$U_{i1} = -\frac{d\phi_{m_1}}{dt} = -\frac{\mu\mu_0 S N_1}{l}\left(N_1 \dot{I}_1 + N_2 \dot{I}_2\right) = -L_1\dot{I}_1 - L_{12}\dot{I}_2$$

$$U_{i2} = -\dot{\phi}_{m_2} = -L_2\dot{I}_2 - L_{12}\dot{I}_1$$

• KOMPLEKSNO:

$$\dot{I} = i\omega I$$
$$(I = I_0 e^{i\omega t})$$

1.) $U_g + U_{i1} = 0 \Rightarrow U_g = L_1\dot{I}_1 + L_{12}\dot{I}_2$

2.) $-I_2 R + U_{i2} = 0 \Rightarrow -I_2 R = L_2\dot{I}_2 + L_{12}\dot{I}_1$

$$U_g = i\omega L_1 I_1 + i\omega L_{12} I_2$$
$$-I_2 R = i\omega L_2 I_2 + i\omega L_{12} I_1 \Rightarrow I_2 = -\frac{i\omega L_{12} I_1}{R + i\omega L_2}$$

$$U_g = i\omega L_1 I_1 + \frac{\omega^2 L_{12}^2 I_1}{R + i\omega L_2}$$

$$I_1 = \frac{U_g}{i\omega L_1 + \frac{\omega^2 L_{12}^2}{R + i\omega L_2}} = \frac{U_g}{Z}$$

$$L_{12}^2 = L_1 \cdot L_2$$

• $$Z = i\omega L_1 + \frac{\omega^2 L_{12}^2}{R + i\omega L_2} = \frac{i\omega L_1(R^2 + \omega^2 L_2^2) + \omega^2 L_{12}^2 (R - i\omega L_2)}{R^2 + \omega^2 L_2^2}$$

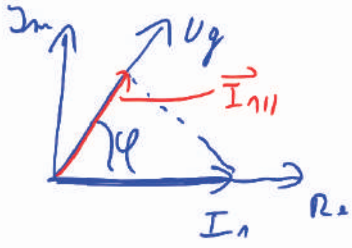
$$= \frac{i\omega L_1 R^2 + \cancel{i\omega^3 L_1 L_2^2} + \omega^2 L_{12}^2 R - \cancel{i\omega^3 L_{12}^2 L_2}}{R^2 + \omega^2 L_2^2}$$

$$Z = R\omega\,\frac{\omega L_{12}^2 + i L_1 R}{R^2 + \omega^2 L_2^2} = \underbrace{\frac{R\omega\sqrt{\omega^2 L_{12}^4 + L_1^2 R^2}}{R^2 + \omega^2 L_2^2}}_{|Z|} \cdot e^{i\varphi}$$

$$\operatorname{tg}\varphi = \frac{L_1 R}{\omega L_{12}^2} = \frac{R}{\omega L_2}$$

$U_g = Z \cdot I_1 \Rightarrow \varphi = 79^\circ$

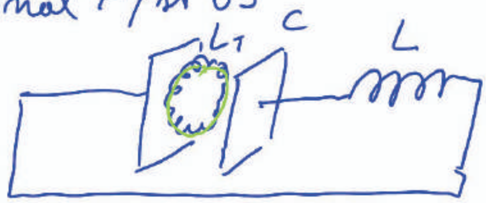
$I_{1\parallel} = I_1 \cdot \cos\varphi \Rightarrow \frac{I_1}{I_{1\parallel}} = \frac{1}{\cos\varphi} = 2{,}7$



13.8 Premikalni tok

ZBIRKA 9

nal 7 / st 59



C: $S_c = 100\,cm^2$

$a = 4\,cm$

L: $N_L = 100$

$l_L = 9\,cm$

$2r_L = 9\,cm$

$I_0 = 1\,mA$

T: $N_T = 500$

$S_T = 2\,cm^2$

$2r_T = 8\,cm$

$|U_T| = ?$

$$C = \frac{\varepsilon_0 \cdot S_c}{a} = 2{,}2 \cdot 10^{-12}\,F$$

$$L = \frac{\mu_0 N_L^2 \pi r_L^2}{l_L} = 8{,}9 \cdot 10^{-4}\,H$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2{,}26 \cdot 10^{7}\,s^{-1}$$

$$\oint \vec{H} \cdot d\vec{s} = \underbrace{\tilde{I}}_{0} + \frac{d\phi_e}{dt}$$

$$H 2\cancel{\pi} r_T = \frac{\cancel{\pi} r_T^{\cancel{2}}}{S_c} I$$

$$H = \frac{r_T I}{2 S_c}$$

MAG. V TOROIDU

$$\phi_e = \vec{D} \cdot \vec{S} = \varepsilon_0 S E$$

$$\dot{\phi}_e = \varepsilon_0 S \dot{E} = \frac{S}{S_c} \cdot I$$

$$E = \frac{e}{S_c \varepsilon_0}$$

V KONDENZATORJU

$$U_T = -\frac{d\phi_{m_T}}{dt} =$$

$$= -\frac{N_T S_T \mu_0 r_T}{2 S_c} \cdot \dot{I}$$

$$\phi_{m_T} = N_T S_T B$$

$$I = I_0 \cos \omega t$$

l_L: $2\pi r_T N_T$

$2\pi r_T \cdot N_T >> l_L$

$$U_T = +\frac{\mu_0 N_T S_T r_T \omega I_0}{2 S_c} \sin \omega t$$

$$|U_T| = 11{,}4\,mV$$

PONEDELJEN 11:05 → 13:10

14 Elektromagnetno valovanje

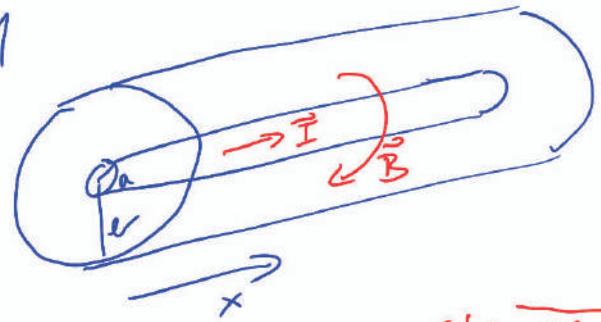
14.1 Koaksialni vodnik

(14.1) nal 1

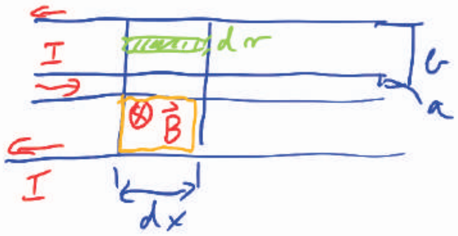
a, b, ε

$\tilde{C} = \frac{dC}{dx} = ?$

$\tilde{L} = \frac{dL}{dx} = ?$



UD STRANI:



- $\tilde{C} = \frac{dC}{dx} = \frac{2\pi\varepsilon\varepsilon_0 \cancel{dx}}{\ln\frac{b}{a} \cdot \cancel{dx}}$ $= dC$ ZA VALJASTI KONDENZATOR DOLŽINE dx

$$\boxed{\tilde{C} = \frac{2\pi\varepsilon\varepsilon_0}{\ln\frac{b}{a}}}$$

- $\tilde{L} = \frac{dL}{dx} = \frac{\cancel{dx}\,\mu_0 \ln\frac{b}{a}}{\cancel{dx}\; 2\pi}$

$$\boxed{\tilde{L} = \frac{\mu_0}{2\pi}\ln\frac{b}{a}}$$

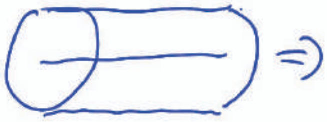
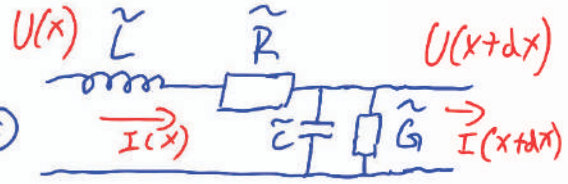
$\Phi_m = L \cdot I \Rightarrow L = \frac{\Phi_m}{I}$

$\Phi_m = \int \vec{B}\, d\vec{S} = dx \int_a^b \frac{\mu_0 I\, dr}{2\pi r}$

- $dS = dr \cdot dx$
- $B = \frac{\mu_0 I}{2\pi r}$ ← ZA ŽICO

$\Phi_m = dx\, I\, \frac{\mu_0 \ln\frac{b}{a}}{2\pi}$

(14.1) mal 2,3

 $\Rightarrow$ 

$\tilde{G} = \frac{dG}{dx} \leftarrow$ PREVODNOST DIELEKTRIKA $\left[\frac{1}{\Omega}\right]$

$$dU = U(x+dx) - U(x) = -L\dot{I} - RI \quad / \cdot \frac{1}{dx}$$

$$dI = I(x+dx) - I(x) = -C\dot{U} - GU \quad / \cdot \frac{1}{dx}$$

$e = CU$

$\frac{de}{dt} = I = C\dot{U}$

$$\frac{dU}{dx} = -\tilde{L}\dot{I} - \tilde{R}I$$

$$\boxed{\begin{matrix}\tilde{R} \to 0 \\ \tilde{G} \to 0\end{matrix}} \to \quad \frac{dI}{dx} = -\tilde{C}\dot{U} - \tilde{G}U$$

$$\frac{d^2U}{dx^2} = -\tilde{L}\frac{d^2 I}{dx\,dt} = +\tilde{L}\tilde{C}\frac{d^2U}{dt^2} \Rightarrow \boxed{\frac{d^2U}{dx^2} = \tilde{L}\tilde{C}\frac{d^2U}{dt^2}}$$

$$\frac{d^2 I}{dx\,dt} = -\tilde{C}\ddot{U}$$

• VALOVNA ENAČBA:

$$\tilde{L}\tilde{C} = \frac{1}{c^2} \Rightarrow c = \sqrt{\frac{1}{\tilde{L}\tilde{C}}}$$

• REŠITVE:

$$\boxed{\begin{matrix} U(x) = U_{\to 0}\, e^{i(\omega t - kx)} + U_{\leftarrow 0}\, e^{i(\omega t + kx)} \\ k = \frac{\omega}{c}\ ; \ c = \sqrt{\frac{1}{\mu\mu_0 \varepsilon\varepsilon_0}} \end{matrix}}$$

$\leftarrow$ HITROST

$\hookrightarrow$ POVADI $\mu \sim 1$

• IMPEDANCA:

$$\frac{dU}{dx} = -\tilde{L}\cdot\dot{I}$$

$$\cancel{-i}kU = \cancel{-}\tilde{L}\cancel{i}\omega I$$

$$\frac{U}{I} = \frac{\tilde{L}\omega}{k} = \boxed{Z = \frac{\tilde{L}\omega c}{\omega} = \frac{\tilde{L}}{\sqrt{\tilde{L}\tilde{C}}} = \sqrt{\frac{L}{C}}}$$

SPLOŠNO:

$\tilde{R} \neq 0,\ \tilde{G} \neq 0$

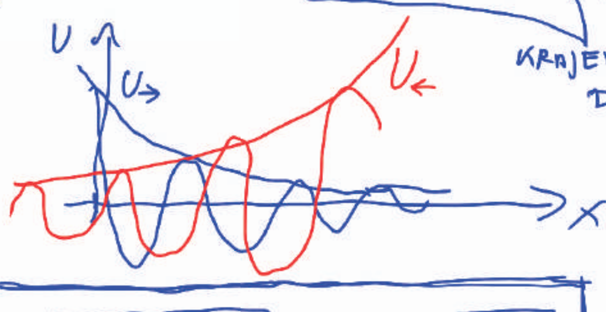
$\hookrightarrow$ DUŠENJE:

$$U(x) = \left(U_{\to 0}\, e^{-ik_{Re}x}\, \underline{e^{-k_{Im}x}} + U_{\leftarrow 0}\, e^{+ik_{Re}x}\, \underline{e^{+k_{Im}x}}\right) e^{i\omega t}\, \underline{e^{-\beta t}}$$

$\omega = Re(\omega) + i\,Im(\omega)$

$k = Re(k) + i\,Im(k)$

KRAJEVNO ČASOVNO DUŠENJE



IMPEDANCA:

$$\boxed{Z = \sqrt{\frac{R + i\omega L}{G + i\omega C}} \xrightarrow[G \to 0]{R \to 0} \sqrt{\frac{L}{C}}}$$

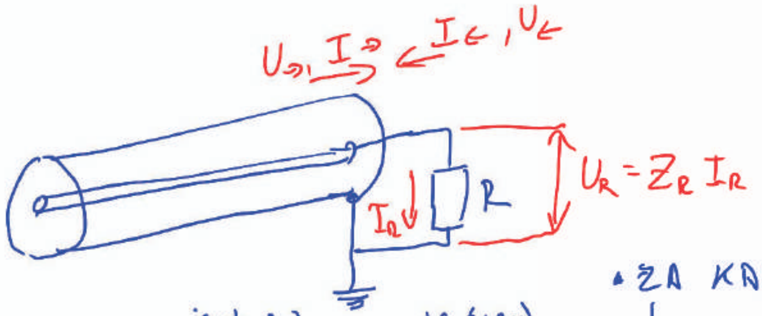
(14.1) vol 5

a, b, ε

$R, U_{\rightarrow 0}, \omega$

$U_{\leftarrow 0} = ?$

a) $R = 0$
b) $R = \infty$
c) $R(U_{\leftarrow 0} = 0) = ?$
d) SPLOŠEN R



$$U = U_{\rightarrow 0} e^{i(\omega t - kx)} + U_{\leftarrow 0} e^{i(\omega t + kx)}$$
$$I = I_{\rightarrow 0} e^{i(\omega t - kx)} - I_{\leftarrow 0} e^{i(\omega t + kx)}$$

• ZA KABEL:

$$\boxed{\frac{U_{\rightarrow}}{I_{\rightarrow}} = Z_K = \frac{U_{\leftarrow}}{I_{\leftarrow}}}$$

$$I_{\rightarrow} = \frac{U_{\rightarrow}}{Z_K}, \quad I_{\leftarrow} = \frac{U_{\leftarrow}}{Z_K}$$

• NA ZAKLJUČKU:

$$\boxed{\begin{aligned} I_R &= I_{\rightarrow} - I_{\leftarrow} \\ U_R &= U_{\rightarrow} + U_{\leftarrow} \\ U_R &= Z_R \cdot I_R \end{aligned}}$$

$$\hookrightarrow I_R = \frac{U_{\rightarrow} - U_{\leftarrow}}{Z_K}$$

$$U_R = \frac{Z_R}{Z_K}(U_{\rightarrow} - U_{\leftarrow})$$

$$U_{\rightarrow} + U_{\leftarrow} = \frac{Z_R}{Z_K}(U_{\rightarrow} - U_{\leftarrow})$$

$$\boxed{U_{\leftarrow} = \frac{Z_R - Z_K}{Z_R + Z_K} \cdot U_{\rightarrow}} \quad ; \quad \Gamma = \frac{Z_R - Z_K}{Z_R + Z_K}$$

a) $Z_R = 0$: $\Gamma = -1 \Rightarrow U_{\leftarrow} = -U_{\rightarrow} \Rightarrow$ FAZNI ZAMIK $\varphi = 180°$
$\hookrightarrow$ FAZA SE OBRNE

b) $Z_R = \infty$: $\Gamma = 1 \Rightarrow U_{\leftarrow} = U_{\rightarrow} \Rightarrow \varphi = 0°$

c) $U_{\leftarrow 0} = 0$: $Z_R = Z_K \rightarrow$ NIČ SE NE ODBIJE

d) SPLOŠEN Z_R: $Z_R = \mathrm{Re}(Z_R) + i\,\mathrm{Im}(Z_R)$

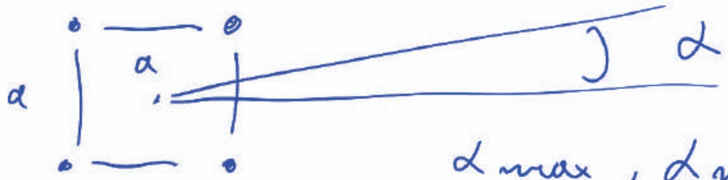
$\rightarrow$ ZA IDEALEN KABEL $Z_K = \sqrt{\frac{L}{C}}$

POGLEJ KAJ TI DA $\rightarrow$ DN

$\Gamma = |\Gamma| e^{i\varphi}$ $\leftarrow$ SPLOŠNO

14.2 Interferenca

DN ZBIRKA 9 nal 21/str 60



α_{max}, α_{min}

ZBIRKA 9 nal 20/str 60

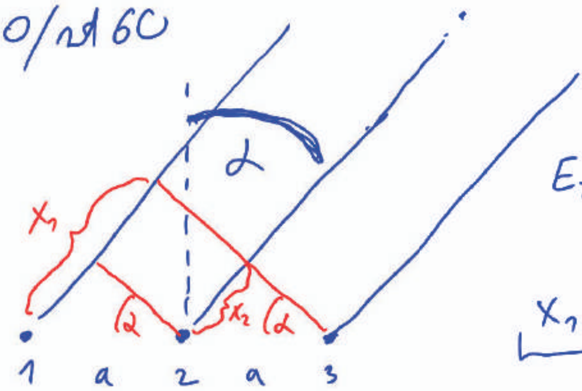
$a = 5\,m$

$\nu = 1{,}5 \cdot 10^8\,s^{-1}$

$\varphi_1 = \frac{2\pi}{8} = \frac{\pi}{4}$

$\varphi_2 = \frac{2\pi}{16} = \frac{\pi}{8}$

$\alpha_{max} = ?$



$E_2 = E_{20}\, e^{i(\omega t - k r_2 + \varphi_2)}$

$\underline{x_1 = 2 x_2}$

RAZDALJE: $x_1 = 2a \sin\alpha$

$\underline{x_2 = a \sin\alpha}$

FAZNI ZAMIKI:

$$x_1 = \left(\frac{\varphi_1}{2\pi} + N_1\right)\lambda = \left(\frac{1}{8} + N_1\right)\lambda$$

$$x_2 = \left(\frac{\varphi_2}{2\pi} + N_2\right)\lambda = \left(\frac{1}{16} + N_2\right)\lambda$$

$$\Rightarrow \frac{x_1}{x_2} = 2 = \frac{\left(\frac{1}{8} + N_1\right)\cancel{\lambda}}{\left(\frac{1}{16} + N_2\right)\cancel{\lambda}}$$

$$\cancel{\frac{2}{16}} + 2N_2 = \cancel{\frac{1}{8}} + N_1$$

$$\underline{N_1 = 2N_2}$$

$$a \sin\alpha = \left(\frac{1}{16} + N_2\right)\lambda$$

$$\boxed{\sin\alpha = \underbrace{\frac{\lambda}{a}\left(\frac{1}{16} + N_2\right)}_{\leq 1}} \Leftarrow \alpha_{max}$$

$\Rightarrow -2 \leq N_2 \leq 2$

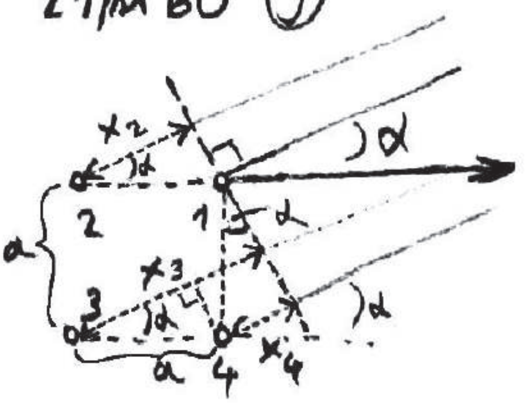
N_2	α_{max}
0	1,4°
1	25°
2	56°

14.2. {1} 21/str 60 (g)

$a = 10\,m$

$\nu_{max} = ?$

$\alpha_{max} = ?$



← DALEČ SO VZPOREDNI ŽARKI

$x_2 = a \cdot \cos\alpha$

$x_4 = a \cdot \sin\alpha$

$x_3 = x_2 + x_4 = a(\cos\alpha + \sin\alpha)$

$\Rightarrow$ MAKSIMUM KO $x_2 = N_2 \cdot \lambda$, $x_4 = N_4 \cdot \lambda$; $N_2, N_4 \in \mathbb{Z}$

$$\frac{x_4}{x_2} = \frac{a \sin\alpha}{a \cos\alpha} = \operatorname{tg}\alpha = \frac{N_4}{N_2} \Rightarrow$$

N_2	N_4	α	λ	ν
0	1	90°	a	$3 \cdot 10^7 s^{-1}$
1	0	0	a	
1	1	45°	$\frac{a}{\sqrt{2}}$	$\sqrt{2} \cdot 3 \cdot 10^7 s^{-1}$
1	2	26,6°	$\frac{a}{\sqrt{5}}$	$\sqrt{5} \cdot 3 \cdot 10^7 s^{-1}$
2	1	63,4°	$\frac{a}{\sqrt{5}}$	

$x_4 = a \cdot \sin\alpha = N_4 \lambda$

$x_2 = a \cdot \cos\alpha = N_2 \lambda$

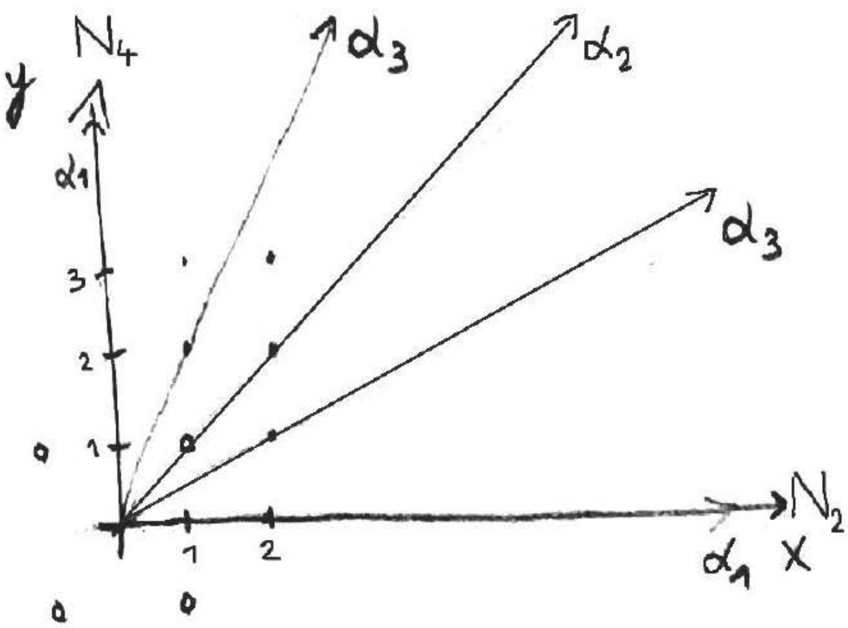
$$\Rightarrow x_2^2 + x_4^2 = a^2 \overbrace{(\cos^2\alpha + \sin^2\alpha)}^{1}$$

$$(N_2^2 + N_4^2)\lambda^2 = a^2$$

$$\boxed{\lambda = \frac{a}{\sqrt{N_2^2 + N_4^2}}} \Rightarrow$$

$\nu = \frac{c}{\lambda} \Rightarrow$

$c = 3 \cdot 10^8 m/s$



ZBIRKA 9 nal 14/str 63

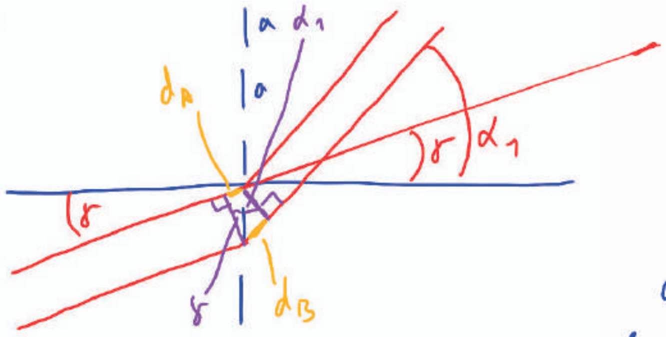
$\lambda = 550\,nm$

$\alpha_1 = 26°$

$\alpha_{-1} = -6°$

$\gamma = ?$

$\frac{1}{a} = ?$



$d_A = a \cdot \sin\gamma$

$d_{B_1} = a \sin\alpha_1$

$d_{B_1} - d_A = 1 \cdot \lambda$

$d_{B_N} - d_A = N\lambda$

$\underbrace{a(\sin\alpha_N - \sin\gamma) = N\lambda}$

$N=1: \quad a(\sin\alpha_1 - \sin\gamma) = \lambda$

$N=-1: \quad a(\sin\alpha_{-1} - \sin\gamma) = -\lambda$

$\Rightarrow \frac{\sin\alpha_1 - \sin\gamma}{\sin\alpha_{-1} - \sin\gamma} = -1$

$\underbrace{\sin\gamma = \frac{\sin\alpha_1 + \sin\alpha_{-1}}{2}}$

$= \underline{9{,}6°}$

ODŠTEJEMO:

$a(\sin\alpha_1 - \sin\alpha_{-1}) = 2\lambda$

$\underbrace{\frac{1}{a} = \frac{\sin\alpha_1 - \sin\alpha_{-1}}{2\lambda}}$

$\frac{1}{a} = \frac{1}{494}\,nm^{-1}$

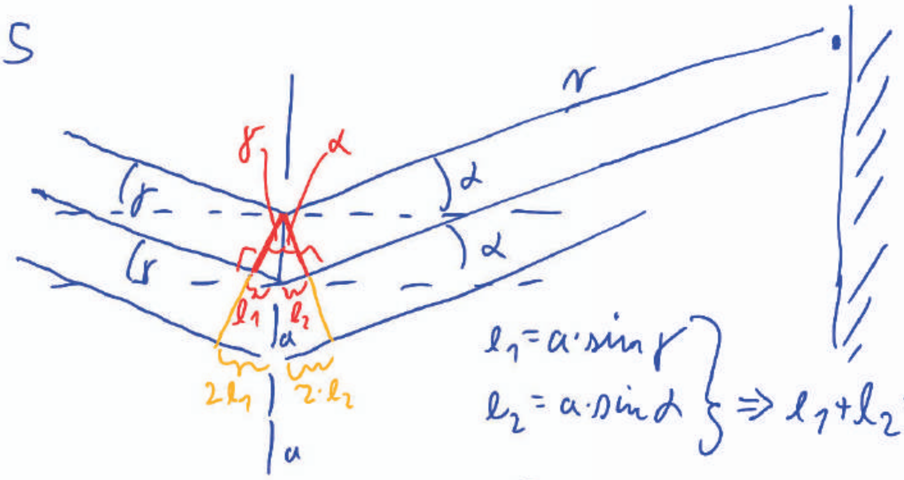
14.2 nal 5

$\lambda = 550\ nm$

$\sin\gamma = 1/4$

$a = 2\lambda$

$j(\sin\alpha) = ?$



$$\left.\begin{aligned} l_1 &= a\cdot\sin\gamma \\ l_2 &= a\cdot\sin\alpha \end{aligned}\right\} \Rightarrow l_1 + l_2 = a(\sin\gamma + \sin\alpha)$$

POLJE NA ZASLONU ZA N REŽ:

$$E = E_1 + E_2 + \dots + E_N = E_0 e^{i(\omega t - kr)} + E_0 e^{i[\omega t - k(r + l_1 + l_2)]} + E_0 e^{i(\omega t - k(r + 2(l_1 + l_2)))} + \dots$$

$$= E_0 e^{i(\omega t - kr)} \underbrace{\left[1 + e^{-i\Delta\phi} + e^{-i2\Delta\phi} + \dots + e^{-i(N-1)\Delta\phi}\right]}_{\sum_{n=0}^{N-1} a_0 q^n = a_0 \frac{1-q^N}{1-q};\ a_0 = 1;\ q = e^{-i\Delta\phi}}$$

$$= \frac{1 - e^{-iN\Delta\phi}}{1 - e^{-i\Delta\phi}}$$

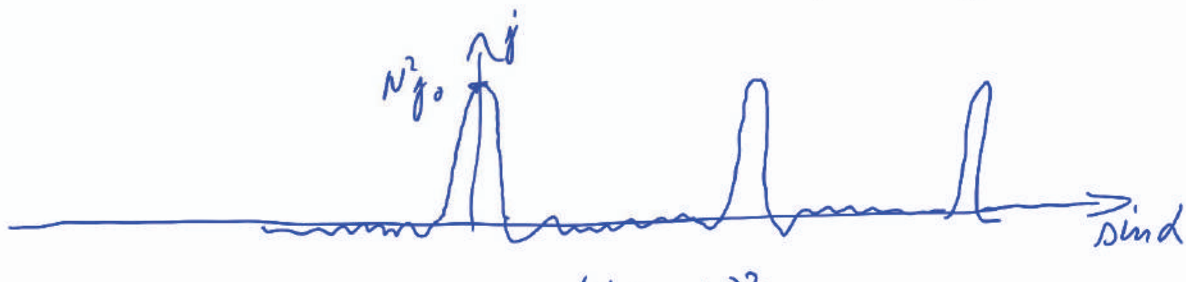
• FAZNI ZAMIK $k(l_1 + l_2) = \Delta\phi$

$$= E_0 e^{i(\omega t - kr)} e^{-i\frac{N}{2}\Delta\phi} e^{i\frac{\Delta\phi}{2}} \frac{e^{+i\frac{N}{2}\Delta\phi} - e^{-i\frac{N}{2}\Delta\phi}}{e^{i\frac{\Delta\phi}{2}} - e^{-i\frac{\Delta\phi}{2}}}$$

$$\frac{e^{ix} - e^{-ix}}{2i} = \sin x$$

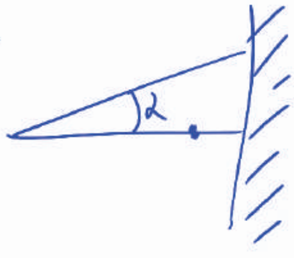
$$E = E_0 e^{i(\omega t - kr - \frac{N-1}{2}\Delta\phi)} \frac{\sin\left(\frac{N}{2}\Delta\phi\right)}{\sin\left(\frac{1}{2}\Delta\phi\right)}$$

$$j = \frac{1}{2}\varepsilon\varepsilon_0 \langle E^2 \rangle c = j_0 \frac{\sin^2\left[\frac{N}{2} ka(\sin\gamma + \sin\alpha)\right]}{\sin^2\left[\frac{1}{2} ka(\sin\gamma + \sin\alpha)\right]}$$



NA MAKSIMUMU: $j(\sin\alpha \to 0, \gamma = 0) = j_0 \frac{\left(\frac{N}{2} ka \sin\alpha\right)^2}{\left(\frac{1}{2} ka \sin\alpha\right)^2} = j_0 \cdot N^2$

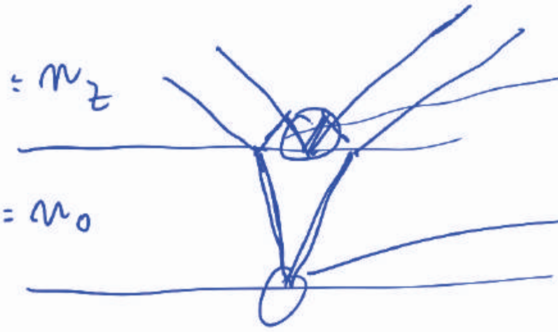
$N \to \infty \to$ δ FUNKCIJE



ZBIRKA 9 zad 2/str 62 → DW

PO ODBOJU:

$1 = n_z$



$n_z < n_0 \Rightarrow$ FAZA SE OBRNE !

$1{,}4 = n_0$

$n_0 > n_z \Rightarrow$ NE OBRNE !

$1{,}33 = n_V$